ELECTROSTATIC PROBE TECHNIQUES

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TABLE OF CONTENTS

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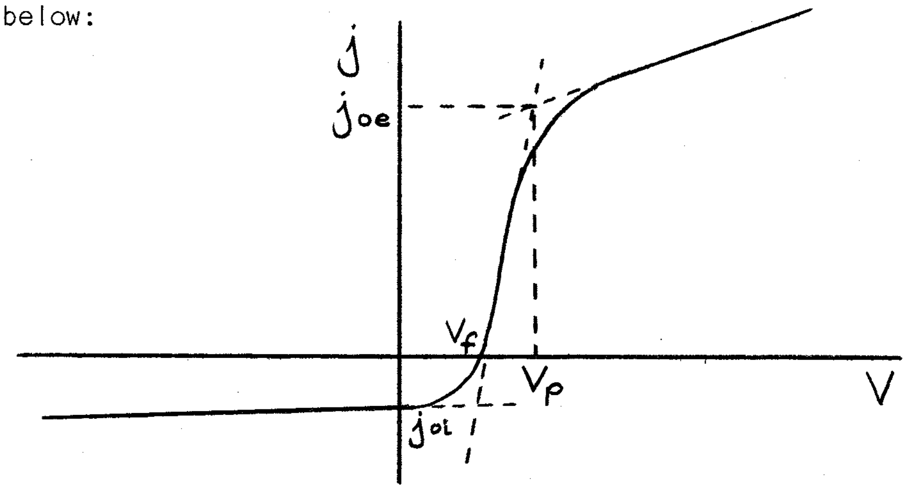
I. INTRODUCTION

Much has been written about the theory of electrostatic probes and the effects of widely varying experimental conditions on the results of probe measurements.[1] The present paper emphasizes circuits and techniques and tries to point out most of the practical difficulties that someone unfamiliar with probes is likely to encounter. No previous experience with probes is assumed, and for this reason, many elementary considerations are discussed in detail. Experience has shown that it is frequently the simple things which are overlooked. Different types of probes are compared, with particular emphasis on frequency and density limitations, and errors which are introduced when the plasma is turbulent. The descriptions are given in terms of pulsed plasmas where frequency components from a few tenths of a kHz to several MHz are present. In most cases, extension of the techniques to D.C. plasmas is straightforward.

II. REVIEW OF PROBE THEORY

Any conductor immersed in a plasma and insulated from ground can be used as a diagnostic probe if properly connected to an appropriate circuit. Probes are capable of measuring plasma density (n), potential (V_p), electron temperature (T_e), distribution function ($f(v_e)$), and in some cases, ion temperature (T_i). Basically, probe measurements consist of biasing the probe to a voltage (V) with respect to some large conductor in contact with the plasma and measuring the current flow (i) to the probe. A typical V_{-i} characteristic for a probe is shown below:



The current density (j = i/A) is plotted instead of i because the collecting area (A) is not in general constant, but increases as the sheath thickness becomes comparable to probe dimensions. The various quantities which are defined on the curve are ion saturation current (j_{oi}), electron saturation current (j_{oe}), plasma potential (V_p), and floating potential (V_f). By plotting out the entire curve, either point by point or with a fast sweep technique, one can ascertain all of the parameters of the plasma. The relevant equations are summarized below:

1. Electron temperature (T_e): If we assume thermal equilibrium, the current in the transition region (the region between V_f and V_p) is given by

$$j(V) = j\ (V_p)\ \exp\ [\frac{e(V-V_p)}{kT_e}]\ .$$

Solving for the electron temperature gives

$$T_e = \frac{k}{e} [\frac{d(\ell n\ j(V))}{dV}]^{-1} \qquad (1)$$

If the distribution is non-Maxwellian, the distribution function can be found as follows:

$$f(v) \propto \frac{d^2 j(V)}{dV^2}\ ,\ V \propto v^2 \qquad (2)$$

For a non-Maxwellian distribution, it has become customary to speak loosely of a "temperature" defined by

$$T = \frac{m\overline{v^2}}{3k} ,$$

where m is the particle mass (electron or ion) and $\overline{v^2}$ is the RMS velocity.

2. Density (n): Once the electron temperature has been determined, the density can be derived in a simple manner from the electron saturation current:

$$j_{oe} = \frac{1}{4} ne\,\overline{v_e} \qquad (3)$$

where $\overline{v}_e$ is the average electron speed. For a Maxwellian distribution,

$$\overline{v}_e = \sqrt{\frac{8kT_e}{\pi m_e}} .$$

If n is known by some other means, this method gives a measure of the electron temperature.

Often, the current j_{oe} is rather large and the plasma is disturbed because of the large flux of electrons to the probe. In this case, it is more convenient to use the ion saturation current j_{oi} to determine the density. At this point, a complication arises, because of the sheath criterion,[2] which states that when $T_i < T_e$, ions are collected at a faster rate than would be predicted by their random velocity. To a good approximation, the ion saturation current is given by

$$j_{oi} = \frac{1}{4} nev^* \qquad (T_i \leq T_e) \qquad (4)$$

where $v^* \approx \sqrt{\frac{8kT_e}{\pi m_i}}$ for a Maxwellian.

Note that the electron temperature and ion mass are used to determine v*. This is physically reasonable since the force driving the ions into the probe is provided by the plasma pressure[3] nkT_e. In the simpler case where $T_i \geq T_e$, the density can be found by the usual relation:

$$j_{oi} = \frac{1}{4} ne\overline{v}_i \qquad (T_i \geq T_e) \qquad (5)$$

where $\bar{v}_i = \sqrt{\frac{8kT_i}{\pi m_i}}$ for a Maxwellian. In either case, if the plasma temperature is constant, the ion saturation current is proportional to the density.

3. Ion Temperature(T_i): For $T_i < T_e$, the sheath criterion prevents us from determining the ion temperature with electrostatic probes. For $T_i \geq T_e$, however, T_i can be determined from equation (5) provided the density is known. For a Maxwellian, the result is:

$$T_i = \frac{2\pi m_i j_{oi}^2}{kn^2e^2} \qquad (T_i \geq T_e) \tag{6}$$

In practice, this method is not too reliable.

4. Plasma Potential (V_p): If the entire probe characteristic is available, the plasma potential can simply be read off the curve. In practice this is not too successful because the knee of the curve is usually not well defined. A more convenient method, from the experimental viewpoint, is to measure V_f, the potential at which no net current is drawn to the probe. At the floating potential, the ion current j_{oi} is given by $j_i \cong j_{oi}$, and the electron current j_e is given by

$$j_e \cong j_{oe} \exp\left[\frac{e(V_f - V_p)}{kT_e}\right].$$

Equating the two currents, $j_i = j_e$,

$$\frac{e(V_f - V_p)}{kT_e} = \log_e\left(\frac{j_{oi}}{j_{oe}}\right) = \log_e\left[\frac{T_i m_e}{T_e m_i}\right]^{-\frac{1}{2}}$$

$$\text{or } V_p = V_f + \frac{kT_e}{2e}\log_e\left[\frac{T_e m_i}{T_i m_e}\right]. \qquad (T_i \geq T_e). \tag{7}$$

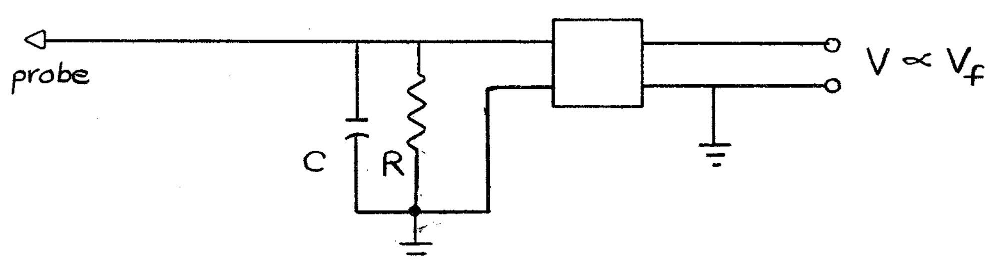
This result, however, is correct only for $T_i \geq T_e$ because of the sheath criterion. For $T_i < T_e$, the T_i in equation (7) should be replaced by T_e.

The electric field in a plasma is given by $\vec{E} = -\nabla V_p$. If the temperature is constant in space, the electric field becomes $\vec{E} = -\nabla V_f$. The drift velocity of a plasma in a magnetic field can then be found from $\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$.

III. TYPES OF PROBES

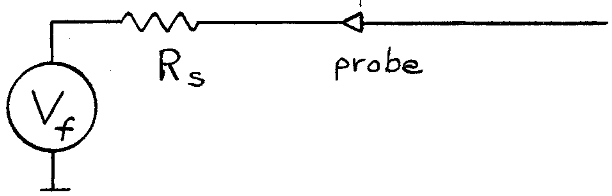
It is generally difficult to obtain an entire V-i characteristic curve for a probe in a pulsed plasma. Poor reproducibility makes the point-by-point method difficult, and rapid time variations complicate the swept-probe technique. Both methods are tediously slow when it is necessary to obtain the variation of some quantity, such as density, in both space and time. To circumvent these difficulties, various probes have been designed which produce an output voltage V(t) which is proportional to V_f, n, or E. These types of probes are described here in increasing order of complexity and versatility. In selecting a probe, it is wise to choose the simplest type which meets the experimental requirements.

1. Single High Impedance Probes (V_f): Perhaps the simplest use to which a probe can be put is the measurement of floating potential. The circuit for such a probe is shown below:



The capacitance C should be made as small as possible and the resistance R should be as large as possible. C includes all cable capacitances and the input capacitance of the black box which may be a scope preamplifier, a pulse transformer, an emitter follower, or some combination. Appropriate black boxes for various applications are discussed at length in section IV. The resistance R is the input resistance of the box.

To understand the limitations of this type of probe, it is helpful to think of the plasma as a voltage source V_f in series with a source resistance R_s:



V_f is the floating potential and R_s is the inverse slope of the V-i curve at the point $V = V_f$. R_s has been calculated by means of a Taylor expansion[4] and the result is given below:

$$R_s = \frac{kT_e}{ne^2 A}\left[\frac{2\pi m_i}{kT^*}\right]^{\frac{1}{2}} = \left(\frac{kT_e}{e}\right)\left(\frac{1}{j_{oi}A}\right) \qquad (8)$$

where $T^* = \begin{cases} T_i, & T_i > T_e \\ T_e, & T_i < T_e \end{cases}$

The resistance per unit area (R_s/A) has been plotted as a function of density for the special case of $T_i = T_e$ in the graph at the end of this paper.

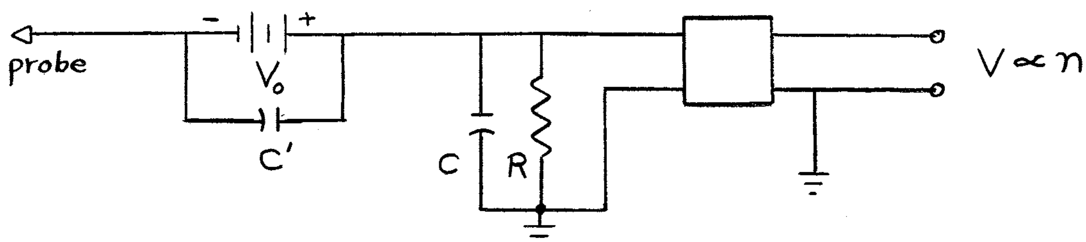
If the probe is to accurately reproduce the floating potential, two conditions must be met:

1) $R >> R_s$
2) $\frac{1}{2\pi fc} >> R_s$.

The first condition sets a lower limit on the density of the plasma with which the probe can be used, while the second condition sets an upper limit on the frequency response of the probe. There is no lower frequency limit for this type of probe, and the maximum density is determined only by effects such as probe arcing and vaporization.

2. Single Low Impedance Probes (n): The simplest type of density measurement consists of biasing a single probe to draw electron or ion saturation current and measuring the current drawn from the plasma. The following discussion will assume that the probe is biased negative to draw ion saturation current. The basic circuit for

such a probe is shown below:



The current is measured by reading the voltage drop across the load resistor R which in this case should be made fairly small. The capacitance C, as usual, includes all cable capacitances and the input capacitances of the black box. Capacitor C' is not critical and merely shorts out the internal resistance of the battery V_o and slightly improves the frequency response.

Consider first the case in which the plasma potential V_p is small ($\lesssim kT_e/e$) and constant in time. Proper operation of the single low impedance probe requires that the following conditions be met:

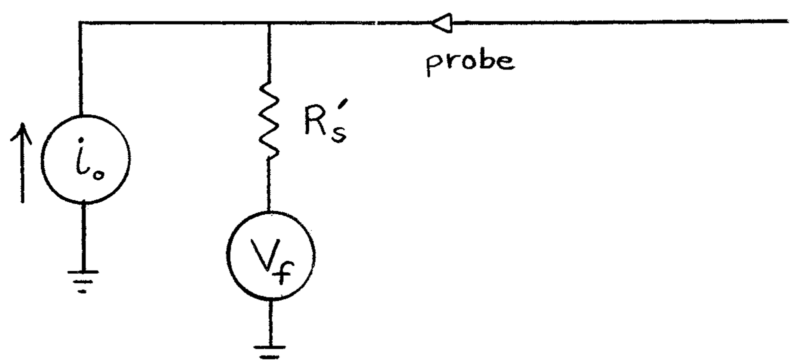
1) $V_o >> kT_e/e$

2) $R << V_o/i_o$

3) $\frac{1}{2\pi fc} >> R$

The first condition insures that the probe is biased to saturation. Because of sheath expansion and arcing, V_o should not be too large. Three to ten times the electron temperature in eV is usually satisfactory. The second condition requires that the voltage at the probe tip remains at V_o even when the current i_o through the resistor R is large. Condition 2) is equivalent to requiring that the voltage drop across R be small in comparison to V_o. The lower limit on R is set by the sensitivity of the detector and the background noise. The third condition defines the upper frequency limit of the probe which can be quite high since R is small.

The single serious disadvantage of this type of probe in most plasmas is that fluctuations of floating potential produce an output signal which cannot be distinguished from the signal due to density. Consider the following equivalent circuit for the plasma:



This time, R_s' is used as the effective sheath resistance because it is not the same resistance as R_s which was calculated at the floating potential. R_s' is the inverse slope of the V-i characteristic at the point $V = V_o$. This slope depends on the density and probe geometry and is best determined experimentally. Typically, R_s' is about 10 times larger than R_s. Proper operation of this type of probe in the presence of potential fluctuations demands that two additional conditions be met:

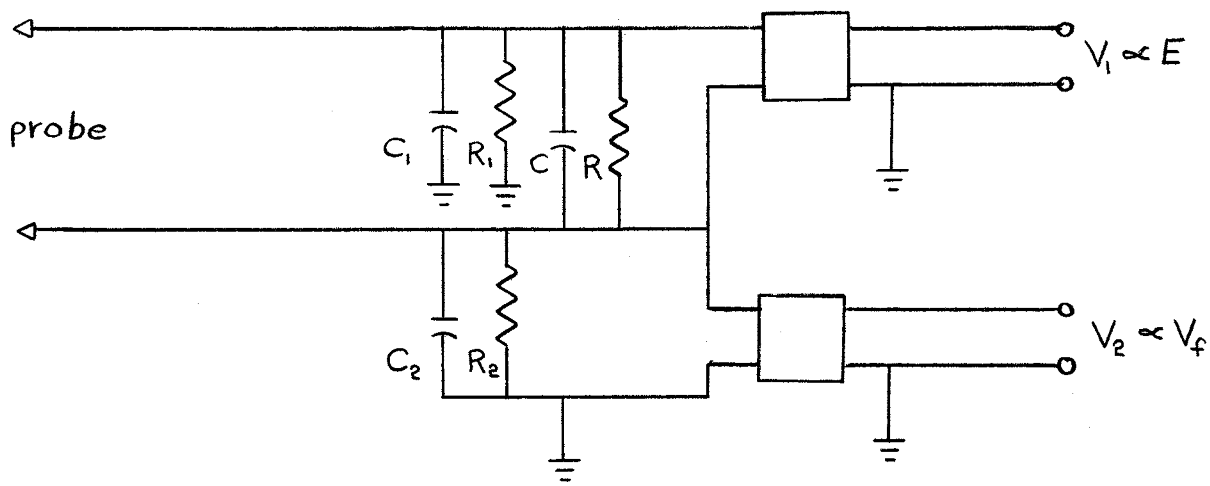
4) $V_o >> V_f$

5) $i_o R_s' >> V_f$

Condition 4) insures that the bias point remains in the saturated part of the curve. Condition 5) is that the density signal dominates the signal due to potential fluctuations. Condition 4) can always be satisfied by making V_o large enough but condition 5) is a fundamental limitation which cannot be overcome with this type of probe. When V_f is sufficiently small to satisfy the required conditions, the output voltage V is proportional to the density according to equation (4) or (5).

3. Double High Impedance Probes (V_f,E): Electric fields in a plasma can be measured with a single high impedance probe by measuring the potential at two nearby points in the plasma on successive shots. The irreproducibility of the plasma, especially at high frequencies, and the difficulty of subtracting

two nearly equal large voltages makes it highly desirable to have a probe which measures potential gradients directly. A double probe does this by electrically subtracting the floating potential of two electrodes spaced a given distance apart. The equivalent circuit is shown below:



As usual, with high impedance probes, all capacitances should be as small as possible and all resistances as large as possible. In particular the following conditions must be met:

1) $R_2 >> R_s$

2) $R >> R_s$

3) $\frac{1}{2\pi f c_2} >> R_s$

4) $\frac{1}{2\pi f c} >> R_s$

Condition 1) insures proper floating potential measurement and condition 2) insures proper electric field measurement. Condition 3) sets an upper limit on the frequency response of the floating potential and condition 4) sets an upper limit on the frequency response of electric field measurements.

In general, the voltage difference between tips ΔV is small compared to the voltage of either tip, V_f. This imposes additional restrictions on the circuit:

5) $$\frac{R_s |R_1 - R_2|}{R_1 R_2} << \frac{\Delta V}{V_f}$$

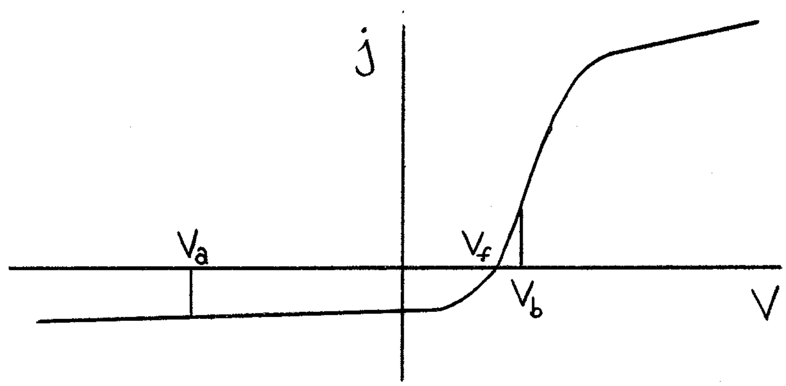
6) $$\frac{|C_1 - C_2|}{C_2} << \frac{\Delta V}{V_f} \quad .$$

The upper black box must be a differential amplifier or transformer with a high common mode rejection ratio (CMRR) if V_1 is to be proportional to ΔV, independent of V_f at all frequencies. In particular,

7) $$\text{CMRR} >> \frac{V_f}{\Delta V} \quad .$$

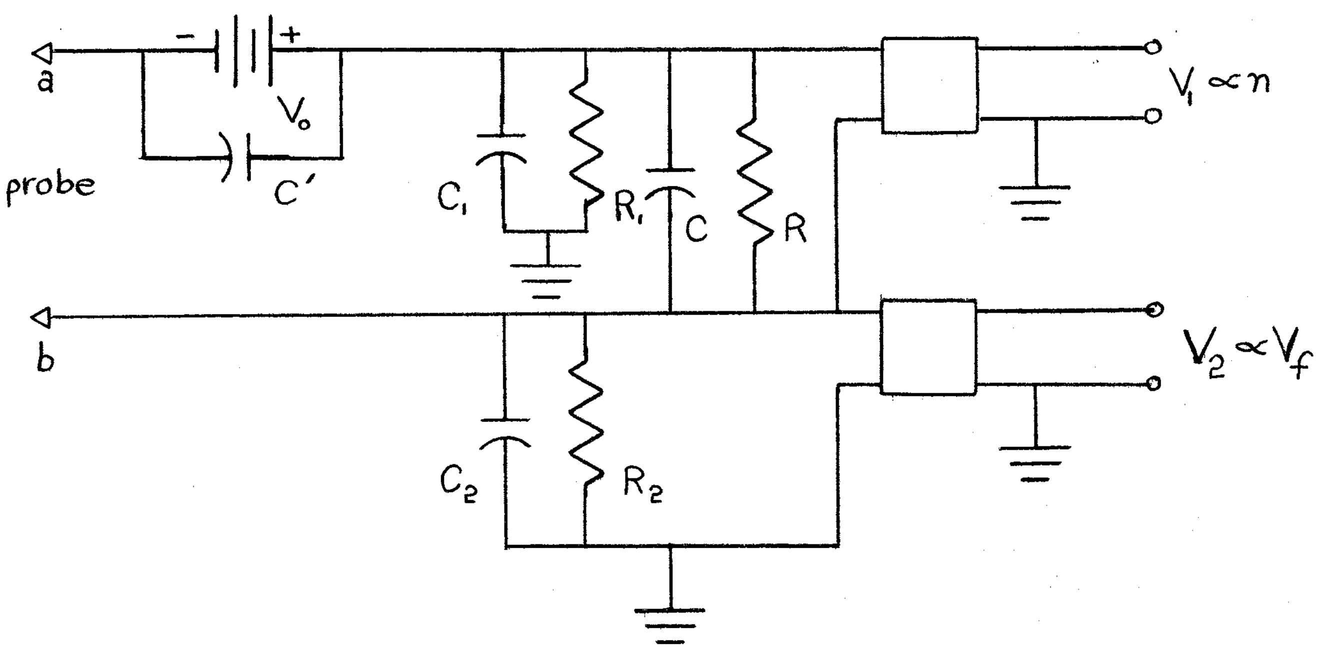
The difficulty in using this type of probe lies primarily in meeting condition 7). Some techniques for achieving adequate CMRR's are discussed in section IV. The CMRR can be measured on the bench by applying a constant sine wave voltage to both probe tips in parallel and then between the tips. The ratio of the two output voltages, $V_{1\ diff}/V_{1\ error}$, gives a measure of the CMRR. The CMRR should be measured as a function of frequency and applied voltage to verify that condition 7) is met for all experimental conditions.

4. Double low impedance probes (n, V_f): To overcome the interference from floating potential variations which plague the single low impedance probe, a double probe has been developed[5] to measure the ion saturation current. The basic concept of the double probe is that the tips can be biased with respect to one another and the current read between them, while both tips float near the floating potential. Consider the V-i characteristic of a single probe:



The probe tip voltages, V_a and V_b, are uniquely defined by the conditions that $V_b - V_a = V_o$ and $j(V_a) - j(V_b) = 0$ (i.e., no net current to the probe). We have assumed that the probe tips have equal area. Note that if $V_o >> kT_e/e$, then $j(V_a)$ is just the ion saturation current. The voltage V_b is usually only slightly greater than V_f because of the steep slope of the curve at that point. This fact makes it possible to read floating potential by measuring the voltage of tip b.

A suitable circuit for a low impedance double probe is shown below:



As usual, the resistors R_1 and R_2 should be large and equal, and C_1 and C_2 should be small and equal, although this requirement is not as important as it was with the double high-impedance probe because of the small value used for R. As with the single low impedance probe, the following conditions must be met:

1) $V_o >> kT_e/e$

2) $R << V_o/i_o$

3) $\frac{1}{2\pi fc} >> R$

In addition, adequate rejection of variations of V_1 due to V_f requires the following:

4) $\text{CMRR} >> \frac{V_f}{i_o R}$

5) $i_o >> V_f/R_1$

6) $i_o >> 2\pi f C_1 V_f$

Conditions 2), 3), and 4) together give an optimum value for R. Conditions 5) and 6) result from the fact that the positive tip (b) is more strongly driven by the plasma than the negative tip (a). Condition 6) is probably the most severe limitation because it sets an upper limit on the frequency of potential fluctuations which can be tolerated.

Proper floating potential measurement requires two additional conditions:

7) $\frac{R_1 R_2}{R_1+R_2} >> R_s$

8) $\frac{1}{2\pi f(C_1+C_2)} >> R_s$

R_s is the sheath resistance given in the graph at the end of this paper. Recall that the output is not exactly proportional to V_f but that $V_b \simeq V_f + \frac{1}{2} i_o R_s$. Fortunately, however, since $i_o \propto n$ and $R_s \propto {}^1/_n$, V_b turns out to be independent of density, and is approximately given by

$$V_b \simeq V_f + \frac{kT_e}{2eA} \quad . \tag{9}$$

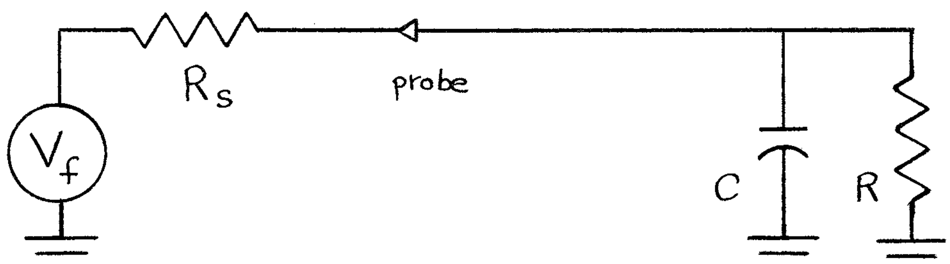
This fortunate circumstance permits measurement of the floating potential with a double low impedance probe in the presence of density variations, provided the electron temperature remains constant.

Finally we mention one other type of error which can occur with this type of probe. If the probe is in a strong electric field, such as to give a voltage difference ΔV between the tips, with $R = \infty$, a voltage proportional to ΔV is impressed on V, unless the following condition is satisfied:

9) $\Delta V \ll I_o R_s'$

R_s' is the inverse slope of the V-i curve at $V = V_a$. Recall that R_s' is typically 10 times larger than R_s.

5. Attenuated high impedance probes (V_f, E): When a high impedance probe is used to measure floating potential, the most serious limitation is the fact that the sheath resistance R_s can be too large to permit the probe to follow rapid potential variations. The equivalent circuit looks as follows:



The condition that $R \gg R_s$ is usually easily satisfied by using an amplifier with high input resistance. There is a limit to how much C can be reduced, however, since it is usually necessary to shield the lead to the probe. 52 Ω polyethylene cable, for example, has about 30 pF/ft. The improvement with higher impedance cable is slight because of the logarithmic dependence on radii. Furthermore, most amplifiers have input capacitances of at least 20 pF. The net result is to place an upper limit on the frequency response of the probe given by

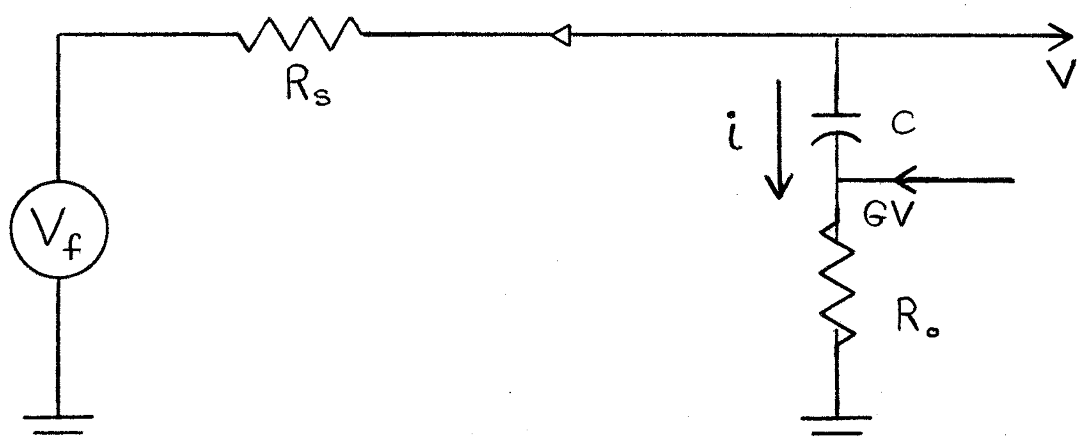
$$f_c = \frac{1}{2\pi R_s C} . \qquad (10)$$

The cutoff frequency f_c can be increased by three methods:

1) increase the probe area, A
2) reduce the effect of the capacitance, C
3) attenuate the signal at the probe tip.

The first method reduces the sheath resistance R_s at the expense of spatial resolution. In addition, large probes tend to perturb the plasma and give off impurities.

The second method consists of driving the shield with a low impedance emitter follower with a voltage and phase equal to that of the probe signal.[6] The appropriate equivalent circuit for understanding this effect is shown below:



The voltage at the bottom end of the capacitor is held at GV by an emitter follower whose voltage gain is G. The current flow, i, through the capacitance C is

$$i = 2\pi fC(V-GV) = 2\pi fCV(1-G),$$

and the output voltage V is

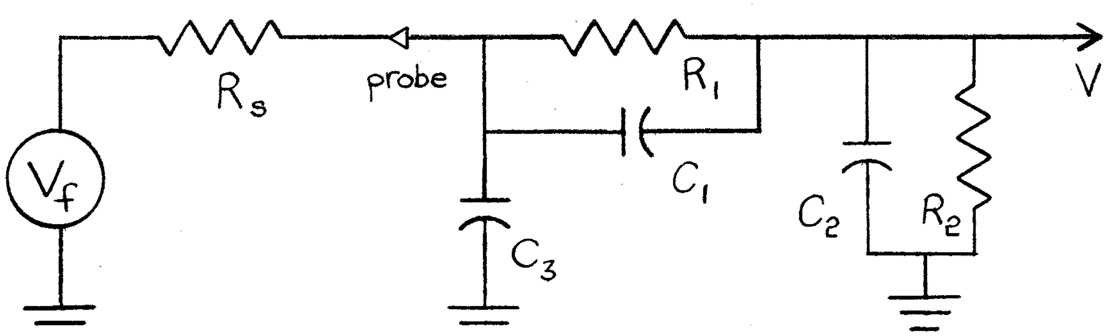
$$V = V_f - iR_s = V_f - 2\pi fCVR_s(1-G).$$

Solving for V gives

$$V = V_f/[1 + 2\pi fCR_s(1-G)] . \qquad (11)$$

In the case of G = o (no emitter follower), equation (10) is recovered by setting $V = \frac{1}{2} V_f$. The ideal situation results when G = 1, in which case, the frequency response is limited only by the input capacitance of the amplifier which has not been included here. G >1 corresponds to positive feedback and in general causes the system to oscillate. The resistance R_o is the output resistance of the emitter follower and should be quite low if the shielding is to be effective. The drawbacks of this system are two-fold: 1) It is difficult to design emitter followers which operate linearly over the wide range of potentials encountered in most plasmas, 2) All amplifiers have a maximum bandwidth above which the output is no longer in phase with the input. In addition, when this method is applied to double probes, the magnitude and phase of the feedback voltages have to be identical to a high degree of accuracy, if common mode effects are to be avoided.

The third method, that is, attenuating the signal at the probe tip, has proved to be the most successful.[7] With this system a high resistance R_1 is placed at the probe tip to attenuate the floating potential signal:



With the resistance R_1 very near the tip, the capacitance C_3 is very small. C_1 is the intrinsic capacitance associated with the geometry of the resistor, or it may be a small capacitor in parallel with the resistor. Most small resistors are observed to have a few tenths of a pF parallel capacitance. With this type of probe, one chooses $R_1 >> R_s$, and the upper limit of the frequency response is

$$f_c = \frac{1}{2\pi R_s (C_1 + C_3)}. \tag{12}$$

f_c can be made much higher than the f_c given by equation (10) because $C_3 << C_2$. For frequencies below f_c, V is proportional to V_f provided $R_1C_1 = R_2C_2$. This condition is easily derived by requiring that the attenuation be the same at zero frequency (where the capacitances can be neglected) and at infinite frequency (where the resistances can be neglected). The voltage gain of the probe G_p (which for this type of probe is less than one) is given by

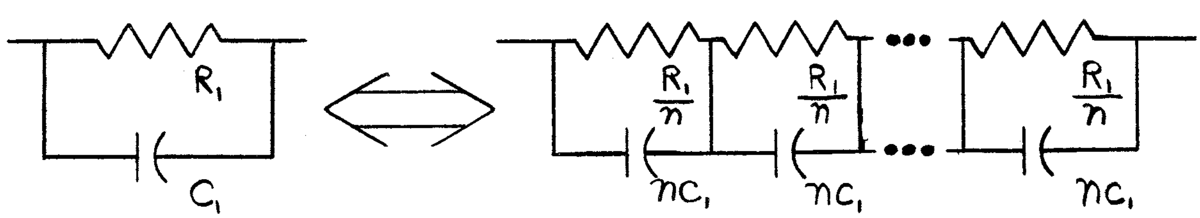
$$G_p = \frac{R_2}{R_1 + R_2} \cong \frac{R_2}{R_1} \quad (R_1 >> R_2) \cdot \tag{13}$$

An alternate description of the attenuated probe is that greater bandwidth is achieved with a sacrifice of gain, the gain - bandwidth product remaining constant. The gain - bandwidth product is given by

$$G_p f_c = \frac{1}{2\pi R_s C_2} \ . \tag{14}$$

The frequency response of this probe can be increased indefinitely, until the gain is so low that the system is noise limited.

Unfortunately, the capacitance C_1 associated with the resistor R_1 is not perfectly lumped but is distributed throughout the resistor. It can be shown that a pure capacitance in parallel with a pure resistance is electrically equivalent to a series of equal capacitors connected across a series of equal resistors, provided the total resistance and capacitance are the same:

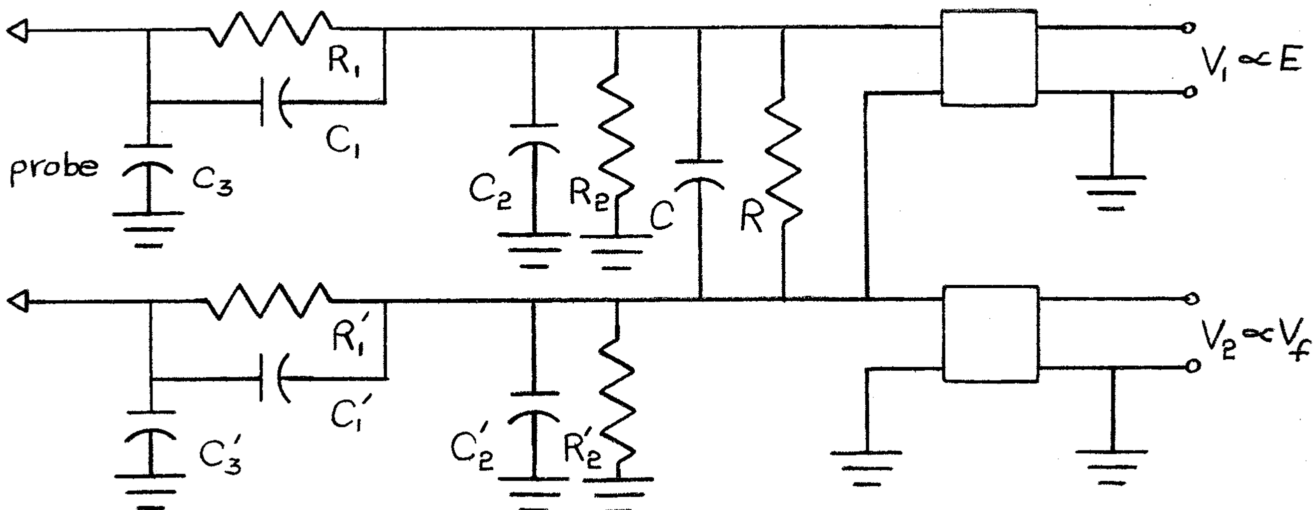


In particular, this is also true for an infinite series where the resistance and capacitance are distributed evenly throughout the resistor. In a real resistor, however, the end effects cause some distortion and the impedance varies by about 10 or 20% from that of an ideal lumped RC. This problem can be overcome by shunting the resistor with a capacitance of a few pF or by using a special balancing circuit such as described in section IV, 5.

Finally, the question of whether to shield the resistor must be answered. Shielding tends to increase the capacitance C_3 and thus lowers the frequency response. On the other hand, if the resistor is left unshielded, it may pick up capacitive signals from the plasma which are larger than signals at the probe tip. Generally, it is safest to shield the resistor because the capacitance added is rather small and the impedance characteristic of the resistor is not appreciably changed. If the shield is left off, it is wise to bench test the probe by wrapping aluminum foil around the probe, near the resistor and driving it with a signal of the same amplitude and frequency as the fluctuations expected from the plasma. The output should be small compared to the signal that results when the probe tip is driven.

The extension of the attenuated probe technique to electric field measurements in tenuous plasmas is straightforward. Two tips are placed close together and each connected to a high resistance. Adequate common mode rejection requires perfect geometrical symmetry at the probe tip. In practice, the probes must be balanced by applying the same signal to the two tips and moving the resistors relative to one another until the difference signal is minimum. Shielding the resistors makes this adjustment less critical, but it is important for the shields to be identical.

With all this as an introduction, we are at last in a position to describe a circuit suitable for a double attenuated probe:



Constant attenuation at all frequencies requires:

1) $R_1 C_1 = R_2 C_2$

2) $R_1' C_1' = R_2' C_2'$

Proper floating potential reproduction requires:

3) $R_1' \gg R_s$

4) $\dfrac{1}{2\pi f(C_1' + C_3')} \gg R_s$

As is generally the case, $R \gg R_2$, R_2' and $C \ll C_2, C_2'$. Then the frequency response for electric fields is the same as for floating potentials. Adequate CMRR requires:

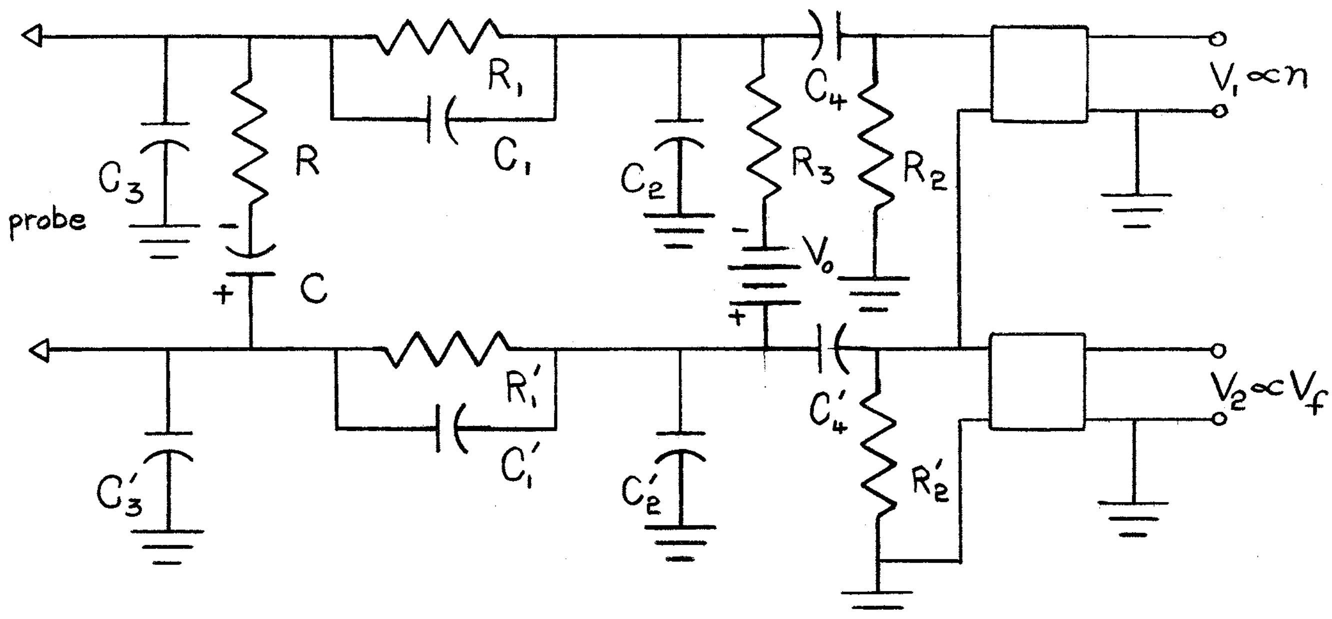
5) $\dfrac{|R_2 - R_2'|}{R_2} \ll \dfrac{\Delta V}{V_f}$

6) $\dfrac{|C_2 - C_2'|}{C_2} \ll \dfrac{\Delta V}{V_f}$

7) $\text{CMRR} \gg \dfrac{V_f}{\Delta V}$

The best CMRR is achieved by balancing the probe on the bench by methods described earlier.

6. Attenuated Low Impedance Probes (n,V_f): The attenuated probe technique described in the previous section can also be applied to the double low impedance probe for density measurement. Its chief advantage lies in the fact that it permits density measurements in the presence of rapidly changing floating potentials. Its chief disadvantages are difficulty of construction and balancing. A suitable circuit for an attenuated low impedance double probe is shown below:



The components R, C, R_1, and R_1' should be as close to the probe tip as possible. C_1 and C_1' are stray capacitances associated with R_1 and R_1'. C_3 and C_3' are stray capacitances from the probe tips to ground . The unique feature of this type of probe is the resistors R_1 and R_1' which serve the dual function of attenuating the signal to increase the bandwidth and of charging the capacitor C to a voltage V_o. The capacitor C should have a voltage rating greater than V_o and as large a capacitance as possible. Its internal leakage resistance should be much greater than $R_1 + R_1' + R_3$.

It should be pointed out that this type of probe differs from all others discussed in that it will work only in a pulsed plasma. The duration of the plasma pulse must be sufficiently short that the voltage V_o between the tips does not change appreciably. Furthermore, the time lapse between shots must be long enough to allow the capacitor C to charge through resistors R_1, R_1' and R_3. The first condition is equivalent to requiring that the charge which leaks off the capacitor be small compared to $Q_o = CV_o$. The change in charge is $\Delta Q = \int i_o(t)dt$ where $i_o(t)$ is the time dependent ion saturation current, $i_o(t) = \frac{1}{4}e\bar{v}_i An(t)$. Hence, a minimum value is set for C:

$$1)\ C \gg \frac{e\bar{v}_i A}{4V_o} \int n(t)dt$$

If the time between shots is T, the capacitor will be fully charged provided

$$2)\ T \gg (R_1 + R_1' + R_3)C.$$

As with other low impedance probes, R must be small enough that no appreciable voltage drop exists across it:

$$3)\ R \ll V_o/i_o\ .$$

R_3 should be large to prevent unnecessary attenuation of the difference signal:

$$4)\ R_3 \gg R_2 + R_2'\ .$$

The voltage V_o is chosen somewhat greater than the electron temperature in eV:

$$5)\ V_o \gg kT_e/e\ .$$

The capacitors C_4 and C_4' prevent the battery V_o from discharging through R_2 and R_2', and isolate the differential amplifier from the DC bias:

$$6)\ \frac{1}{2\pi fC_4} \ll R_2$$

$$7)\ \frac{1}{2\pi fC_4'} \ll R_2'$$

In conditions 6) and 7), the frequency f is the lowest fourier component of the density signal. Constant attenuation at all frequencies requires:

$$8)\ R_1C_1 = R_2C_2$$

$$9)\ R_1'C_1' = R_2'C_2'$$

Proper floating potential reproduction requires

10) $R_1 >> R_s$

11) $\dfrac{1}{2\pi f(C_3 + C_3' + C_1 + C_1')} >> R_s$

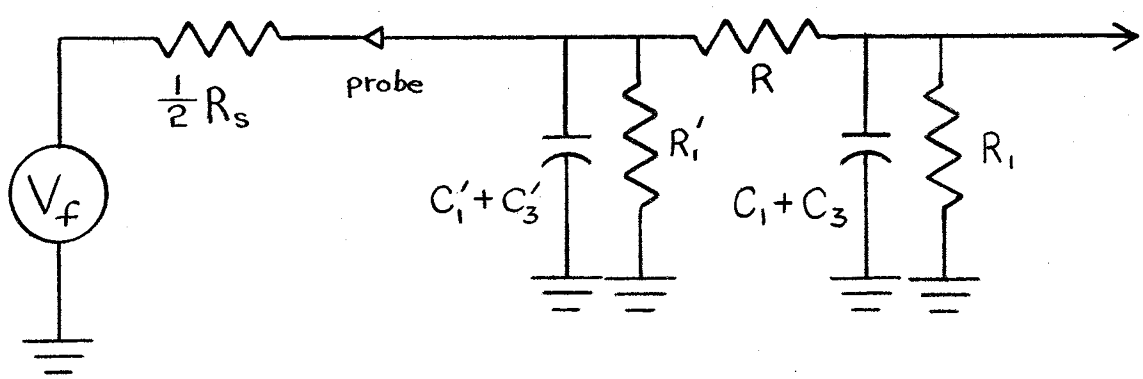
Adequate CMRR requires:

12) $\dfrac{|R_2 - R_2'|}{R_2} << \dfrac{i_o R}{V_f}$

13) $\dfrac{|C_2 - C_2'|}{C_2} << \dfrac{i_o R}{V_f}$

14) CMRR $>> V_f/i_o R$.

This type of probe is subject to one rather subtle type of floating potential interference. Since the positive tip is more effectively driven by the plasma because of its lower sheath resistance ($\sim \frac{1}{2}R_s$), a current proportional to V_f will flow through R. The equivalent circuit which shows this effect is shown below:



It is usually the case that $R << R_1'$. Also, condition 11) insures that $C_1' + C_3'$ can be neglected. Then the conditions that the current through R due to V_f be small compared to the ion saturation current i_o, are:

$$15)\quad \frac{2V_f}{R_s + 2R + 2R_1} \ll i_o$$

$$16)\quad \frac{2V_f}{R_s + 2R + 1/\pi f(C_1 + C_3)} \ll i_o$$

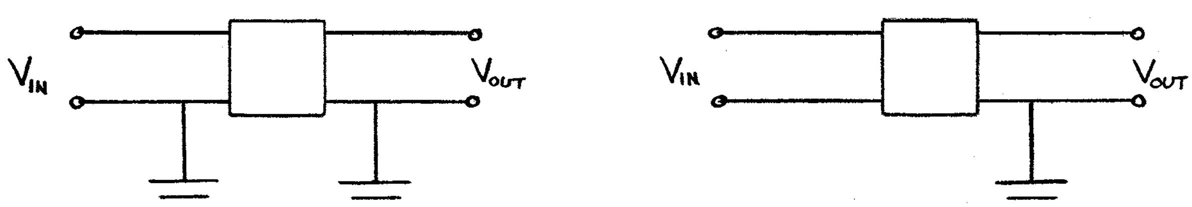
These restrictions can be quite severe in a low density plasma with a large floating potential.

With four components at the end of the probe, the question of shielding becomes much more serious. Shielding tends to greatly increase C_3 and C_3', but in cases where the capacitive pickup is large, it is necessary. Sometimes a compromise is made by shielding only R_1 and R_1'.

As usual, the circuit must be balancing on the bench before being put into service. When R_1 and R_1' are physically small, their resistance is observed to be slightly voltage dependent. If the CMRR is tested by driving both tips in parallel, it is necessary to reduce V_o to zero if the probe is to remain balanced in the plasma. A more physically realistic way to balance the probe would be to drive only the positive tip, but this may cause some trouble at high frequencies where the plasma couples evenly to both tips by displacement currents through the sheaths. (To be discussed in Section V, 4.)

IV. COUPLING CIRCUITS

In this section we will discuss various circuits which may be used to couple the probes previously described to the vertical amplifier of an oscilloscope. These circuits were represented as black boxes in the previous section. It should be noted that all the black boxes fall into one of two categories: those with a common connection at the input and output (single-ended) and those whose output is isolated from their input (differential):



single - ended   differential

All the circuits discussed here are linear; that is, their output V_{out} is proportional to their input, V_{in}. In some cases, non-linear circuits are useful, such as to record the log of the ion saturation current in plasmas where the density varies over several orders of magnitude. The Tektronix type O-plug-in is capable of logarithmic amplification. In the circuits discussed here, a high degree of linearity is considered desirable. The differential circuits are by far the more difficult to design and adjust because of the requirement of a large common mode rejection ratio (CMRR). If the input has a common mode signal V_{CM} which is large compared to the difference signal ΔV, the CMRR must be much greater than $V_{CM}/\Delta V$ if the circuit is to work properly.

1. Direct Coupling: In cases where considerable capacitance can be tolerated, direct coupling between the probe and the scope can be used whenever a single-ended circuit is required. The scope should be as close to the probe as practical. Noise pickup often demands that the scope be some distance from the experiment, however. Recall that 52Ω polyethylene cable has about 30 pF capacitance per foot, and that the capacitance goes inversely as the impedance. RG114 is useful because of its high impedance (180Ω) and low capacitance (6 pF/ft). An open ended transmission line will exhibit resonances at frequencies where the electrical length of the cable is an integral multiple of a quarter wave. A transmission line looks capacitive at lengths much less than $\frac{1}{4}$ wave. As the quarter wave point is approached the cable inductance becomes important giving an effective series resonance at $\lambda/4$. Between a quarter and a half wave, the impedance increases again, becoming infinite at $\lambda/2$. The electrical length of polyethylene coaxial cable is 5 nsec/meter. If the line is terminated with its characteristic impedance, the termination is reflected back to the input, and the line can be arbitrarily long without adding any capacitance. Such a line, whose impedance is constant for all frequencies, is said to be "flat." For very long lines there are dielectric and ohmic losses, but for lengths commonly used in the laboratory ($\lesssim$ 100 ft.), these can be neglected.

Several types of Tektronix probes which are useful for reducing capacitance while maintaining high impedance are listed below:

Type	Length	R_{in}	C_{in}	Attenuation
P6028	6'	1MΩ	95pF	X1
P6006	6'	10MΩ	11pF	X10
P6013	10'	100MΩ	3pF	X1000

The last two types, incidentally, use the same attenuation technique discussed in section III, 5.

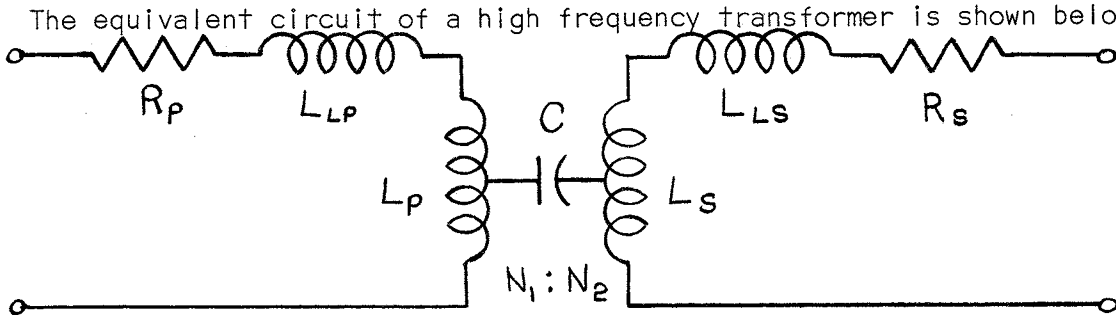
2. Emitter Followers: When it is necessary to drive a long cable, it is best to terminate the cable in its characteristic impedance and drive it with an emitter follower located near the probe. An emitter follower is an amplifier with a low output impedance, a high input impedance, and near unity voltage gain. A very useful emitter follower for driving low impedance cables is the Navy's preferred semiconductor circuit PSC-22 (shown in drawing no. 4048).[8] The PSC-22 is a complimentary symmetry circuit, which enables it to drive a low impedance (~ 50 Ω) without drawing appreciable current (~4 ma) from the battery since neither transistor conducts except during the short time when a signal is present. The complimentary symmetry circuit has one drawback in that for the low signal levels (a few millivolts) it is non-linear. The PSC-22 has a 1KΩ, 50 pF input and a 50 Hz to 5 MHz bandpass. If a higher input impedance is desired, a Los Alamos preamplifier (no. 4039) can be used. It has an input impedance of about 10MΩ, an output impedance of 1KΩ, a gain of 100, and a bandwidth of about 5 MHz. It is suitable for driving a PSC-22. Another useful circuit is the cathode follower in drawing no. 4040. It has an input resistance of 1MΩ, an output of 50Ω, a gain of 0.1, and a bandwidth of 10 MHz. The three circuits previously described are compared in the following table:

Circuit	R_{in}	R_{out}	Voltage Gain	Bandwidth	Output
4048	1K	50Ω	1	5 MHz	± 10V
4039	10M	1K	100	5 MHz	± 10V
4040	1M	50Ω	0.1	10 MHz	-.5 to + 2V

The circuits described here are all single-ended and will not work in applications requiring differential amplification. The bandwidth is usually limited by stray capacitance and transistor response time. The output voltage is usually limited to about $\frac{1}{2}$ the power supply voltage, but non-linearities may become significant at considerably lower voltages.

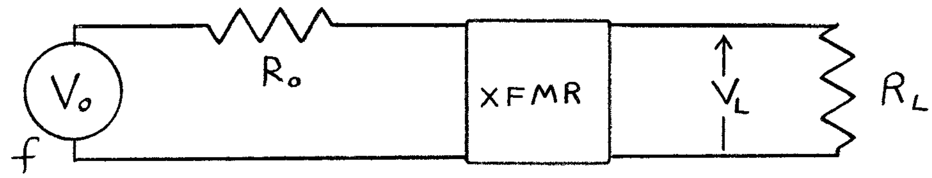
3. Transformer Coupling: A transformer is essentially a device for transforming power from one impedance level to another, determined by the turns ratio. A transformer is probably the simplest type of differential circuit which one can use. The subject of transformer design and application is a complex one, and only those topics which are of particular interest to probe circuits are discussed here. In particular, we will be concerned with the factors which limit bandwidth, amplitude, and CMRR.

The equivalent circuit of a high frequency transformer is shown below:



R_p and R_s are the respective DC winding resistances. L_p and L_s are the primary and secondary winding inductances. L_{Lp} and L_{Ls} are primary and secondary leakage inductances (caused by magnetic flux linking only one transformer winding). C is the winding-to-winding capacitance, and $N = N_1/N_2$ is the turns ratio.

When a transformer is placed in a circuit, its primary is driven by some source resistance R_o and its secondary drives a load R_L:



What is usually desired is that the output voltage V_L be proportional to V_o independent of the frequency f. For this to be true, the following conditions must be met:

1) $R_o + R_p << 2\pi f L_p$

2) $R_s + R_L << 2\pi f L_s$

3) $R_o + R_p >> 2\pi f L_{Lp}$

4) $R_s + R_L >> 2\pi f L_{Ls}$

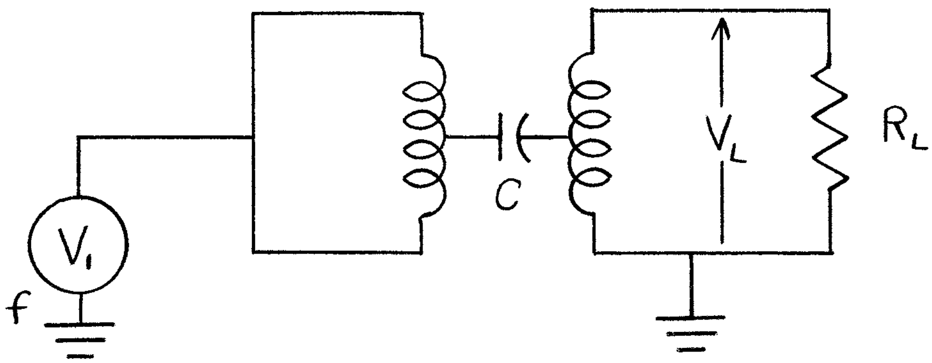
These conditions set lower and upper limits on the frequencies f over which the transformer works properly. If f is within the allowed range,

$$V_L = \frac{R_L V_o}{R_o + R_p + R_s + R_L} .$$

The common mode rejection ratio is limited by two effects. The first is the series resonance which occurs when f becomes sufficiently large:

$$5)\ f << \frac{1}{2\pi\sqrt{(L_{Lp} + L_{Ls})C}} .$$

For f below this value, the approximate CMRR of the transformer can be derived by considering what happens when both primary leads are tied together and driven:



The capacitance C is actually distributed along the windings, but we will consider it to be concentrated at the center. In this case the output voltage is given by

$$V_L \cong \frac{2R_L V_1}{R_L + 1/2\pi fC} = \frac{4\pi fR_L C V_1}{2\pi fR_L C + 1}$$

$$\cong 4\pi fR_L C V_1 .$$

The CMRR is defined by

$$CMRR = \frac{V_1}{V_L} \cong \frac{1}{4\pi fR_L C} . \qquad (15)$$

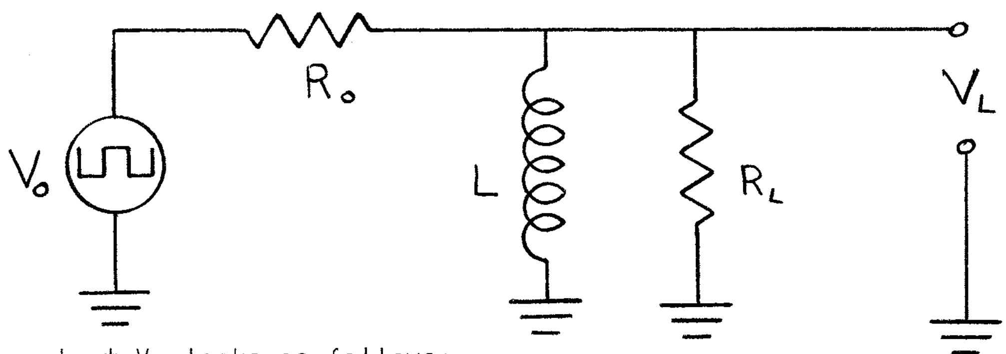
This expression has been experimentally confirmed.[9] The CMRR becomes worse as the frequency gets higher. Good CMRR requires a small interwinding capacitance and low value load resistor.

Another factor to consider when selecting a transformer is core saturation. As the magnetic field in the core increases, a point is reached where further increase in primary current does not increase the magnitude of the field. For small fields, B is proportional to the primary current i_p. Since the primary is inductive,

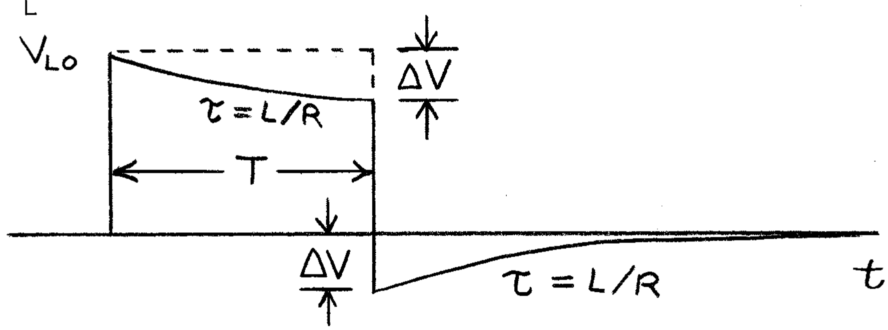
$$B \propto i_p = \frac{1}{L_p} \int V_p \, dt \quad .$$

The quantity $\int V_p\, dt = Et$ for which the core saturates is an important property for small pulse transformers. For sufficiently low frequency, high amplitude signals, the coupling becomes non-linear and harmonics are introduced. Core saturation is generally not a serious limitation because the signal can be attenuated before it reaches the transformer. When a transformer is used in a circuit where a DC current flows through the windings, care must be taken to insure that the current is not large enough to saturate the core.

One other effect can be annoying when using a pulse transformer. Consider the highly idealized transformer connected to the square wave source shown below:



The output V_L looks as follows:



The droop ΔV is given by

$$\Delta V = V_{Lo} e^{-\frac{RT}{L}} \tag{16}$$

where

$$V_{Lo} = \frac{R_L V_o}{R_L + R_o} \quad \text{and} \quad R = \frac{R_o R_L}{R_o + R_L} .$$

The condition that the fractional droop $\Delta V/V_{Lo}$ be small is the same as conditions 1) and 2) discussed earlier. This overshoot can be particularly annoying when it is necessary to measure a small signal immediately preceded by a large signal of long duration. Such a situation is encountered, for example, in the octupole, where a large density burst during filling is followed by a period of low density during confinement.

Finally, we list a number of pulse transformers which are typical of what is commercially available. Those listed were taken from Catalog 206 of Pulse Engineering, Inc., which also contains much useful information about testing transformers.

Number	Turns Ratio	L_p (mhy)	E_t (V-μsec)
5268	1:2:1	15	5000
4034	2:1:1	120	2000
5163	1 ct: 1ct	10	252
5158	1 ct: 1ct	1	77

cont'd.

Rise Time (nsec)	C(pF)	L_L (μhy)
220	55	41
220	75	300
60	1.23	14.0
21	0.96	2.5

The last two types have good CMRR because of an electrostatic shield between windings which reduces the winding capacitance C to a very low value.

4. Differential Amplifiers: Usually the CMRR of even the best transformer is not sufficient when used with double high impedance or attenuated probes. Also, it is difficult to find transformers which work well over more than two decades of frequency. On the other hand, differential amplifiers can be built which are flat from DC to many MHz with a very high CMRR (> 10^4). Unfortunately, the construction of differential amplifiers

is very difficult because of the high degree of electrical and physical symmetry required. For someone inexperienced in circuit design, the best approach is to try to use one of the Tektronix preamplifiers. A number of these are listed below, to facilitate selection:

Type	Price	Sensitivity	Bandwidth	CMRR
D	$170	1mV/cm	DC-300KHz	10^4
E	190	50μV/cm	.06-20KHz	5×10^4
G	190	50mV/cm	DC-20MHz	10^2
W	575	1mV/cm; 50mV/cm	DC-8MHz;DC-23MHz	2×10^4
1A7	425	10μV/cm	DC-500KHz	5×10^4

Frequently it is useful to have some small, cheap, home-made differential amplifiers which can be placed near the probe. Over the past few years, a large number of such amplifiers have been built and tested. The simplest types (nos. 4046, 4036, 4037, 4033) have a floating emitter follower which drives a pulse transformer. The most successful designs which have been used are listed below:

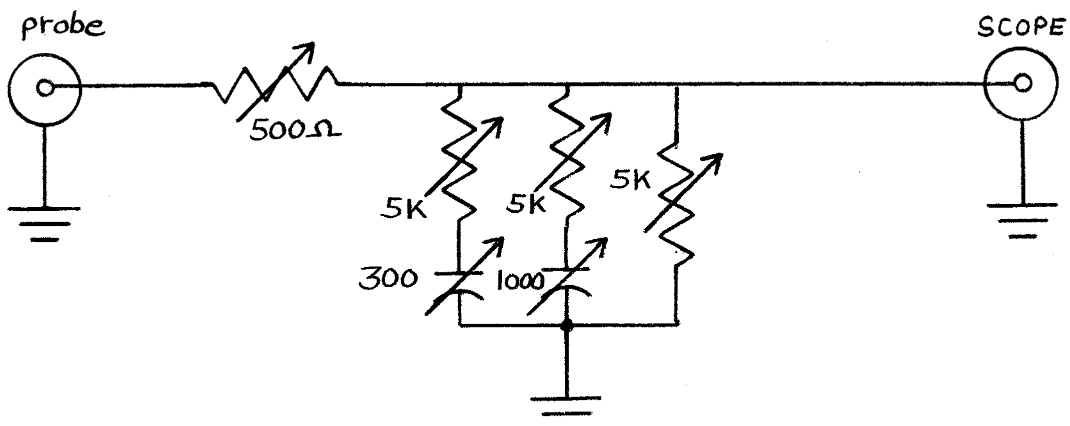
No.	Bandwidth	CMRR	R_{in}	Gain
4046	10MHz	100	12K	10^{-2}
4036	8MHz	100	5K	1
4037	10KHz-16MHz	1	1K	4
4033	1KHz-40MHz	100	5K	10^{-3}
4044	10MHz	200	2K	10
4047	1MHz	100	5K	10

In addition to these circuits, several integrated operational differential amplifiers have given good results.

Finally, we mention the Tektronix type P6023 probes which are especially good for use with differential preamplifiers because they have adjustments to match input resistance and capacitance to compensate for differences in amplifier inputs. The input is 10MΩ, 12 pF, and the attenuation factor is 10.

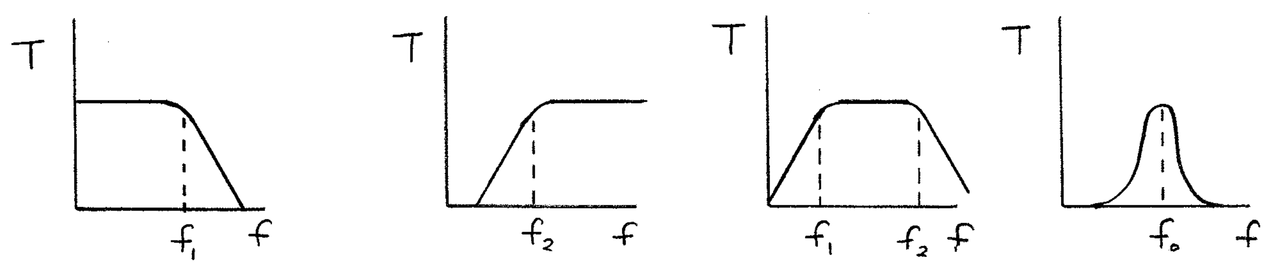
With any of the amplifiers discussed here, it is essential to balance the amplifiers for maximum CMRR before attempting to make measurements.

5. Passive Balancing Networks: It was mentioned in section III, 6, that an ordinary resistor cannot be exactly represented by a parallel RC circuit. Hence if the probe circuit has a simple RC input, the attenuation will not be the same at all frequencies. To overcome this problem, the method used by Tektronix in their high voltage probe has been applied to attenuated plasma probes. The method consists essentially of using a series of RC circuits adjusted in such a way as to give an impedance which is proportional to the series probe impedance at all frequencies. Of course, with a finite number of RC's, it is never possible to attain this goal, but a very close approximation (within a few percent) is achieved with a circuit such as that shown below:



This circuit was found suitable for 10M Ω shielded probes with 10 ft. of RG58 cable, and gives an overall attenuation of 10^4. The circuit is adjusted essentially by trial and error with a long square wave with fast rise time driving the probe. If the best adjustment occurs at the end of the range of any of the pots or trimmers, their values should be changed accordingly. In some cases, it may be desirable to add more RC sections.

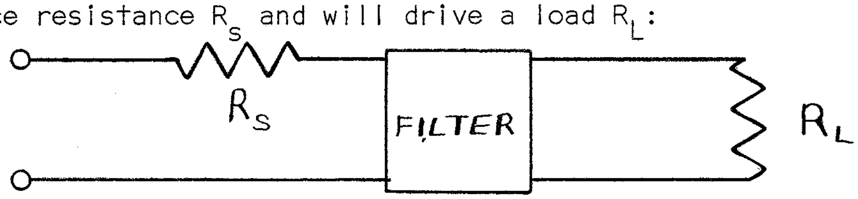
6 Filters:[10] It is often desirable to limit the frequency response of a probe circuit to one frequency or to a range of frequencies. For example, one may wish to measure a small high frequency density fluctuation in the presence of a large slowly varying background density, or if the CMRR is very bad at high frequencies, one might wish to electrically ignore the error signal. Devices which perform such duties are called filters. We will discuss four basic types: 1) low-pass filters; 2) high-pass filters; 3) band-pass filters; and 4) resonant filters. They are distinguished by the frequency dependence of their transmission properties:



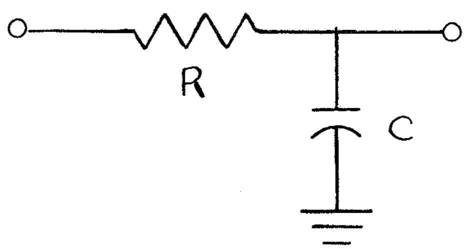
low-pass high-pass band-pass resonant

Another type of filter, one which rejects signals at a given frequency, will not be discussed.

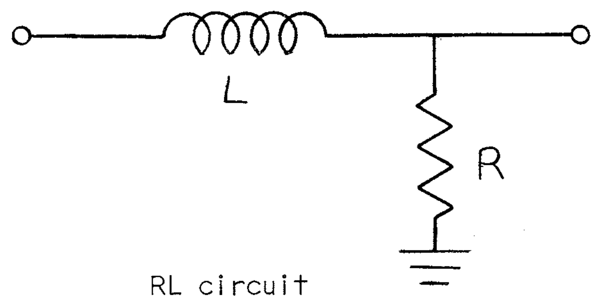
In the following discussion, f_1 will represent the high frequency cutoff and f_2 will represent the low frequency cutoff. The filter will be driven by a source resistance R_s and will drive a load R_L:



The simplest types of low pass filters are shown below:



RC circuit



RL circuit

LOW PASS FILTERS

These circuits are also useful as integrators when used well above their cut-off frequency. The cutoff frequency f_1 (3db down) is given by

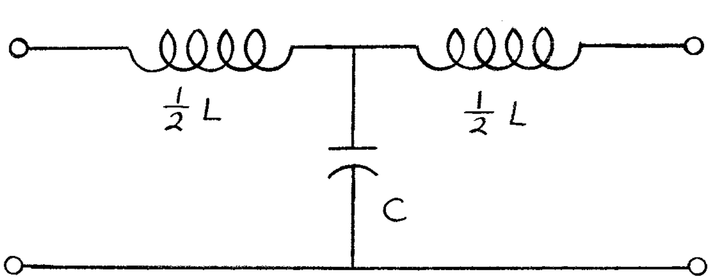
$$\left.\begin{aligned} f_1 &= \frac{R_L - R - R_s}{2\pi R_L C(R + R_s)} \\ \text{and} \qquad f_1 &= \frac{RR_L - R_s(R + R_L)}{2\pi L\ (R + R_L)} \ . \end{aligned}\right\} \qquad (17)$$

The equations look much simpler for the usual case of $R_s << R$, $R_L >> R$:

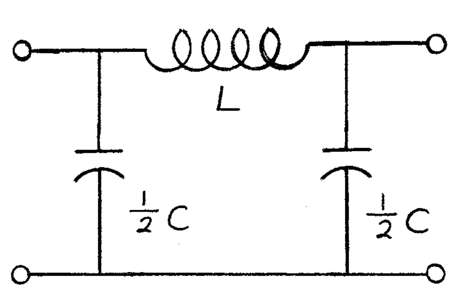
$$\left.\begin{aligned} f_1 &= \frac{1}{2\pi RC} \\ f_1 &= \frac{R}{2\pi L} \ . \end{aligned}\right\}$$

Generally, the RL circuit is preferable because R can be made equal to the cable impedance and the cable will be properly terminated over the filter pass-band. Above f_1, the transmission decreases a factor of 10 for each decade of frequency.

If a sharper cutoff is desired, an LC filter can be used. There are two types:



T Section



π Section

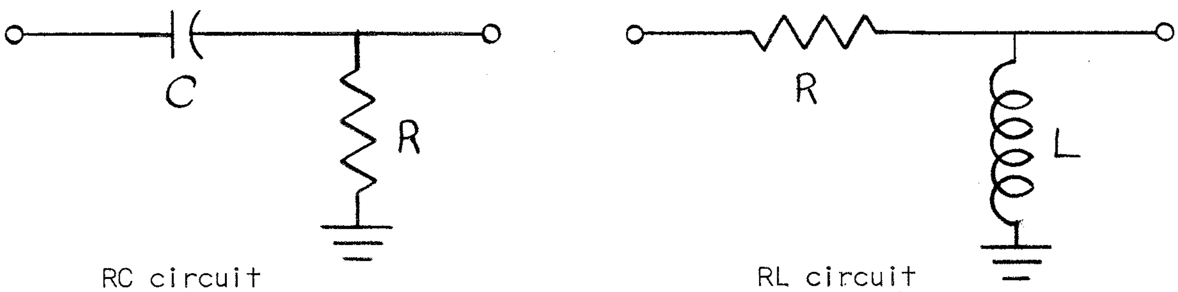
LOW PASS FILTERS

The values L and C are chosen as follows:

$$\left.\begin{aligned} L &= \frac{R_L}{\pi f_1} \\ C &= \frac{1}{\pi f_1 R_L} \end{aligned}\right\} \qquad (18)$$

where R_L is the load resistance, which in general is made equal to the cable impedance. The choice between the T and π section is made purely on the basis of convenience.

High pass filters are designed in a similar way:



RC circuit RL circuit

HIGH PASS FILTERS

Well below cutoff, these circuits act as differentiators. The cutoff frequency f_2 is given as follows:

$$\left.\begin{aligned} f_2 &= \frac{R + R_L}{2\pi C[R R_L - R_s (R + R_L)]} \\ &\text{and} \\ f_2 &= \frac{R_L (R_s + R)}{2\pi L \ (R_L - R_s - R)} \end{aligned}\right\} \qquad (19)$$

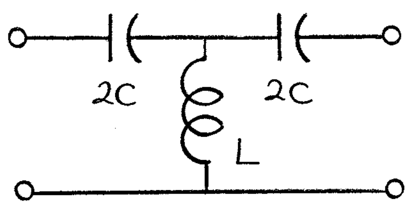
For $R_L >> R$ and $R_s << R$, these become:

$$f_2 = \frac{1}{2\pi RC}$$

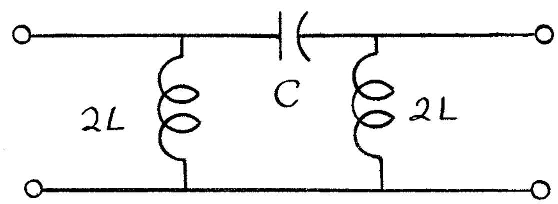
$$f_2 = \frac{R}{2\pi L} \ .$$

These conditions are the same as those derived for the low-pass filter. For the high-pass filter, however, the RC circuit is preferable because of the parallel resistive termination. Again, the transmission decreases by 10 for each decade of frequency.

Better high-pass filters are shown below:



T section



π section

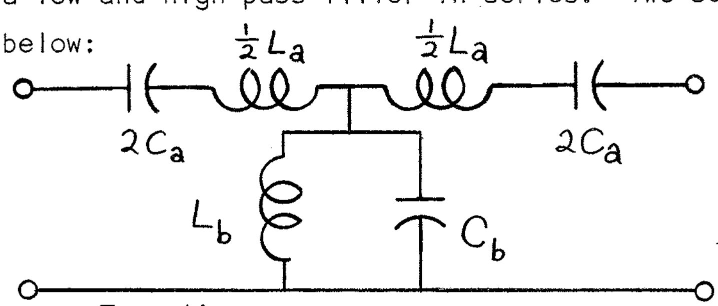
HIGH PASS FILTERS

L and C are given by:

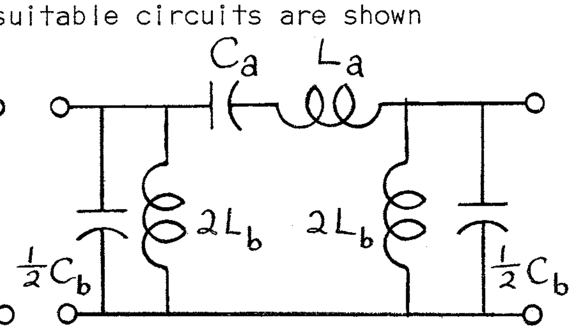
$$\left.\begin{aligned} L &= \frac{R_L}{4\pi f_2} \\ C &= \frac{1}{4\pi f_2 R_L} \end{aligned}\right\} \qquad (20)$$

If a very sharp cutoff is desired, filters can be connected in series.

Band-pass filters have a flat response over a range of frequencies and a sharp lower and upper cutoff. The simplest band-pass filter is a low and high-pass filter in series. Two suitable circuits are shown below:



T section



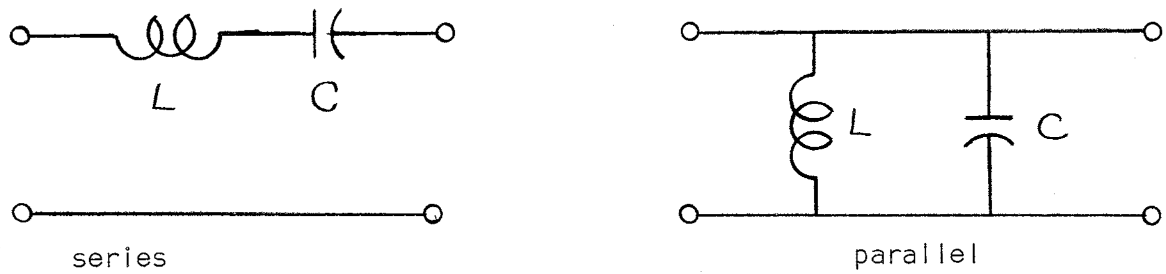
BAND PASS FILTERS

π section

The values are selected according to the following table:

$$\left.\begin{aligned} L_a &= \frac{R_L}{\pi(f_2-f_1)} & \qquad C_a &= \frac{f_2-f_1}{4\pi f_1 f_2 R_L} \\ L_b &= \frac{R_L(f_2-f_1)}{4\pi f_1 f_2} & \qquad C_b &= \frac{1}{\pi(f_2-f_1) R_L} \end{aligned}\right\} \qquad (21)$$

Resonance filters consist of a single capacitance and inductance, and have a response curve which is sharply peaked at the resonant frequency, f_o. There are two types of resonance filters:



RESONANCE FILTERS

The resonant frequency f_o is given by:

$$f_o = \frac{1}{2\pi\sqrt{LC}} \qquad (22)$$

The Q of the resonance $\frac{f_o}{\Delta f}$ is a quantity of considerable importance. Δf is the resonance width at the half power points. The Q is given by:

$$\left.\begin{aligned} Q &= \frac{2\pi f_o L}{R_s + R_L + R_o} \quad \text{(series)} \\ Q &= \frac{R_s + R_L}{2\pi f_o L} \quad \text{(parallel)} \end{aligned}\right\} \qquad (23)$$

where R_o is the DC resistance of the inductor and all other symbols are as defined before. Hence, if a high Q is desired, the series circuit works best at low impedances and the parallel circuit works best at high impedances. Note that the Q can be changed by varying the L/C ratio. For ordinary LC circuits, Q's greater than a few hundred are difficult to achieve.

A useful device for producing a very high Q (10^5 - 10^6) is a quartz crystal. Crystals have a parallel and a series resonance very near each other, and can be operated in either mode. Unfortunately, crystals are rather expensive and it is very difficult to change their resonant frequency.

V. Miscellaneous Considerations

1. Johnson noise:

A fundamental limitation to the sensitivity of wide-band, high impedance amplifiers and probes is noise voltage due to the thermal motion of electrons. The RMS voltage fluctuation was calculated by J. B. Johnson[11] and is given by:

$$\overline{V} = \sqrt{4kTR\Delta f} \quad , \tag{24}$$

where k is Boltzmann's constant, T is the temperature, R is the input resistance, and Δf is the bandwidth. As an example, consider the Tektronix type L plug-in which has an input resistance of 1 megΩ and bandwidth of 25 MHz. The calculated Johnson noise at room temperature is about 0.65 mV. This is just about the level of noise observed with a type L. In general, good amplifiers approach the Johnson noise level rather closely. High impedance, wide-band plasma probes are subject to the same limitation.

2. Electrode size and shape:

The probe theory derived in section II assumed that the Debye sheath thickness λ_d, given by

$$\lambda_d = \sqrt{\frac{kT}{4\pi ne^2}} \quad , \tag{25}$$

was small compared to probe dimensions. This requirement, together with the fact that sensitivity increases with probe area, places a practical lower limit on probe size. Remember also that the sheath resistance R_s decreases with increasing probe area. On the other hand, large probes reduce spatial resolution and tend to disturb the plasma.

Except for spherical probes, it should be noted that probes have a directional effect. A plane probe weighs particle fluxes according to the cosine of the direction to the normal. Hence a plane probe would be useful, for example, in measuring currents parallel and perpendicular to field lines. A cylindrical probe also has a cosine weighing factor, but has azimuthal symmetry. A spherical probe collects particles isotropically. When the sheath becomes comparable to probe dimensions, the plane probe is superior because the collecting area does not increase. Cylindrical probes represent a compromise and are most commonly used because of their convenience.

3. Sheath Expansion:

When the sheath thickness becomes appreciably larger than probe dimensions, collection is no longer limited by space charge effects, and orbital motions become important.[12] In the limit of very thick sheaths, it can be shown that the saturation current is independent of bias voltage only for a plane probe. For spheres, the saturation current varies with V, and for cylinders, as $V^{\frac{1}{2}}$.

For most laboratory plasmas, the probe size is chosen large enough that orbital motions can be neglected. In this case, the collecting area may still increase as the density goes down. The collecting area of a cylindrical probe is given approximately by

$$A = 2\pi(r + \lambda_D)\ (\ell + \lambda_D) \tag{26}$$

where r is the probe radius, ℓ is the probe length, and λ_D is the Debye distance. Note that three regions can be distinguished.

1) $\lambda_D << r$, $A = 2\pi r\ell$, $i_o \propto n$

2) $r << \lambda_D << \ell$, $A = 2\pi\lambda_D\ell$, $i_o \propto \sqrt{n}$

3) $r >> \ell$, $A = 2\pi\lambda_D^2$, $i_o = \text{const.}$

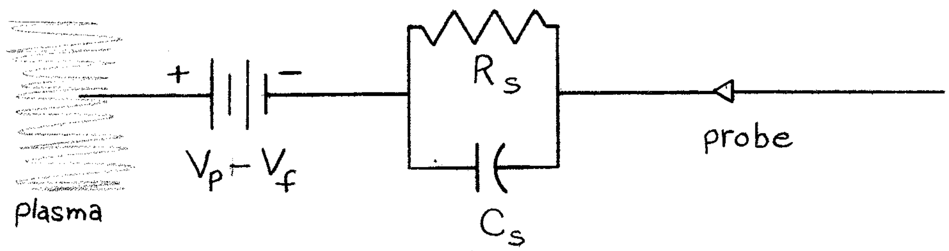
Hence the cylindrical probe behaves like a plane, cylinder, or sphere, depending on the density. Data interpretation is simplest if one of the above cases is used. Usual operation, however, is between cases 1) and 2) in which case:

$\lambda_D \sim r << \ell$, $A = 2\pi(r + \lambda_D)\ell$

and $$i_o = \frac{\pi}{2} n e \bar{v}_i \left(r + \sqrt{\frac{kT}{4\pi n e^2}}\right)\ell. \tag{27}$$

4. High Frequency Sheath Properties:[13]

It has been assumed in the previous discussion that the sheath can be represented electrically as a D.C. resistance between the plasma and the probe, with magnitude R_S given in the graph at the end of this paper. The value of R_S is most easily determined by dividing the electron temperature in eV by the ion saturation current. As the frequency becomes sufficiently high, displacement currents across the sheath become important, and the sheath looks capacitive. An electrical equivalent circuit for the plasma sheath valid at high frequencies is shown below:



$V_p - V_f$ is the potential drop across the sheath as given in equation (7), and R_S is the sheath resistance given in equation (8). It can be shown[14] that the capacitance C_s is approximately the same as would be derived for a capacitor of the appropriate geometry with plates separated one Debye length and filled with a dielectric $\varepsilon_o = 1$. In plane geometry

$$C_S = \frac{kA}{\lambda D} \tag{28}$$

where k is given by

$$k = \frac{1}{2}\left[\frac{2T_e}{T_i \log_e (T_e m_i / T^* m_e)}\right]^{\frac{1}{4}} . \tag{29}$$

In a hydrogen plasma with $T_e = T_i$, k = 0.36.

The RC time constant of the sheath is a quantity of some interest:

$$R_S C_S = \left[\frac{\sqrt{2\pi m_i kT}}{n e^2 A}\right]\left[\frac{kA}{\lambda D}\right] = 2\pi k\sqrt{\frac{2 m_i}{ne^2}}$$

Or in terms of the ion plasma frequency f_{pi},

$$R_S C_S = \sqrt{8\pi}\,\frac{k}{f_{pi}} \tag{30}$$

where

$$f_{pi} \equiv \frac{\omega_{pi}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4\pi n e^2}{m_i}} \tag{31}$$

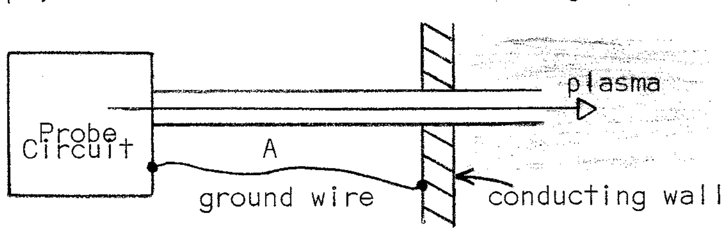
Using the value 0.36 for k, $R_S C_S = 1.4/f_{pi}$.

Hence $R_S C_S$ is about equal to the transit time of ions across the sheath. The capacitive component of the sheath becomes important at frequencies above the ion plasma frequency. When looking at potential fluctuations in this range, it should be noted that the phase shifts by 90° near the ion plasma frequency.

If a floating probe is to give accurate results near and above f_{pi}, it would be ideal to have $R_S C_S$ equal to the product of the probe input resistance and input capacitance. Then the signal attenuation through the sheath is constant for all frequencies. Unfortunately $R_S C_S$ varies with the square root of the density, so this condition cannot be universally satisfied, but it can be closely approximated if the density is fairly constant in a particular experiment.

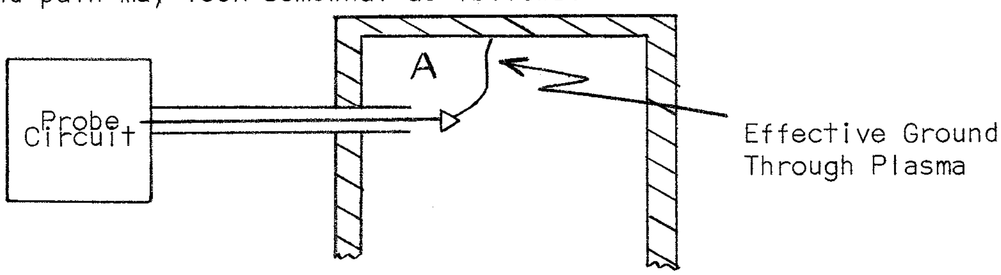
5. Grounding Considerations:

The method of grounding the probe shield is of considerable importance for a floating probe if the output is to be proportional to the floating potential. Aside from the usual precautions of avoiding long ground leads and ground loops, two effects are worth mentioning:



Since the probe shield is usually inside an insulator, the shield cannot be grounded where it enters the vacuum system. An external ground wire is usually placed between the probe circuit and the tank wall. If the area A is large, a considerable inductance will be introduced in the ground lead and the plasma will be able to capacitively drive the probe shield at high frequencies. The inductance should be kept sufficiently low so that the resonant circuit formed by the ground inductance and the plasma-to-shield capacitance is resonant at a frequency much higher than the bandwidth of the amplifier circuit. Usually this requires returning the ground wire back along the probe to the vacuum tank wall.

A second consideration is the fact that the plasma may not make good electrical contact to ground at the point where the probe enters the vacuum tank. The ground path may look somewhat as follows:



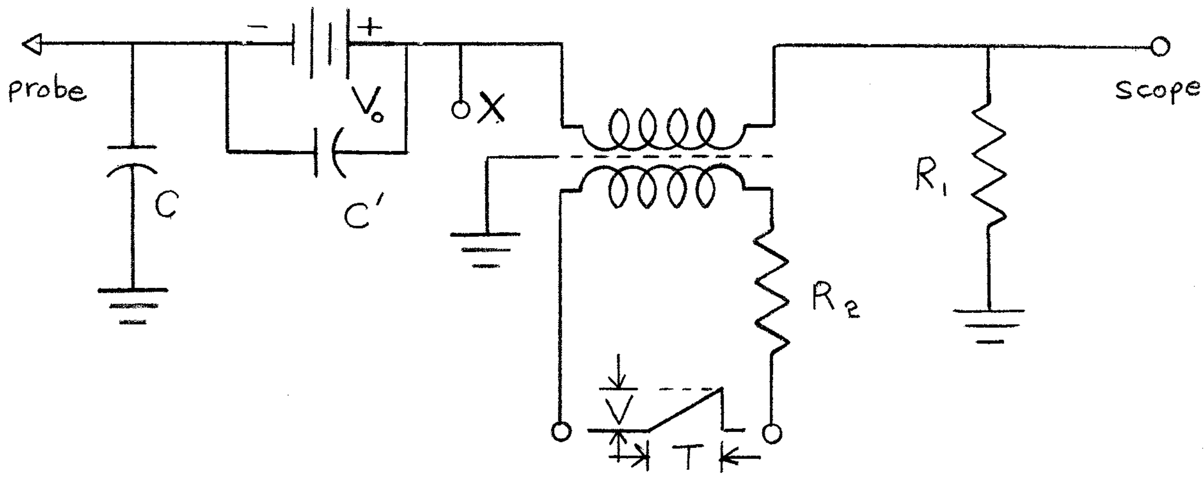
In this case there may be a large area A around which current must flow. In the presence of time varying magnetic fields, the voltage at the probe circuit is given by

$$V = V_f + \frac{\partial}{\partial t} \iint_A \vec{B} \cdot d\vec{S} \qquad (32)$$

With large, time-varying, magnetic fields, or with high frequency field perturbations caused by currents within the plasma, this second term can become important. Since it is difficult to estimate the effective area A, all that is usually possible is to calculate an upper limit on the magnitude of this effect. In most experiments this contribution is negligible.

6. <u>Swept Probe Technique:</u>

Sometimes it is desired to obtain an entire V-i characteristic at a particular time. This can be done by sweeping the probe voltage and measuring the current drawn. The sweep time must, of course, be faster than the time during which the parameters of the plasma, such as n, T, and V_f, change appreciably. A suitable circuit for sweeping a probe is shown below:



R_2 is the source resistance of the sawtooth generator[15] and C is the stray capacitance from the probe to ground (which is considerable when the probe is shielded). V should be large enough to allow the entire characteristic to be swept, that is, several times the electron temperature in eV. V_o merely provides a bias so that a positive-going sawtooth can be used. The pulse transformer must be capable of passing the major Fourier components of the sawtooth and should have a sufficiently large Et constant, in accordance with section IV,3. If the probe is to follow the sweeping voltage, the following relation must hold:

$$1)\ (R_1 + R_2)\, C \ll T$$

R_2 should be as small as possible, preferably less than R_1. R_1 is chosen according to considerations in section III, 2.

Proper operation of the probe can be verified by measuring the time dependence of the voltage at point X to see that it does resemble the sawtooth input.

If the waveform at X is appreciably distorted, the method can still be used by feeding the signal from X to the horizontal amplifier of the oscilloscope.

For very fast sweep times, the AC current through C due to the sawtooth can become important. Proper operation of the probe requires that this parasitic current be much less than some characteristic current such as the ion saturation current i_o:

2) $\frac{CV}{T} \ll i_o$.

Finally, we have the requirement that the sheath be always in equilibrium. The sheath relaxation time is the order of the ion transit time across the sheath[16] which is about $1/f_{pi}$. Hence,

3) $T \gg 1/f_{pi}$.

These three conditions set a lower limit on the duration of the sweep, T.

EFFECT OF FLUCTUATIONS

When a probe is used to measure current in a fluctuating plasma, the potential fluctuations can cause a DC shift in the current drawn. This rectifying action results from the non-linear nature of the V-i curve, and shows up when the frequency of the fluctuation is above the cutoff frequency of the probe circuit. Assuming a sine wave fluctuation, $V = V_o + \Delta V \cos \omega t$, integration of the current drawn over one period $T = 2\pi/\omega$ leads to the following approximate result which is valid in the transition region:

$$\frac{\Delta j}{j_e} = 1 - \mathcal{I}_o \left(\frac{e\Delta V}{kT_e}\right) \qquad (33)$$

Δj is the DC current shift, j_e is the electron current, and $\mathcal{I}_o$ is the zero order Bessel function of the first kind, with $\mathcal{I}_o(o) = 1$. Beyond the transition region, no shift is expected because of the linear relationship between V and i in the saturation regions. This result is strictly valid only for frequencies much below the ion plasma frequency, since it has been assumed that the sheath has ample time to reach equilibrium. Above f_{pi}, the current shift is actually greater than predicted here and increases linearly with frequency, reaching a maximum near the electron plasma frequency.

Rectification also occurs with high impedance probes measuring floating potential. Equating the ion current j_{oi} and electron current $j_e - \Delta j = \mathcal{I}_o (\frac{e\Delta V}{kT_e})$, gives:

$$j_{oi} = j_{oe} \mathcal{I}_o (\frac{e\Delta V}{kT_e}) \, e^{\frac{e(V-V_p)}{kT_e}} .$$

But the floating potential V_f in the absence of fluctuations is given by:

$$j_{oi} = j_{oe} \, e^{\frac{e(V_f - V_p)}{kT_e}} .$$

Equating the two expressions gives:

$$e^{\frac{eV_f}{kT_e}} = e^{\frac{eV}{kT_e}} \mathcal{I}_o (\frac{e\Delta V}{kT_e}) .$$

Or

$$V - V_f = - \frac{kT_e}{e} \log_e \mathcal{I}_o (\frac{e\Delta V}{kT_e}) . \quad (34)$$

Since $\mathcal{I}_o \leq 1$, $V - V_f$ is positive. Hence we see that fluctuations, which have amplitudes comparable to the electron energy, shift the probe voltage up from the floating potential in cases where the detailed character of the fluctuation cannot be followed. In effect, this process is simply a non-linear averaging.

FOOTNOTES

1. Probably the most thorough such treatment is F. F. Chen in *Plasma Diagnostic Techniques*, R. H. Huddlestone and S. L. Leonard, Eds. (Academic Press Inc., New York, 1965), Chapter 4.

2. D. Bohm, E. H. S. Burhop, and H. S. W. Massey in *Characteristics of Electrical Discharges in Magnetic Fields*, A. Guthrie and R. K. Wakerling, Eds. (McGraw-Hill Book Company, Inc., New York, 1949), Chapter 2.

3. F. F. Chen, Princeton University, Plasma Physics Laboratory Report MATT-77.

4. J. C. Sprott, Rev. of Sci. Instr. 37, 897, (1966).

5. E. O. Johnson and L. Malter, Phys. Rev. 80, 58 (1950).

6. For Example, See PLP 43.

7. PLP 48 compares these last two methods.

8. The drawing numbers refer to drawings on file at the BARN.

9. See PLP 43.

10. For a very thorough discussion, see H. H. Skilling, *Electrical Engineering Circuits*, Chapter 19.

11. J. B. Johnson, Phys. Rev. 32, 97 (1928).

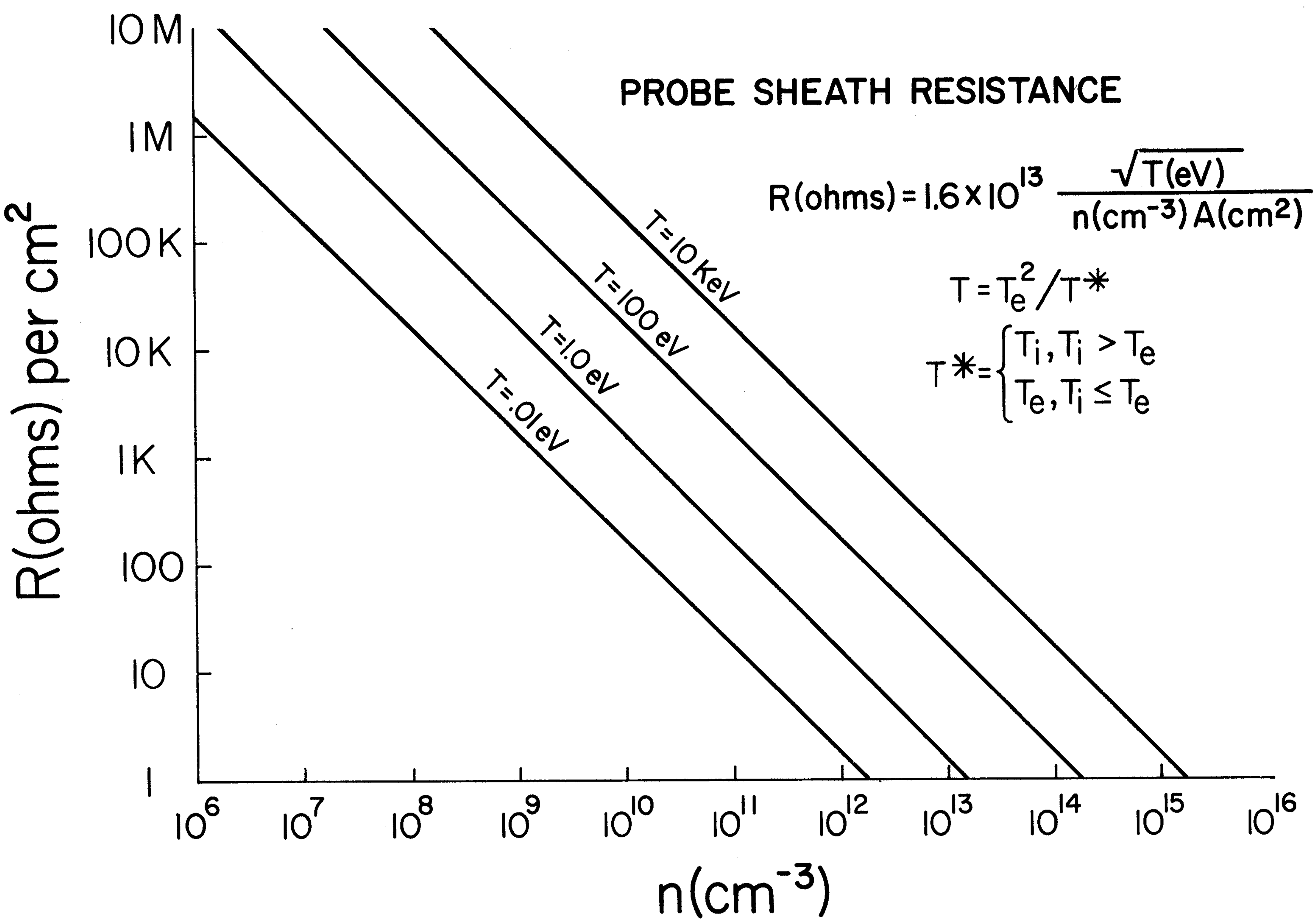
12. For a more detailed discussion, see F. F. Chen, *op cit*.

13. See PLP 61.

14. F. W. Crawford and R. Grard, Journ. Appl. Phys. 37, 180 (1965).

15. In some cases the transformer winding resistance must also be included.

16. Actually, in some cases it may be as short as $1/f_{pe}$. See F. F. Chen in Huddlestone & Leonard, page 187.


PROBE SHEATH RESISTANCE
R(ohms) = 1.6 X 10^13 √T(eV) / n(cm^-3) A(cm^2)
T = Te^2/T*
T* = { Ti, Ti > Te ; Te, Ti ≤ Te
T=10 KeV
T=100 eV
T=1.0 eV
T=.01 eV
R(ohms) per cm^2
10 M
1 M
100 K
10 K
1 K
100
10
1
10^6
10^7
10^8
10^9
10^10
10^11
10^12
10^13
10^14
10^15
10^16
n(cm^-3)