SCALING OF SYNCHROTRON RADIATION WITH MULTIPOLE ORDER

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Kerst (PLP 819) has estimated the synchrotron radiation loss for a low beta multipole of arbitrary order. This note addresses the same problem using a simpler and more general flux-space argument. Also included are some useful, numerically-derived, scaling laws for maximum field and stable flux as a function of multipole order.

The synchrotron radiated power from a volume dV containing plasma with density n, non-relativistic electron temperature T_e, and magnetic field B is given by (Rose and Clark, page 251):

$$dP = 6.2 \times 10^{-20} B^2 n T_e dV$$

All units are mks except T_e which is in eV. Averaging over a flux tube containing magnetic flux $d\psi$ gives

$$\frac{dP}{d\psi} = 6.2 \times 10^{-20} B^2 n T_e \frac{dV}{d\psi} = 6.2 \times 10^{-20} \oint B^2 n T_e \frac{d\ell}{B}$$

where $d\ell$ is a unit length parallel to $\vec{B}$.

If n and T_e are constant along a field line, the radiated power can be written as

$$\frac{dP}{d\psi} = 6.2 \times 10^{-20} n T_e \oint B d\ell$$

From Ampere's law, $\oint B d\ell$ is just μ_o times the total hoop current I_T, provided β is negligible. (For high β, one would also include the diamagnetic current enclosed by the flux surface.) Hence the total synchrotron radiated power is

$$P = 6.2 \times 10^{-20} \mu_o I_T \int nT_e d\psi$$

which can be written in a convenient, machine-independent form:

$$P = 6.2 \times 10^{-20} \mu_o I_T \langle nT_e \rangle \psi_T$$

where the brackets < > denote a flux space average, and ψ_T is the total flux in the device. This relation has been verified by numerical integration of the Larmor radiation formula for a particle trapped near the separatrix of various order multipoles.

In order to proceed further with a determination of the influence of multipole order on radiation losses, it is necessary to have relations between I_T, ψ_T, and maximum field. A numerical calculation was done for an ideal, linear multipole in which N current filaments with equal currents were placed uniformly around a circle of radius a. The maximum field on the separatrix and the flux per unit length L between the separatrix and ψ-critical were then calculated for various N. The total flux ψ_T was assumed equal to twice the flux between the separatrix and ψ-critical, in order to account for the private flux. The results, normalized to the same maximum field on the separatrix, are listed in the table on page 3, assuming a constant ψ-space distribution of electron energy density.

N		I_T(MA)/Ba	ψ_T(webers)/BaL	P(watts)/$nT_eB^2a^2L$
2	(quadrupole)	3.55	0.127	3.52×10^{-20}
3	(hexapole)	3.17	0.176	4.35×10^{-20}
4	(octupole)	3.00	0.177	4.14×10^{-20}
5		2.92	0.168	3.82×10^{-20}
6	(dodecapole)	2.85	0.157	3.48×10^{-20}
8		2.79	0.137	2.98×10^{-20}
10		2.75	0.126	2.69×10^{-20}
20		2.70	0.080	1.68×10^{-20}
50		2.62	0.041	0.84×10^{-20}
	(tokamak)			19.48×10^{-20}

The tokamak case was calculated simply from

$$P = 6.2 \times 10^{-20} B^2 n T_e \pi a^2 L$$

and so it is representative of any circular cross-section device in which the magnetic field is essentially uniform. Low order multipoles have typically 5 to 10 times less synchrotron radiation than the corresponding tokamak, and the advantage increases linearly with order at very high order. An additional improvement of $\langle nT_e \rangle_\psi / \langle nT_e \rangle_V$ could be expected if the ψ-space averaged electron density in a multipole is smaller than the volume-averaged electron energy density in a tokamak. A first order relativistic correction can be included by replacing T_e with $T_e(1 + T_e/204000)$ everywhere. Finite β and reabsorption of the radiation will, of course, alter these conclusions.