

Electron Cyclotron Heating in Toroidal Octupoles

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Microwave frequencies of 0.4–24 GHz at powers up to 100 kW have been used to heat electrons in regions of cyclotron resonance in a supported and in a levitated toroidal octupole magnetic field. A theoretical model has been developed to predict the heating rate of a cold, tenuous plasma in an arbitrary, nonuniform, magnetic field. The model predicts strong heating at places where $\nabla_{\parallel} B = 0$ at resonance, and heating efficiencies approaching 100% for sufficiently high densities ($\omega_p^2 > \omega^2/Q$). Scintillator probes have been used to verify the predicted localized heating and to measure the density at which total absorption occurs. Gun injected plasmas and microwave produced plasmas with $n \sim 10^9 \text{ cm}^{-3}$ and $kT_e \sim 5 \text{ eV}$ are heated to $\lesssim 1 \text{ keV}$ in the supported octupole and to $\lesssim 10 \text{ keV}$ in the levitated octupole. Upper off-resonance heating is also observed.

I. INTRODUCTION

Techniques of charged-particle acceleration by rf electric fields in certain magnetic field geometries such as cyclotrons are widespread and highly developed. The rf heating of magnetoplasmas is more complicated for two reasons: (1) The nonuniform fields required for plasma confinement are not suitable for simple cyclotron acceleration. (2) The particles in a plasma are more uniformly distributed in phase space.

Electron cyclotron heating has been used extensively in magnetic mirror devices, particularly at Oak Ridge,^{1,2} where it was observed that the heating is surprisingly efficient in spite of the small size of the region in which cyclotron resonance occurs. Typically, these experiments use microwave frequencies of 3–10 GHz and power densities of the order of a few W/cm³. Background pressures are typically 10^{-5} – 10^{-3} Torr, and the plasmas produced consist of a cold (few eV), dense (10^9 – 10^{12} cm^{-3}) component and a more tenuous, hot component with an energy as high as a few hundred keV. Several theories have been developed to explain the heating rate in these mirror machines.^{3–7} The theories generally approach the problem by solving the equation of motion of a single electron moving adiabatically along the axis of an idealized mirror geometry. The electron periodically passes through the resonance regions and experiences a stochastic acceleration.

Electron cyclotron heating has also been used in more complicated geometries such as stellarators, levitrons, spherators, and multipoles.⁸ Interest in these experiments has been more with plasma confinement than with the heating mechanism. Microwaves are used to produce as well as to heat the plasmas, and so high-background gas pressures are required. The experiments are generally done in the afterglow of the discharge.

In this paper a theoretical model is proposed to describe the cyclotron heating of a cold, tenuous plasma in an arbitrary, nonuniform, magnetic field. The model is completely general in the sense that, if the electron density distribution is specified, the average electron heating rate can be calculated, at

least in the nonrelativistic limit. The calculated heating rate is equivalent to that obtained by others using different methods, but the description in terms of a density distribution is especially suited for plasmas in complicated magnetic fields where the trajectories of single particles cannot be calculated. Experiments are described in which gun injected and microwave produced plasmas are heated by pulsed microwave in a supported and in a levitated toroidal octupole. The behavior of the cold ion, afterflow plasma produced by pulsed microwave heating at high-background pressure was described elsewhere.⁹

II. APPARATUS

The Wisconsin supported toroidal octupole is described elsewhere.¹⁰ A computer calculated, magnetic flux plot of the octupole is shown in Fig. 1. The light lines are magnetic field lines (ψ lines), and the heavy lines are contours of constant magnetic field strength (arbitrarily normalized to 1 at the outside wall mid-plane). The aluminum walls form a vacuum (and microwave) cavity with a volume of $3 \times 10^6 \text{ cm}^3$, evacuated to a base pressure of 5×10^{-7} Torr. The levitated toroidal octupole¹¹ has a similar flux plot, a slightly higher magnetic field (14 kG maximum), a much larger volume ($8.6 \times 10^6 \text{ cm}^3$), and a somewhat lower-base pressure (2×10^{-7} Torr). The magnetic field in both devices is a half-sine wave with a 5-msec duration for the supported and a 43-msec duration for the levitated device.

Microwave power is produced by any of twelve different sources covering the range 0.4 to 24 GHz. Continuous wave sources produce up to 1 kW, and pulsed sources produce up to 100 kW for 144 μsec . Power is coupled to the cavity through holes in the wall at high frequencies ($\gtrsim 3 \text{ GHz}$) or with quarter wave stubs or loops at low frequencies.

For a cavity large compared with the rf wavelength λ , the mode number can be approximated by¹²

$$N \simeq 8\pi V / 3\lambda^3,$$

where V is the cavity volume. The average mode spacing

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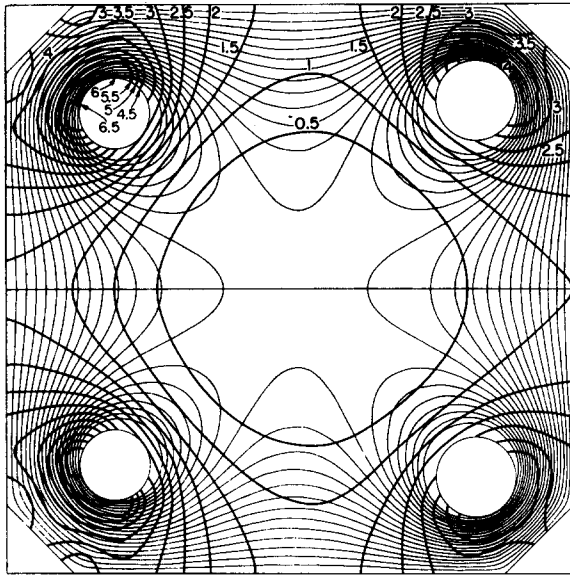


FIG. 1. Computer calculated magnetic flux plot of the octupole field. The light lines are magnetic field lines, and the heavy lines are contours of constant magnetic field strength.

at frequency f is then

$$\frac{df}{dN} = \frac{\lambda^3 f}{8\pi V}.$$

At short wavelengths the mode spacing is small, and the modes overlap for

$$\lambda^3 < 8\pi V/Q.$$

When the modes overlap, the cavity impedance is relatively insensitive to frequency, and the reflected power is small. The unloaded cavity Q measured at 9 GHz is ~ 4000 for the supported octupole and $\sim 50\,000$ for the levitated octupole. Good matching of the microwaves to the cavities is therefore expected and observed for frequencies above about 2 GHz in both devices.

The mean-square electric field in the cavity can be determined from the input power P_0 by

$$\bar{E}^2 = P_0 Q / 2\pi\epsilon_0 fV. \quad (1)$$

At sufficiently high densities, plasma loading lowers the Q , and the electric field is correspondingly reduced.

III. THEORETICAL MODEL

Consider a plasma confined in a cavity of arbitrary size and shape by a static nonuniform magnetic field $\mathbf{B}(\mathbf{r})$ in the presence of an rf electric field $\mathbf{E}_0(\mathbf{r}, t) \exp(i\omega t)$. Assume a plasma density sufficiently low so that the waves freely penetrate the plasma and a plasma pressure sufficiently low so that the magnetic field is unperturbed ($\beta = 2\mu_0 p / B^2 \ll 1$).

We define a function ψ which is constant on a flux surface and has a value equal to the magnetic flux enclosed by that surface. The magnetic field lines lie in the surfaces and may or may not be confined in the cavity. An orthogonal coordinate system will be used in which $\hat{\psi}$ is a unit vector normal to a flux surface and \hat{l} is a unit vector in the direction of \mathbf{B} . This system is convenient because plasma energy flows readily in the \hat{l} direction and because in many axisymmetric and linear geometries the coordinate orthogonal to \hat{l} and $\hat{\psi}$ is ignorable.

The power absorbed per unit volume in a resistive, dielectric medium can be expressed in terms of a real, tensor conductivity $\vec{\sigma}$ as

$$\frac{dP}{dV} = (\vec{\sigma} \cdot \mathbf{E}) \cdot \mathbf{E},$$

or in our coordinates,

$$\frac{dP}{d\psi} = \int (\sigma_{\perp} E_{\perp}^2 + \sigma_{\parallel} E_{\parallel}^2) \frac{dl}{B}. \quad (2)$$

The perpendicular and parallel conductivities of a cold, tenuous, uniform plasma can be expressed in terms of a collision frequency ν for $\nu \ll \omega$ as

$$\sigma_{\perp} = \epsilon_0 \omega_p^2 \nu (\omega^2 + \omega_c^2) [(\omega^2 - \omega_c^2)^2 + 4\omega^2 \nu^2]^{-1}$$

and

$$\sigma_{\parallel} = \epsilon_0 \omega_p^2 \nu / \omega^2,$$

where ω_p is the plasma frequency and ω_c is the cyclotron frequency. For $\nu \ll \omega$, most of the contribution to the integral in Eq. (2) comes from the resonance at $\omega = \omega_c$, and $B(l)$ can be expanded in a Taylor series about its resonance value, retaining only the first-order term,

$$B(l) = B_0 + l |\nabla_{\parallel 0} B|,$$

where $l=0$ is taken where $B = m\omega/e$, and the subscript 0 refers to the value of a quantity at $l=0$. If the electric field and density vary slowly over the width of the resonance, the integral in Eq. (2) can be evaluated for $-\infty < l < +\infty$ to get the result

$$\frac{dP}{d\psi} = \frac{\pi n_0 e E_{\perp 0}^2}{2B_0 |\nabla_{\parallel 0} B|} = \frac{1}{2} \pi n_0 e E_{\perp 0}^2 \frac{d^2 V}{dB d\psi} \Big|_{B_0}. \quad (3)$$

The corresponding heating rate is

$$\frac{d\bar{W}}{dt}(\psi) = \frac{dP}{d\psi} \left(\int \frac{n dl}{B} \right)^{-1} = \frac{\pi n_0 e E_{\perp 0}^2}{2 \int n dl / B} \frac{d^2 V}{dB d\psi} \Big|_{B_0}. \quad (4)$$

For n constant along the field line ($\nabla_{\parallel} n = 0$), the heating rate reduces to

$$\frac{d\bar{W}}{dt}(\psi) = \frac{\pi e E_{\perp 0}^2}{2V'} \frac{d^2 V}{dB d\psi} \Big|_{B_0}, \quad (5)$$

where $V' = dV/d\psi = \mathcal{F}(dl/B)$ is the volume of a unit flux tube. An equivalent result has been obtained by Kuckes¹³ by solving the equation of motion of an elec-

tron that passes through the resonance region. Equation (5) has the features of a stochastic acceleration since the heating rate is mass independent and since the energy increases linearly in time.

Note that the heating rate is independent of ν and is proportional to the fractional volume of the flux shell in which the magnetic field is within dB of the resonance value. The collision time ν^{-1} is the time during which the phase of an electron remains stationary with respect to the electric field and may result from real-particle collisions or from other mechanisms such as the phase coherence of the rf field and the nonzero parallel velocity in a nonuniform magnetic or electric field. It can be shown that the requirement $\nu \ll \omega$ places an upper limit on the parallel velocity for which Eq. (3) is valid:

$$v_{\parallel} \ll \omega B_0 (|\nabla_{\parallel 0} B|)^{-1}.$$

For typical conditions in the experiment to be described, the above requirement is not very restrictive and is well satisfied for nonrelativistic electrons. Furthermore, the assumption that the electric field is locally unperturbed by the plasma near resonance places a limit on the density by requiring that the width of the resonance zone be much less than the skin depth, or

$$\omega_p^2 \ll \omega c |\nabla B| / B_0.$$

Again, this requirement is not very restrictive for the experiments to be described, since $n \lesssim 10^{10} \text{ cm}^{-3}$. It is also assumed that the electrons are nonrelativistic ($\bar{W}/mc^2 \ll 1$), and that the density is low enough that $\omega_p^2 < \omega^2$.

The total power absorbed by the plasma can be determined by integrating Eq. (3) over all ψ , which for the special case of $n = \text{const}$, gives

$$P = \frac{1}{2} \pi n e E_{\perp 0}^2 \frac{dV}{dB} \Big|_{B_0}. \quad (6)$$

Using Eq. (1) and assuming $E_{\perp 0}^2 = \frac{2}{3} E^2$, the heating efficiency is found to be

$$\eta = \frac{P}{P_0} = \frac{1}{3} \pi \frac{\omega_p^2}{\omega^2} Q \frac{B_0}{V} \frac{dV}{dB} \Big|_{B_0}. \quad (7)$$

The quantity

$$\frac{B_0}{V} \frac{dV}{dB} \Big|_{B_0}$$

is a geometrical factor of order unity. For low densities ($\omega_p^2 \ll \omega^2/Q$) the power absorbed is proportional to density. At higher densities ($\omega_p^2 > \omega^2/Q$) plasma loading dominates the cavity Q , and the efficiency approaches 100%.

Other theoretical treatments have considered mirror fields of the form

$$B(Z) = B(0) (1 + \beta Z^2).$$

For this case, with $\nabla_{\parallel} n = 0$, Eq. (4) predicts a heating

rate of

$$\frac{d\bar{W}}{dt} = \frac{e E_{\perp 0}^2}{2 B_0} (r-1)^{-1/2},$$

where $r = B_0/B(0)$. The results of the single-particle theory of Grawe⁷ can also be approximately reproduced in the nonrelativistic limit by considering a distribution function that consists only of particles that mirror at a particular value of Z . For a distribution in which all the particles mirror exactly at resonance, it is necessary to generalize the method slightly by leaving the density inside the integral in Eq. (2), and by introducing a phenomenological resonance width as suggested by Guest.⁴ The resulting rate is given by

$$\frac{d\bar{W}}{dt} \simeq 0.21 \frac{e E_{\perp 0}^2}{B_0} \left(\frac{\omega}{\omega_B} \right)^{1/3} \left(\frac{r}{r-1} \right)^{2/3},$$

where ω_B is the bounce frequency. Grawe's⁷ result is identical except that his coefficient is 0.79. Other forms of $B(Z)$ and $n(Z)$ can be substituted into Eq. (4) to yield heating rates for a variety of conditions. For example, a magnetic field sinusoidal in l with mirror ratio R and a loss-cone Maxwellian distribution gives a heating rate of

$$\frac{d\bar{W}}{dt} = \frac{e E_{\perp 0}^2}{2 B_0 (R^{1/2} - 1)} \left(\frac{R-r}{r-1} \right)^{1/2}.$$

The formalism developed here is also useful for explaining off-resonance heating.¹⁴⁻¹⁶ In this case, the collision frequency ν appears in the result, and it is necessary to determine the dominant mechanism which destroys the phase coherence. In a multi-mode cavity, an electron moving along a magnetic field line might experience a stochastic electric field with a correlation length of about a wavelength, so that $\nu \simeq v_{\parallel} \omega / c$. Below resonance $\sigma_{\parallel} > \sigma_{\perp}$ and so parallel electric fields should become important and lead to parallel heating.

IV. SUPPORTED OCTUPOLE EXPERIMENTS

Langmuir probes and microwave cavity perturbation techniques have been used to show that the density during microwave heating rises exponentially with a growth time that agrees with the calculated ionization time, until a saturation is reached at an average density that is typically an order of magnitude below that for which $\omega_p = \omega$. The experiments to be described here were done at a sufficiently low pressure ($\sim 10^{-6}$ Torr) so that the density does not change appreciably during the time of the microwave pulse ($\sim 100 \mu\text{sec}$). Hydrogen plasmas with $n \sim 10^9 \text{ cm}^{-3}$, $kT_e \sim 5 \text{ eV}$, and $kT_i \sim 30 \text{ eV}$ are produced by gun injection. The microwave pulse is applied during the quiescent decay of the trapped plasma.

The main diagnostic for studying the energetic electron component produced by microwave heating was a two-channel scintillator probe.¹⁷ The probe con-

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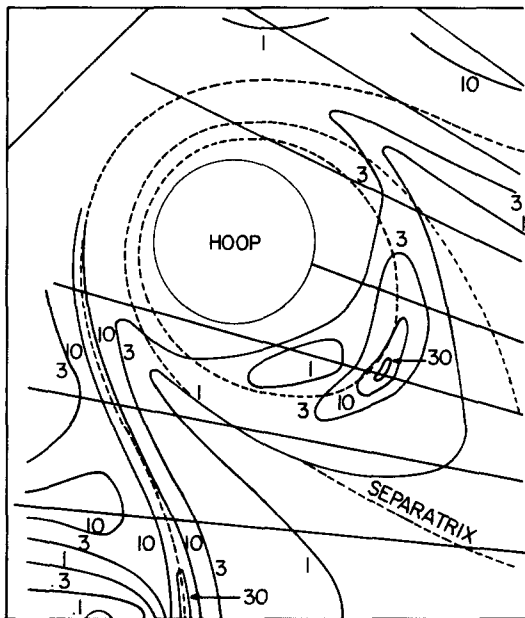


FIG. 2. Spatial contours of constant scintillator-probe output in the upper inside quadrant of the octupole field for resonance at $B_0 \approx 1.1$.

sists of two small plastic scintillators wrapped in lead foil and sealed into the end of a 6-mm-diam aluminum tube which can be inserted into the plasma. The faces of the scintillators are covered with aluminum foils with thicknesses of 0.017 and 0.05 mm. Flexible light guides carry the optical signals to photomultiplier tubes outside the vacuum. The probe was calibrated with an electron beam of variable energy and known intensity. The ratio of the signals from the two channels gives a measure of the average electron energy if the form of the distribution is known. The output signal from either channel is proportional to density and to about the fourth power of the average energy for most reasonable distributions.

Figure 2 shows contours of constant scintillator probe signal during the heating pulse in the upper inside quadrant of the octupole field for resonance at $B_0 \approx 1.1$ in Fig. 1. The diagonal lines show where the probe scans were made. The heating is localized in the resonance regions, as expected, but the energetic electrons flow along the field into regions where $B > B_0$. The small signals on the midplane near the inner wall result from the fact that the probe traverses the resonance and obstructs the heating. The signal from a probe in the resonance region drops two orders of magnitude if a 6-mm-diam probe is moved across the resonance region at another azimuth 1 m away, in-

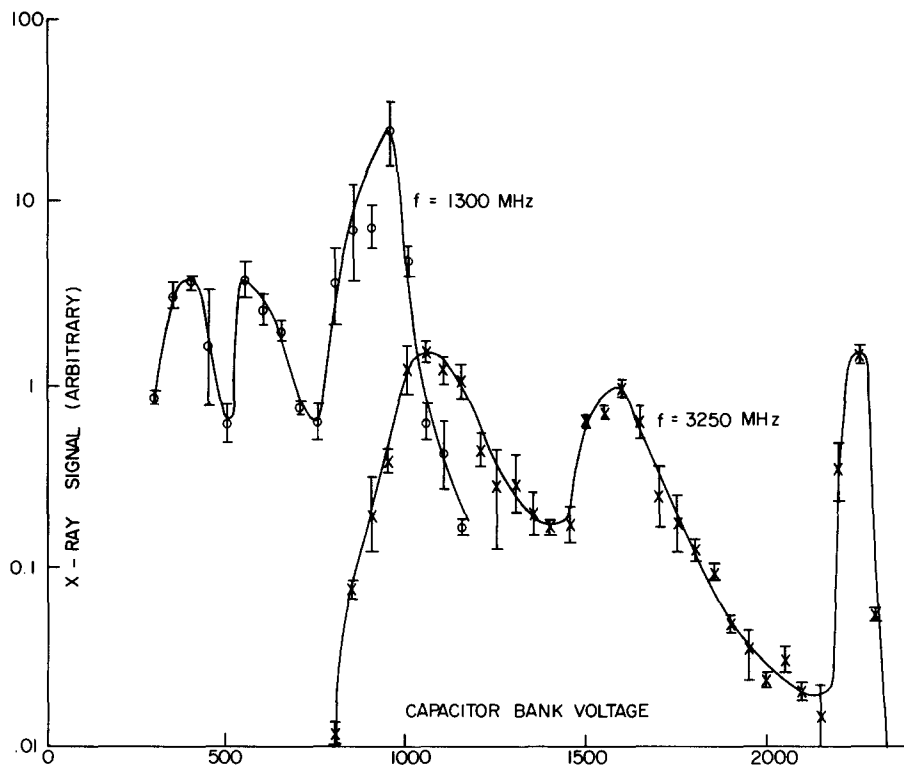


FIG. 3. X-ray signal detected by a scintillator embedded in the vacuum tank wall versus capacitor bank voltage for two different rf frequencies.

dicating the presence of large ∇B drifts as expected, and providing a warning that the probe can perturb the plasma.

The observed gradients in scintillator probe output parallel to \mathbf{B} suggest that these energetic electrons are somewhat anisotropic. There is no evidence, however, of a sharp density gradient in a localized region around $B = B_0$, as assumed in some theories and as sometimes observed in mirror devices.¹⁸ The dominant scattering mechanism is believed to be nonadiabatic effects caused by the highly nonuniform field.¹⁹ There is evidence that the anisotropy decreases with decreasing energy, and Langmuir probes which measure only the cold plasma density do not indicate density gradients parallel to \mathbf{B} . The theoretical model of Eq. (5) is, therefore, thought to be more appropriate for describing these heating experiments than are the single-particle models of other theories.

If the magnetic-field strength is reduced, the resonances move out near the walls and hoops, and we expect to see x rays produced by energetic electrons that strike the walls and hoops. Figure 3 shows the x-ray signal detected by a scintillator embedded in the vacuum tank wall versus voltage on the capacitor bank used to excite the magnetic field. The x-ray signal shows three peaks which correspond, respectively, to resonance at the inner hoops, outer hoops, and wall. A collimated scintillation detector with an acceptance cone of 5° was used to verify the origin of the x rays. The position of the peaks scales with frequency in the expected manner.

The scintillator probe shows a rapid rise of signal when the microwaves are turned on, a relatively constant signal during the heating, and a rapid decay ($\sim 10 \mu\text{sec}$) when the microwaves are turned off. Kuswa²⁰ has shown using electrostatic energy analyzers that the electron distribution function on the zero field axis for a wide range of heating conditions is given by

$$f(W) = (3n/\bar{W}) \exp[-(6W/\bar{W})^{1/2}],$$

where n is typically 10^9 – 10^{10} cm^{-3} and \bar{W} is $\sim 100 \text{ eV}$ – 1 keV . Scintillator probes give comparable, although less reliable, densities and average energies.

The relatively constant energy during the pulse and the rapid decay after the heating suggest the presence of a strong loss mechanism that balances the heating after an initial transient period. At low pressures, the most likely mechanism is the thermal flow of energetic electrons to obstacles in the plasma such as probes and hoop supports. Kuswa²⁰ has shown that the lifetime of electrons after the heating pulse is proportional to $W^{-1/2}$ as expected for obstacle loss, and that the cross section for loss is $\sim 90 \text{ cm}^2$ in reasonable agreement with the geometrical area of the probes and supports.

Equation (7) predicts a heating efficiency approaching 100% at these densities. Equating the microwave power to the power lost by thermal flow of electrons to

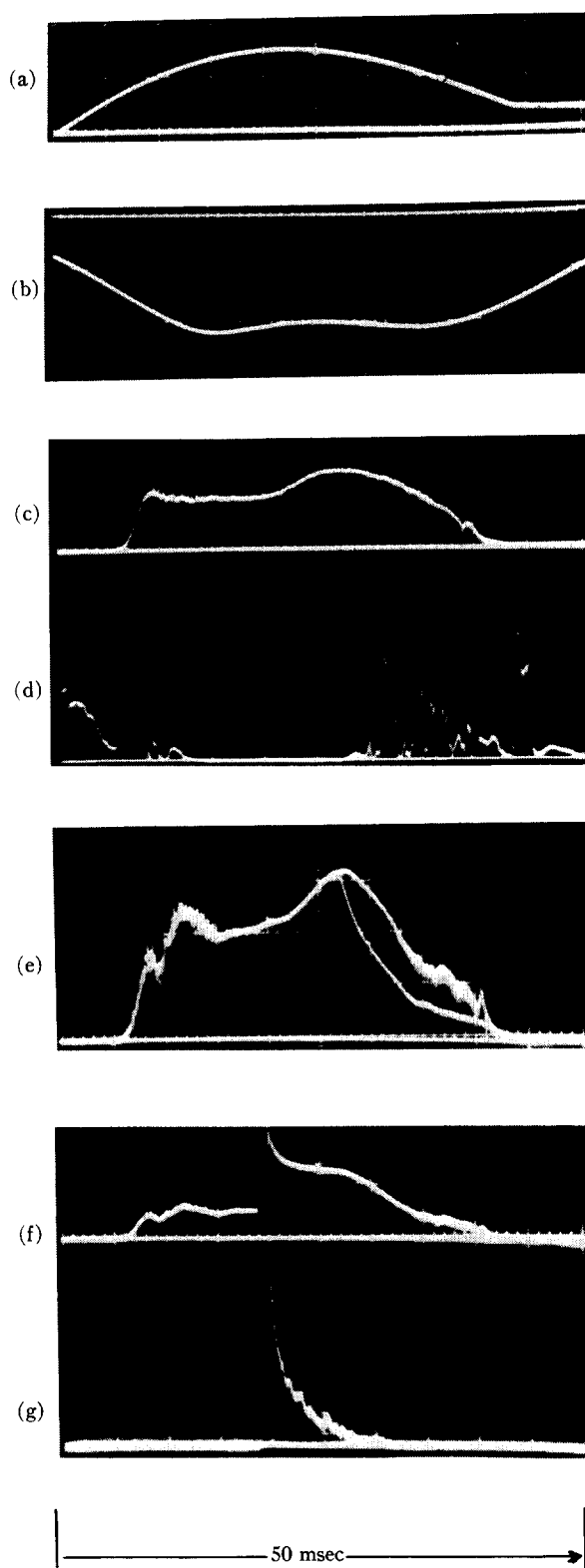


FIG. 4. Typical behavior of cw microwave produced plasmas in the levitated octupole, turned off at 27 msec (e), or heated with a high-power resonant pulse at 20 msec (f) and (g). (a) Magnetic field; (b) levator position; (c) ion-saturation current; (d) 9-GHz diagnostic; (e) ion-saturation current; (f) ion-saturation current; (g) scintillator probe signal.

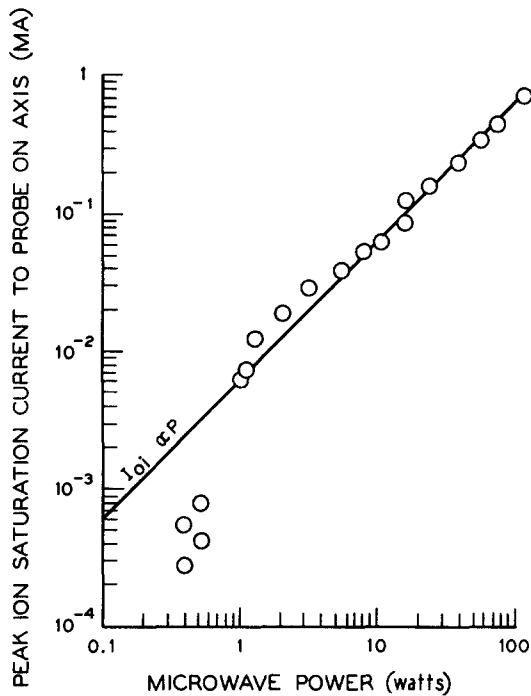


FIG. 5. Ion-saturation current to a probe on axis, showing the linear dependence of plasma density on cw 2.45-GHz microwave power, with hoops supported and a pressure of 1×10^{-6} Torr.

obstacles gives

$$P_0 = \frac{1}{2} A \int_0^\infty W \left(\frac{2W}{m} \right)^{1/2} f(W) dW = n \left(\frac{4\bar{W}}{3m} \right)^{1/2} A \bar{W},$$

provided there is sufficient ion loss or secondary emission to keep the plasma potential from rising excessively. Using analyzer measurements for n and A , the calculated equilibrium energy is ~ 800 eV as compared with the observed value of ~ 700 eV.

Kuswa²¹ has also used electrostatic energy analyzers to show that the average ion energy on axis rises from ~ 20 to ~ 50 eV when 10 kW of 3 GHz microwaves are pulsed on for 144 μ sec. Floating potential differences of ~ 300 V along a field line are observed. These potentials are believed to be caused by abrupt changes in the electron density distribution and lead to electrostatic ion acceleration.

V. LEVITATED OCTUPOLE EXPERIMENTS

Microwave heating in the levitated octupole was done using plasmas produced by gun injection or by low power, cw, microwave preionization. Figure 4 summarizes some typical cases. Figure 4(c) shows the ion saturation current to a probe on axis for 100 W of 2.45 GHz microwaves. The electron temperature is relatively constant at 3–5 eV, and so the ion saturation current is very nearly proportional to density. The current rises rapidly (in a few ionization times) as soon as resonance appears in the machine, is relatively

constant until resonance leaves the machine, and then decays slowly. The 9-GHz cavity perturbation diagnostic in Fig. 4(d) confirms the probe density measurement (each mode is $\sim 5 \times 10^7$ cm^{-3}), and shows that when 9-GHz resonance is present, the signal is strongly damped. The cw microwaves can be turned off abruptly to observe the decay [Fig. 4(e)], or high-power pulsed heating can be added, raising the density slightly [Fig. 4(f)], and producing energetic electrons [Fig. 4(g)].

Varying the cw microwave power provides a convenient means of controlling the density as shown in Fig. 5. Over a wide range, the density is proportional to microwave power. The average density at 100 W is $\bar{n} \approx 10^{10}$ cm^{-3} . When the microwaves are turned off, the initial density and temperature decay rates are $1/\tau_n \approx 10$ msec^{-1} and $1/\tau_T \approx 3$ msec^{-1} . These numbers are consistent with an absorbed power of

$$P = (\bar{n} V k T_e / \tau_T) + [\bar{n} V (k T_e + U_i) / \tau_n] \approx 50 \text{ W}$$

in rough agreement with the prediction of total absorption. Levitation has very little effect on these results.

The validity of the theoretical model is most easily tested by measuring the lowest density at which total absorption occurs. This was done by varying the power

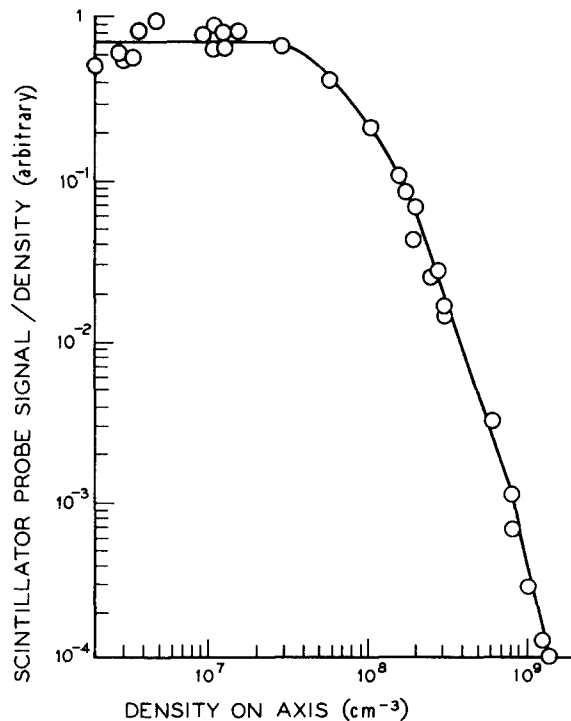


FIG. 6. Scintillator-probe signal/density (which is a monotonic function of average electron energy) versus density, showing that at low densities the heating rate is constant while at high densities the rate is reduced because the plasma lowers the cavity Q . Microwaves are 2.45-GHz continuous wave and 9.0-GHz 100-kW pulse, with hoops supported and a pressure of 2×10^{-6} Torr.

of the cw source and observing the output signal from a scintillator probe on axis during high-power pulsed heating at 9 GHz. By dividing the scintillator probe signal by the density (as measured by a Langmuir probe on axis), a quantity that is a function only of the average energy is obtained. Figure 6 shows how this quantity varies with density. At low densities, the heating rate is density independent, but at high densities the plasma loads the cavity, and the heating rate per particle decreases. Since the scintillator probe signal is roughly proportional to $n\bar{W}^4$, the decrease at high densities is consistent with $n\bar{W} = \text{const}$, as expected theoretically. Furthermore, Eq. (7) predicts that the transition should occur at a density for which $\omega_p^2 \simeq \omega^2/Q$. For $f = \omega/2\pi = 9$ GHz and $Q = 50\,000$, the required density is $\sim 2 \times 10^7$ cm $^{-3}$, in agreement with the result of Fig. 6. The geometrical factor in Eq. (7) raises the critical density to $\sim 1 \times 10^8$ cm $^{-3}$.

The preceding measurements were made with the hoops supported. Figure 7 shows the effect of levitation on the signal from a scintillator probe on axis. The signal with levitation is larger and decays more slowly after the pulse than for the supported case, as would be expected if support losses are present. Even with levitation, the energetic electrons decay more rapidly than can be accounted for by loss to the probe, suggesting that other loss mechanisms are present. The probe signals in the levitated device are typically three or four orders of magnitude larger than in the supported

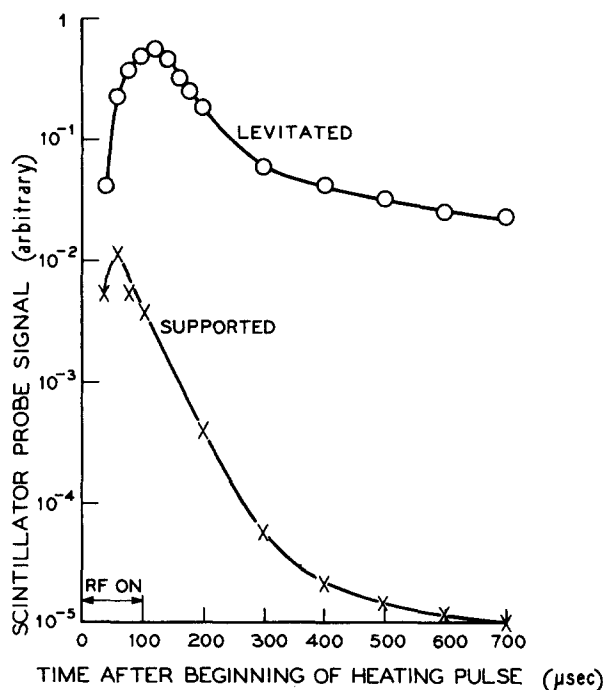


FIG. 7. Decay of energetic electrons after the heating pulse, showing the effect of levitation, using gun preionization and 100 kW for 100 μsec of 9.0-GHz pulsed heating at a pressure of 3×10^{-6} Torr.

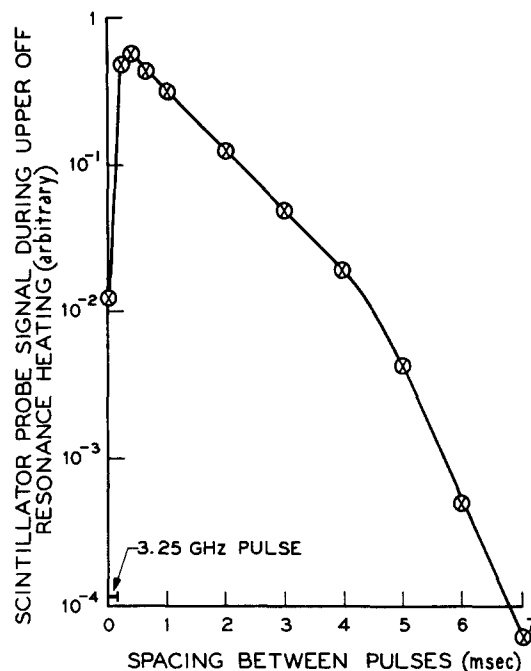


FIG. 8. Scintillator-probe signal during the upper off-resonance heating pulse at various times in the afterglow of the resonance pulse, showing the decrease of off-resonance heating with decreasing electron energy. Microwave sources are 100-W continuous at 2.45 GHz (resonant), 10-kW pulsed at 3.25 GHz (resonant), 100-kW pulsed at 9.0 GHz (off-resonance).

device at the same densities, indicating average electron energies of ~ 1 –10 keV.

Experiments with magnetic mirror devices^{15,16} show that heating can occur when the magnetic field is too small to produce electron-cyclotron resonance. A similar effect was observed here. A 100- μsec pulse of 100 kW, 9-GHz microwaves was used to heat a plasma with $\bar{n} \sim 10^{10}$ cm $^{-3}$ produced with 100-W resonant cw heating at 2.45 GHz. The signal from a scintillator probe on axis was observed as the magnetic field was lowered. When resonance left the machine, the probe signal dropped about four orders of magnitude, but a clearly detectable signal remained. The signal varied linearly with density, in contrast to the resonance case (Fig. 6), indicating that the upper off-resonance heating is less than 100% efficient.

The dependence of the off-resonance heating on electron temperature was determined by pulsing on a 10-kW, 3.25-GHz resonant source at various times prior to the off-resonance pulse, so that the off-resonance heating takes place while the electron temperature is decaying. Figure 8 shows the resulting scintillator probe signal. Since the density is relatively constant during this interval, it appears that the upper off-resonance heating is significantly enhanced by the presence of energetic electrons, as predicted.^{7,16}

Lower off-resonance heating cannot be done in a simple multipole because of the zero-field region. An experiment was performed on the supported octupole

with a superimposed toroidal field. A 400-MHz, 100-W cw microwave source caused no effect on the decay of a 10^9 cm^{-3} , gun injected plasma, although the same source would produce a plasma of comparable density and electron temperature if the magnetic field were suitably reduced. This result is reasonable since any increase in v_{\parallel}/v_{\perp} , as expected for lower off-resonance heating,¹⁶ should not significantly increase the loss from a multipole, in contrast to the enhanced axial diffusion predicted and observed for magnetic mirror devices.

VI. CONCLUSIONS

It has been shown that microwave energy can be effectively transferred to the electron component of plasmas confined in toroidal octupoles. The observed spatial distribution of energetic electrons is in agreement with the predictions of a simple theory of cyclotron heating in a nonuniform magnetic field. The theory is strictly valid only for cold, tenuous plasmas, although the plasmas studied apparently satisfy the requirements, and the prediction of 100% heating efficiency for sufficiently high densities is in agreement with observations. The electrons tend to spread out along the field, in contrast to the sharp parallel density gradients characteristic of similar experiments in mirror machines. Levitation increases the electron energy and lifetime, although other losses not produced by obstacles are apparently present. Upper off-resonance heating is effective with hot-background plasmas, as in mirror machines, but lower off-resonance heating in a multipole with B_{θ} has no effect. The energetic electrons produced by microwave heating should also provide a useful means for studying loss mechanisms from multipoles because of their short lifetime and ease of detection.

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