A MULTICOMPONENT PLASMA POWER BALANCE CALCULATION

J.C. Sprott

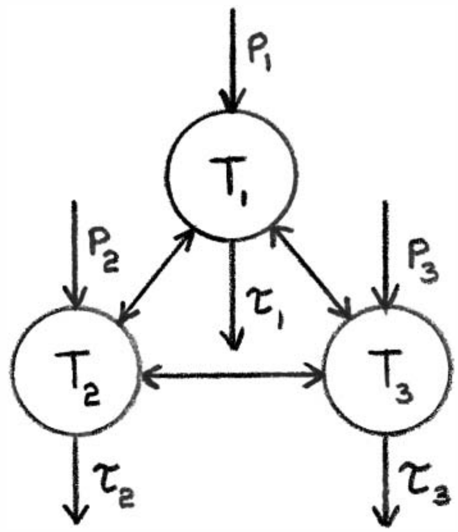
PLP 782

February 1979

Plasma Studies

University of Wisconsin

This note describes a computer code (MSHEX) which solves the steady state power balance equations for a multicomponent plasma of given density, in which the various species interact with one another by coulomb collisions. It was originally written to study minority species ion cyclotron resonance heating, but it also has application in calculating impurity temperatures, and in studying neutral beam heating and alpha particle slowing down. A three-component case is shown schematically below:



The input variables are the density of each specie (n), the input power to each specie (P), the energy confinement time for each specie (τ), and the atomic mass number (M) and charge (Z) of each specie. The output is the steady state temperature of each specie (T).

The equations that are solved simultaneously are:

$$P_i - \frac{1.5 n_i e T_i V}{\tau_i} - \sum_{j=1}^{3} \frac{1.5 n_i e (T_i - T_j) V}{\tau_{ij}} = 0$$

where

$$\tau_{ij} = 4.8 \times 10^5 \; \frac{(M_i T_j + M_j T_i)^{1.5}}{\sqrt{M_i M_j}\; n_j Z_i^2 Z_j^2}$$

and i and j are indices which denote one of the three species. V is the volume of the plasma. Units are cm^{-3}, eV, and sec.

The first case examined was a Tokapole II plasma with $n = 10^{13}$ cm^{-3} and a 1% He^{+} impurity. With τ = 300 μsec for each specie, the electron, ion, and helium temperatures are plotted vs ohmic heating power in Fig. 1. The results agree well with the experiment if the ohmic input power is ~ 400 kW. The helium and hydrogen ions are at essentially the same temperature, so that Doppler broadening of He II ought to give a valid ion temperature. In Fig. 2, the ohmic heating power is held constant at 400 kW, and ICRH power is added to the hydrogen. If the confinement time of each specie is constant (300 μsec), the hydrogen decouples from the He above ~ 50 eV, and the Doppler broadening technique is no longer suitable as an ion temperature diagnostic. The result is insensitive to the He density. If an impurity specie with a higher Z^2/M (such as $^{12}C^{++}$) is used, the technique works to higher temperatures.

Fig. 3 shows what would be expected for a minority specie heating experiment in Tokapole II (95% D, 5% H). Such an experiment should work well, and high minority ion temperatures ought to be achieved with modest power levels. Doppler broadening of impurities would be useless as a diagnostic, however.

Fig. 4 shows the result applied to the Oak Ridge EBT-I device with 99% D and 1% H as a function of ECRH power. The temperatures are in reasonable agreement with the experiment if the absorbed ECRH power is the order of 10 kW. Holding the ECRH power constant at 10 kW and turning up the ICRH power gives the result shown in Fig. 5. With an ICRH power of only 1 kW, the minority ion temperature should reach 4 keV even if the confinement time remains constant (15 msec) rather than increases as neoclassical theory would predict. Such an experiment should provide an excellent test of neoclassical confinement well into the collisionless ion regime. A final case is shown in Fig. 6 where the parameters are chosen to more nearly fit the EBT-S (scaled up EBT-I) device. The result is essentially the same except that a higher ICRH power is required.

The effect of charge exchange losses on the EBT minority specie heating experiment was assessed by adding to the two ion power balance equations a term given by

$$P_{cx} = \frac{1.5 n_i e T_i V}{\tau_{cx}}$$

where

$$\frac{1}{\tau_{cx}} = 4.9 \times 10^{-11}\, n_o \sqrt{T_i}\, (1 + 0.00585\, T_i^{1.5})\, e^{-0.0582\sqrt{T_i}}$$

and n_o is the neutral density (assumed H_2). The result for EBT-S with various values of n_o is shown in Fig. 7. Charge exchange is not a serious consideration for $n_o \lesssim 10^{10}$ cm^{-3}.

A listing of the code is included in the appendix. It makes use of the MACC nonlinear simultaneous equation-solving routine, ZRNEQ. Convergence is rapid, and the cost is low.

```
*   *   *   *   UNIVAC 1110 TIME/SHARING EXEC --- MULTI-PROCESSOR SYSTEM --- VER.  MACC
 CD0536        PROJECT: 02980          USER: 4126810219            CREATED ON FEB 17, 1979
     FILE NAME: PP*000CD0536             PART NUMBER: 00           PRINTED ON FEB 17, 1979



SPROTT,2980,4126810219
S ,MSHEX
N-MACC 1.17S-02/17/79-13:00:01                     MSHEX
          C        PROGRAM MSHEX  -  J.C. SPROTT  -  FEB 10, 1979
                   DIMENSION TINIT(3),T(3),WORK(48)
                   COMMON P(3),DENS(3),TAU(3),AMU(3),Z(3),V
                   EXTERNAL AUXFCN
                   DO 300 I=1,3
                   TAU(I)=3.0E-4
                   Z(I)=1.0
             300   TINIT(I)=10.0
                   F=0.01
                   DENS(1)=1.0E13
                   DENS(2)=(1.0-F)*Z(1)*DENS(1)/((1.0-F)*Z(2)+F*Z(3))
                   DENS(3)=F*Z(1)*DENS(1)/((1.0-F)*Z(2)+F*Z(3))
                   AMU(1)=1.0/1836.0
                   AMU(2)=1.0
                   AMU(3)=4.0
                   V=5.0E5
                   LA='MSHEX '
                   DO 400 I=1,3
                   WRITE(6,350) I,AMU(I),Z(I),DENS(I),TAU(I)
             350   FORMAT(1H ,'       SPECIE ',I1,':    MASS =',F9.6,'    Z =',F4.1,'
                  2DENS =',1PE8.2,'    TAU =',1PE8.2)
             400   CONTINUE
                   WRITE(6,450)
             450   FORMAT(1H0,'      IP    IERR    P1(WATTS)    P2(WATTS)    P3(WATTS)
                  2T1(EV)        T2(EV)         T3(EV)')
                   P(1)=4.0E5
                   P(3)=0.0
                   DO 900 IP=1,26
                   P(2)=10.0*1.5848932**(IP-1)
                   CALL ZRNEQ(TINIT,AUXFCN,3,1.0E-8,3,100,T,0,LA,WORK,IERR,$900)
                   WRITE(6,500) IP,IERR,P(1),P(2),P(3),T(1),T(2),T(3)
             500   FORMAT(1H ,2I6,6F12.1)
             900   CONTINUE
                   END

 END OF COMPILATION:          NO  DIAGNOSTICS.
S   ,AUXFCN
N-MACC 1.17S-02/17/79-13:00:05                     AUXFCN
                   FUNCTION AUXFCN(T,M)
                   DIMENSION T(1)
                   COMMON P(3),DENS(3),TAU(3),AMU(3),Z(3),V
                   AUXFCN=0.0
                   DO 500 N=1,3
                   TEQ=4.8E5*(ABS(AMU(M)*T(N)+AMU(N)*T(M)))**1.5/SQRT(AMU(M)*AMU(N))
                  2/DENS(N)/(Z(M)*Z(N))**2
             500   AUXFCN=AUXFCN+(T(M)-T(N))/TEQ
                   AUXFCN=P(M)-2.4E-19*DENS(M)*V*(T(M)/TAU(M)+AUXFCN)
                   RETURN
                   END
```

END OF COMPILATION: NO DIAGNOSTICS.

(N

2 RLIB35 02/17-13:00:08

SPECIE 1: MASS = .000545 Z = 1.0 DENS = 1.00+13 TAU = 3.00-04
SPECIE 2: MASS = 1.000000 Z = 1.0 DENS = 9.90+12 TAU = 3.00-04
SPECIE 3: MASS = 4.000000 Z = 1.0 DENS = 1.00+11 TAU = 3.00-04

P	IERR	P1(WATTS)	P2(WATTS)	P3(WATTS)	T1(EV)	T2(EV)	T3(EV)
1	5	400000.0	10.0	.0	86.8	13.2	12.8
2	5	400000.0	15.8	.0	86.8	13.2	12.8
3	5	400000.0	25.1	.0	86.8	13.2	12.8
4	5	400000.0	39.8	.0	86.8	13.2	12.8
5	5	400000.0	63.1	.0	86.8	13.2	12.8
6	5	400000.0	100.0	.0	86.8	13.2	12.8
7	5	400000.0	158.5	.0	86.8	13.2	12.8
8	5	400000.0	251.2	.0	86.8	13.2	12.8
9	5	400000.0	398.1	.0	86.8	13.3	12.9
0	5	400000.0	631.0	.0	86.8	13.3	12.9
1	5	400000.0	1000.0	.0	86.9	13.4	13.0
2	5	400000.0	1584.9	.0	86.9	13.5	13.1
3	5	400000.0	2511.9	.0	86.9	13.7	13.3
4	5	400000.0	3981.1	.0	87.0	14.0	13.5
5	5	400000.0	6309.6	.0	87.1	14.5	14.0
6	5	400000.0	10000.0	.0	87.2	15.3	14.7
7	5	400000.0	15848.9	.0	87.4	16.5	15.8
8	5	400000.0	25118.9	.0	87.8	18.5	17.5
9	5	400000.0	39810.7	.0	88.4	21.6	20.1
0	5	400000.0	63095.7	.0	89.2	26.6	24.0
1	5	400000.0	100000.0	.0	90.6	34.5	29.7
2	5	400000.0	158489.3	.0	92.6	47.1	37.5
3	5	400000.0	251188.7	.0	95.6	67.4	46.9
4	5	400000.0	398107.2	.0	100.0	100.0	56.6
5	5	400000.0	630957.4	.0	106.1	152.5	64.0
6	5	400000.0	1000000.1	.0	114.4	237.3	66.6

NID: CD0536   PROJECT: 02980   USER: 4126810219

	AMOUNT	COST
ME	00:00:03.224	$0.06
/O REQUESTS	327	$0.07
/O WORDS	269769	$0.02
EQUESTS	1	$0.00
ORDS	3584	$0.00
USAGE	0.089	$0.04
IN	51	$0.03
PRINTED	1	$0.01
FILES	1	$0.06
RE SUPPORT	14	$0.12
ARGE	1	$0.03
COST		$0.44

E ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR OVERNIGHT RUNS

NO. 340-L35 DIETZGEN GRAPH PAPER
LOGARITHMIC
3 CYCLES X 5 CYCLES

DIETZGEN CORPORATION
MADE IN U.S.A.

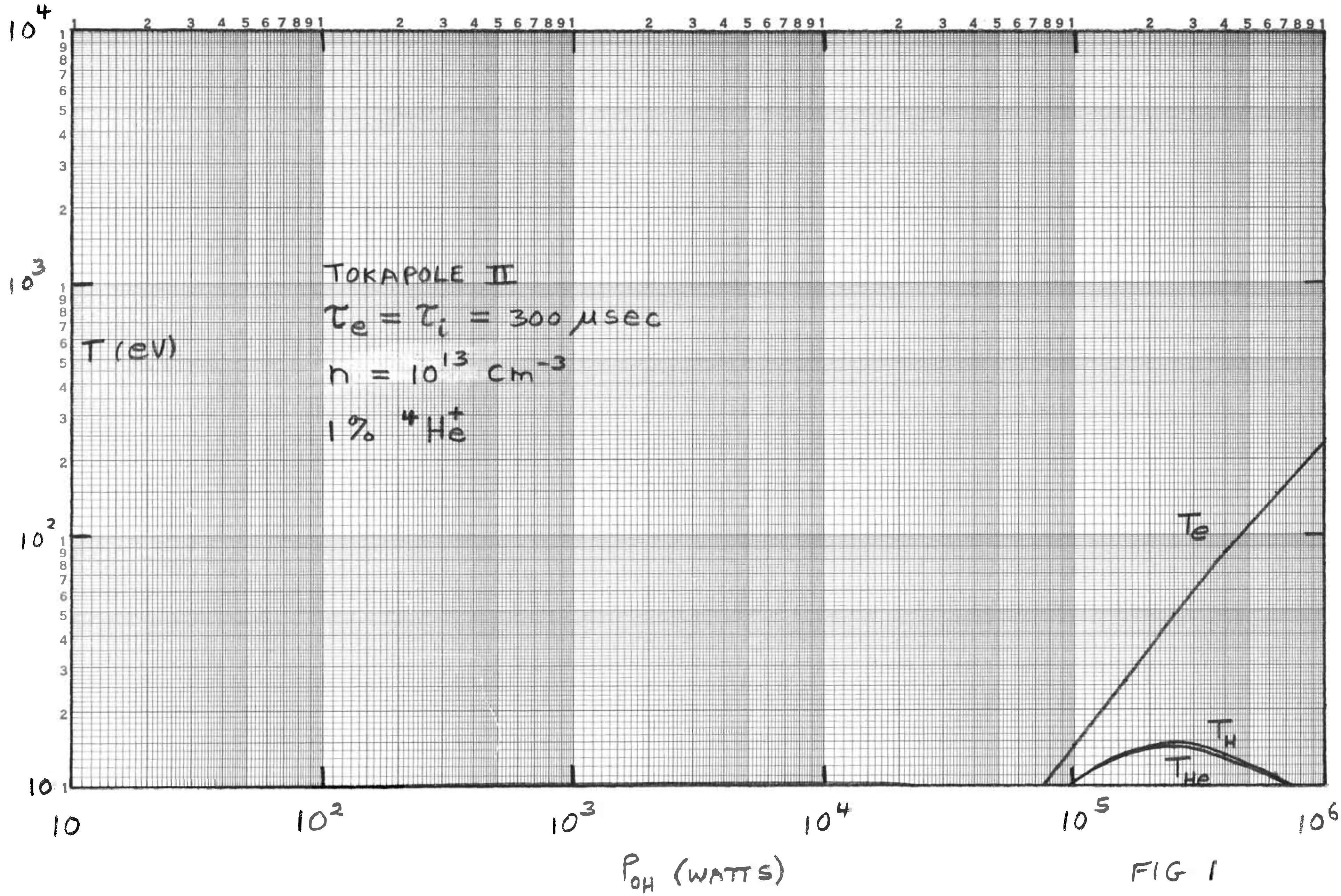


FIG 1

NO. 340-L35 DIETZGEN GRAPH PAPER
LOGARITHMIC
3 CYCLES X 5 CYCLES

DIETZGEN CORPORATION
MADE IN U.S.A.

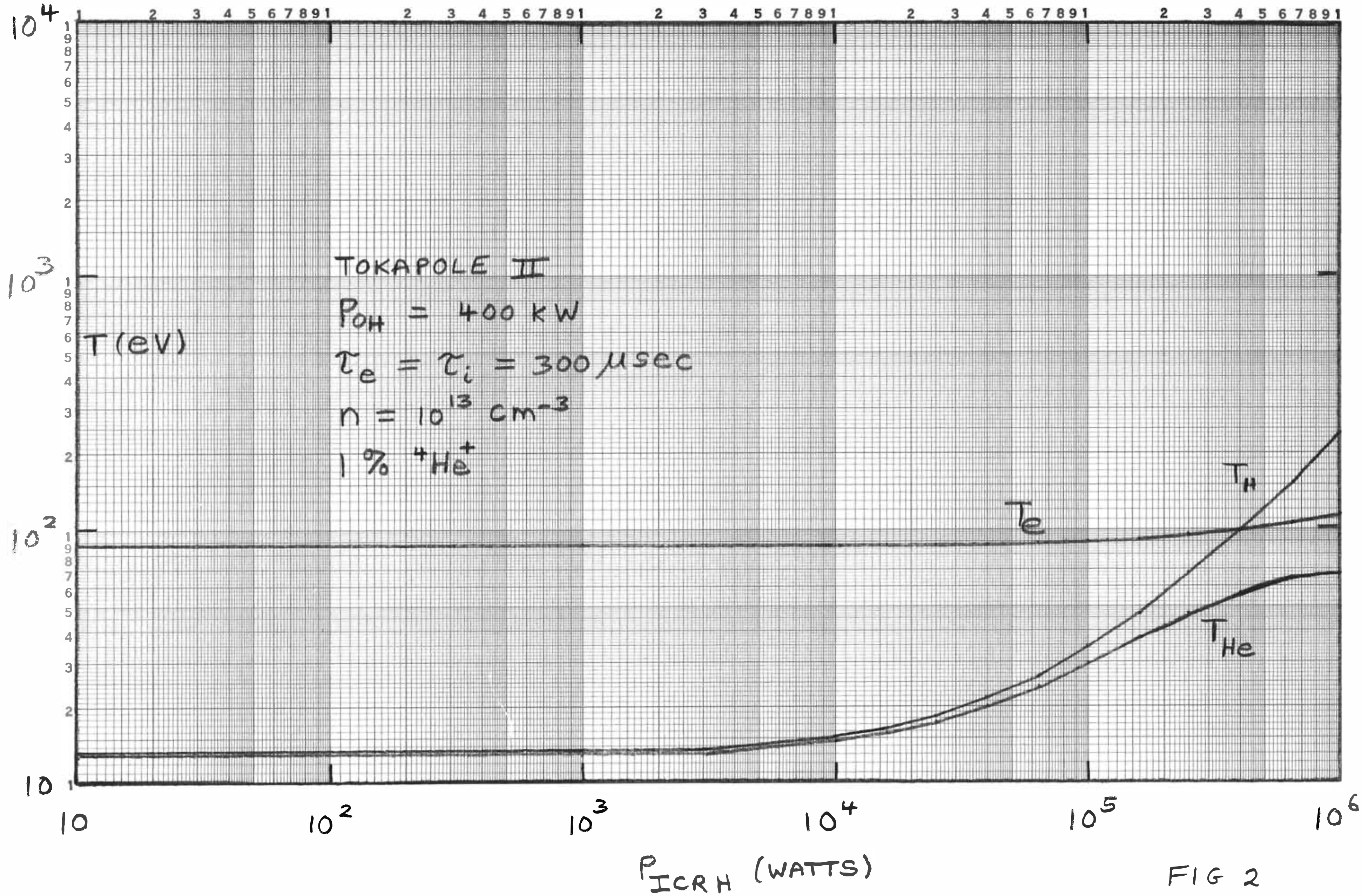


FIG 2

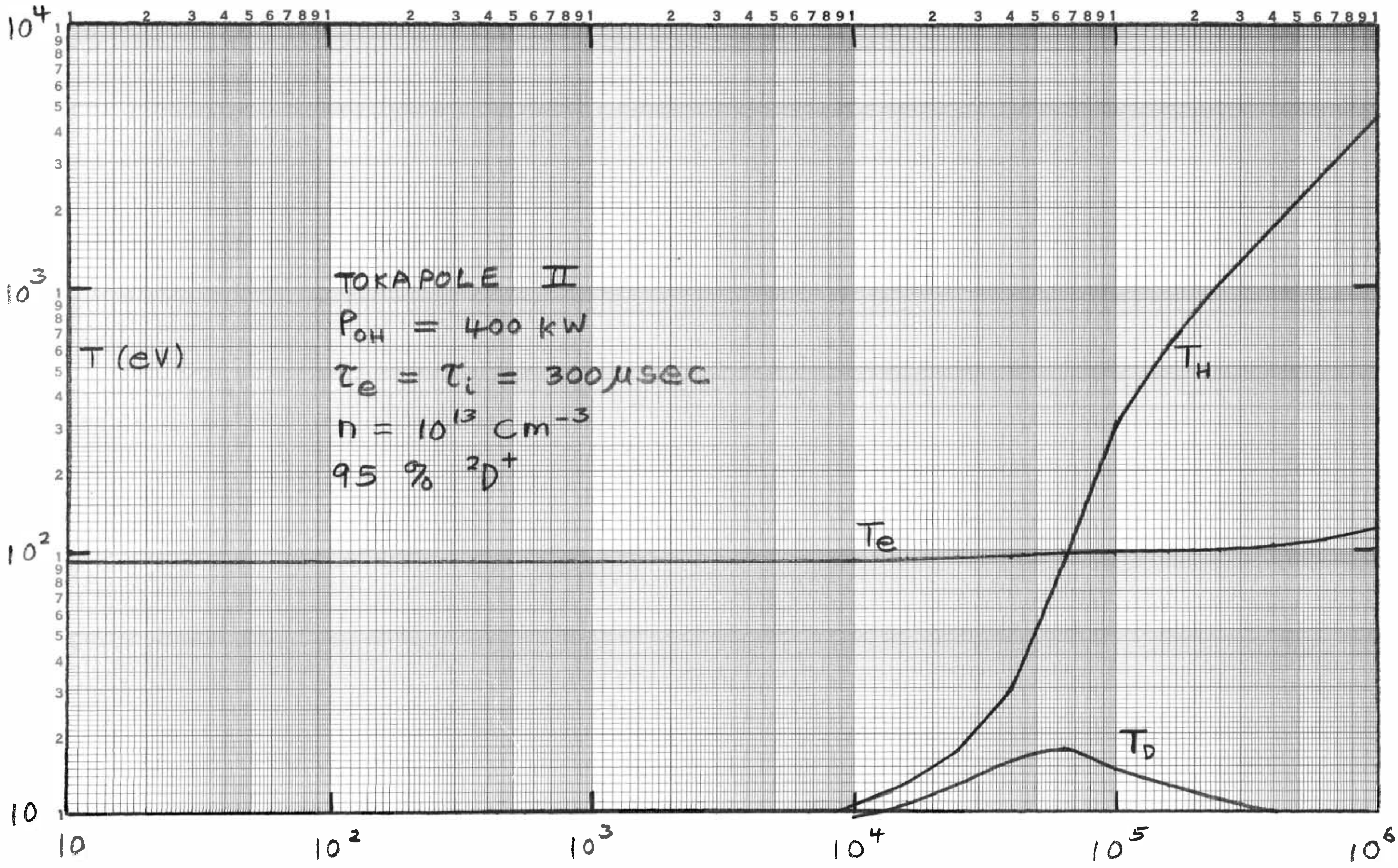

TOKAPOLE II
P_OH = 400 kW
τ_e = τ_i = 300 μsec
n = 10^13 cm^-3
95 % ²D⁺
T (eV)
T_H
T_e
T_D
10^4
10^3
10^2
10
10^5
10^6
P_ICRH (WATTS)

FIG 3

NO. 340-L35 DIETZGEN GRAPH PAPER
LOGARITHMIC
3 CYCLES X 5 CYCLES

DIETZGEN CORPORATION
MADE IN U.S.A.

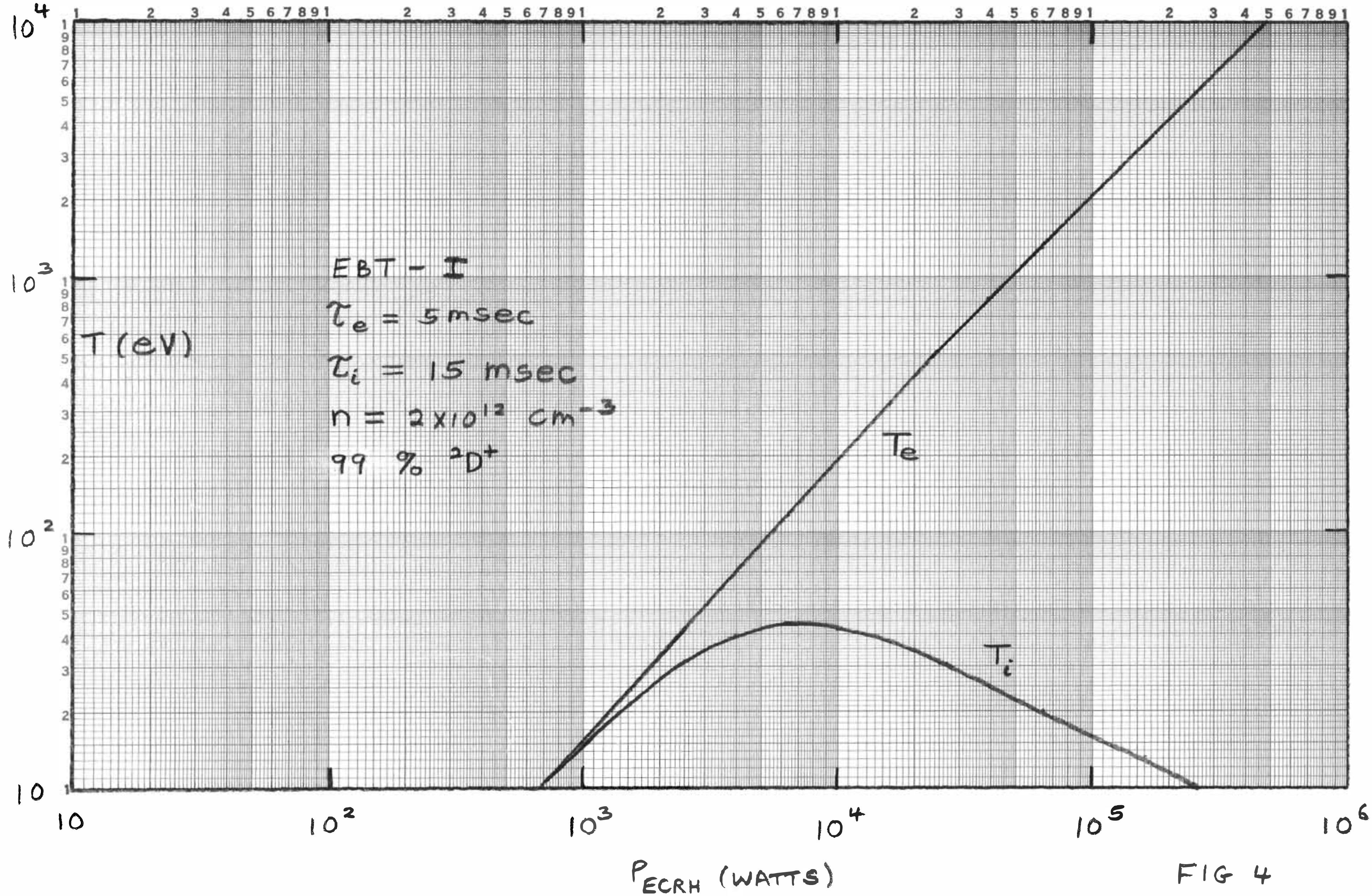


FIG 4

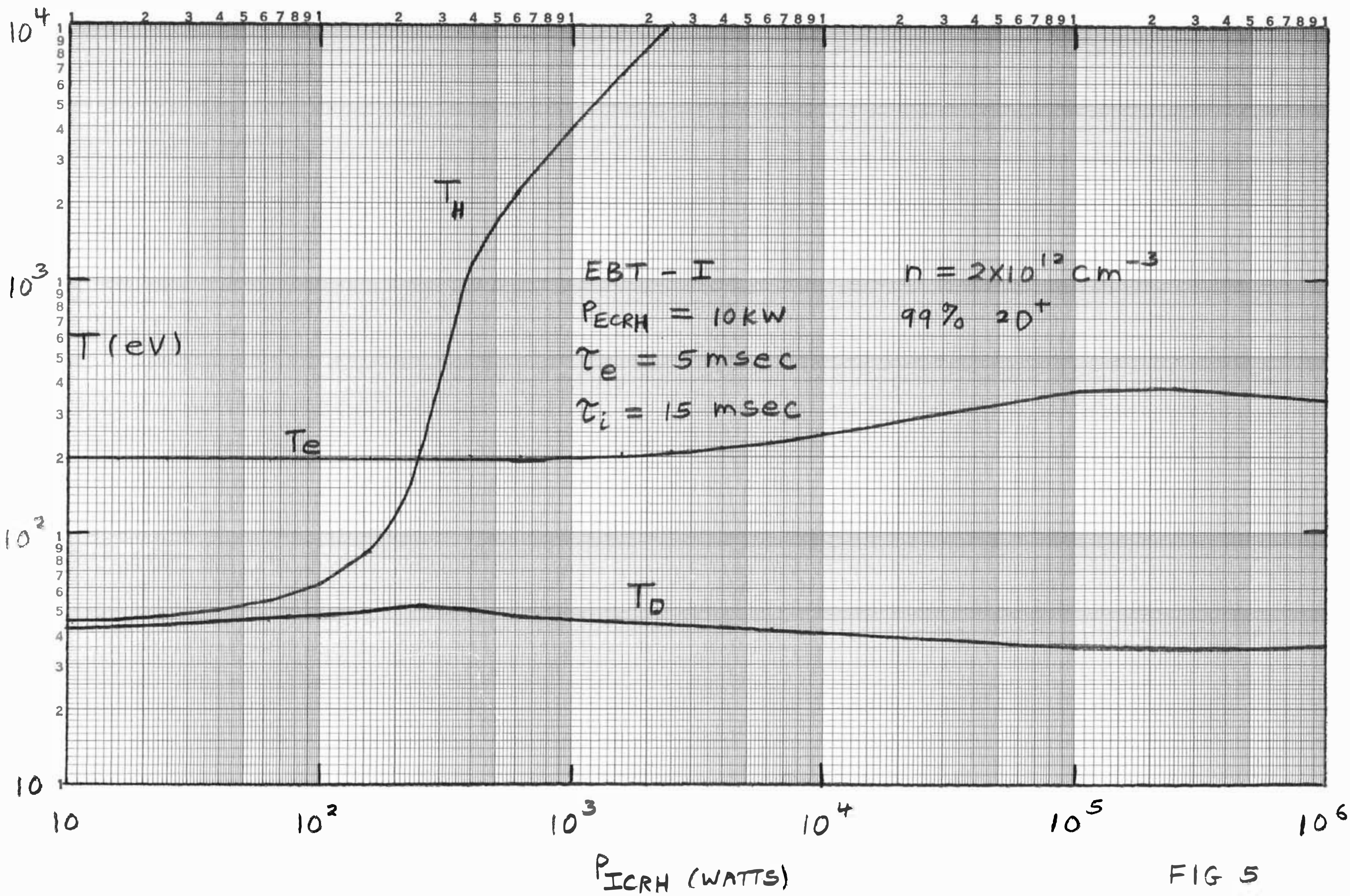

T_H
EBT - I
$n = 2 \times 10^{12}\ cm^{-3}$
$P_{ECRH} = 10KW$
99% $2D^+$
$\tau_e = 5\ msec$
$\tau_i = 15\ msec$
T (eV)
T_e
T_D
10^4
10^3
10^2
10
10
10^2
10^3
10^4
10^5
10^6
P_{ICRH} (WATTS)

FIG 5

NO. 340-L35 DIETZGEN GRAPH PAPER
LOGARITHMIC
3 CYCLES X 5 CYCLES

DIETZGEN CORPORATION
MADE IN U.S.A.

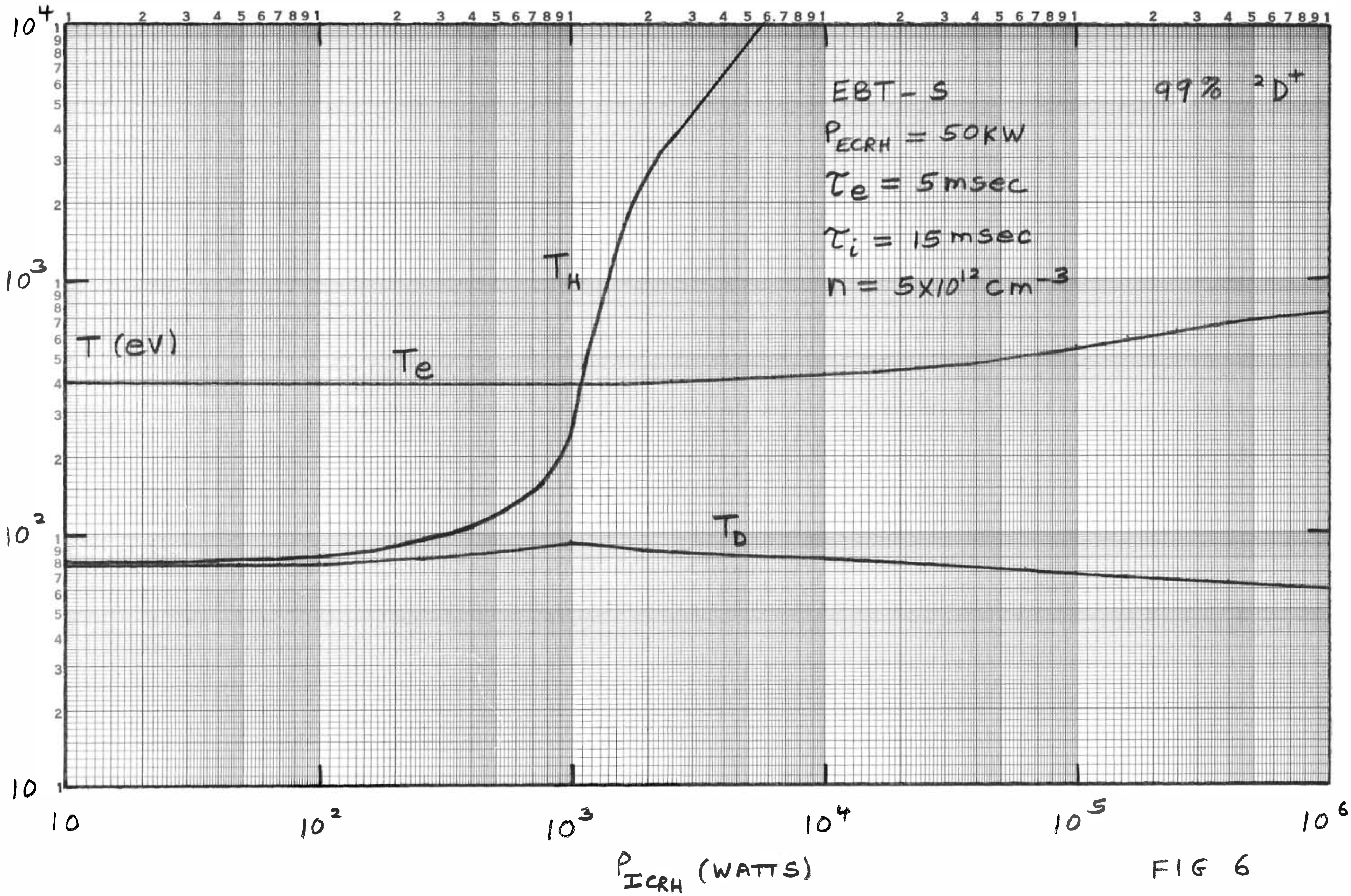


FIG 6

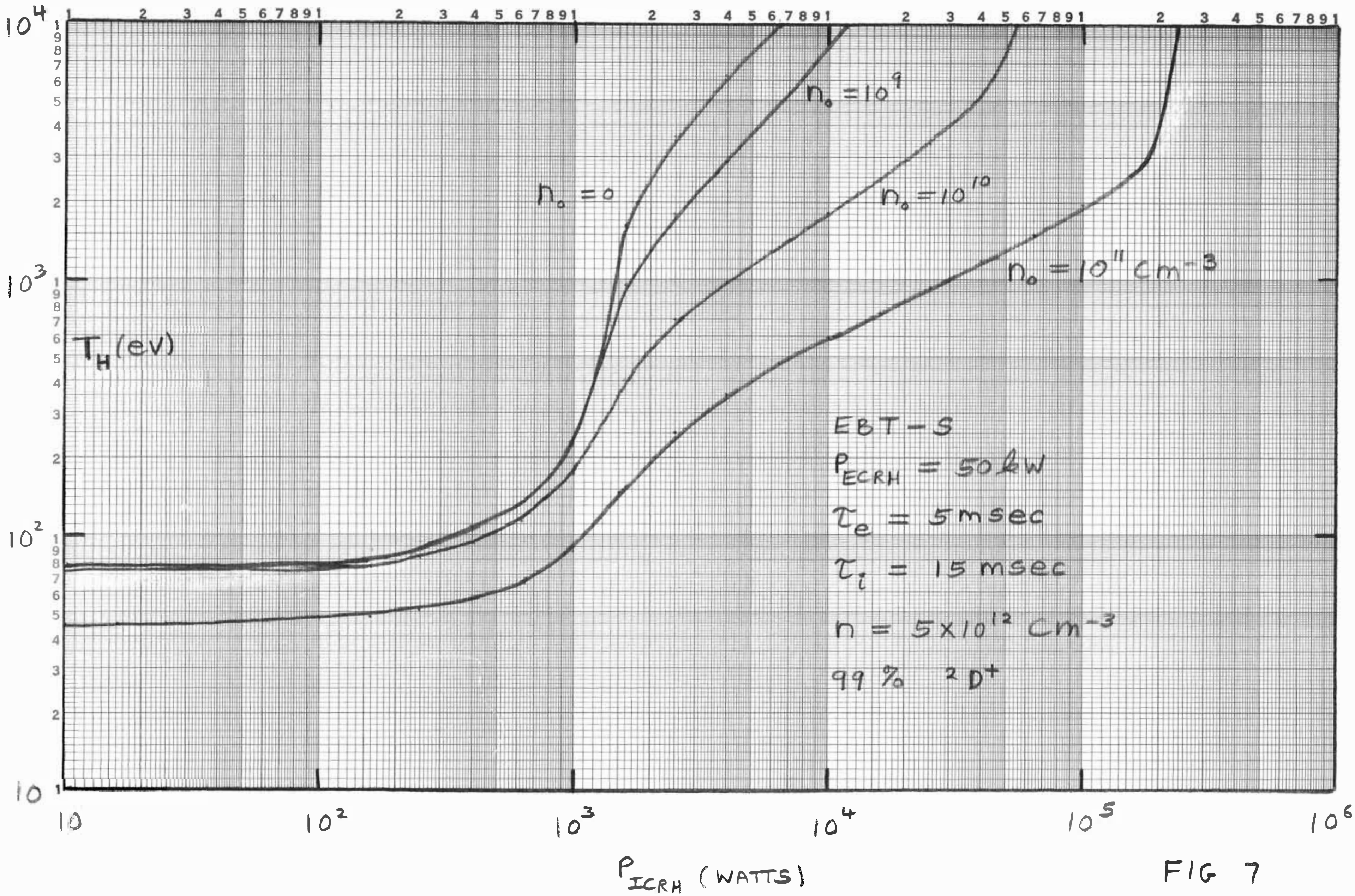

10^4
10^3
10^2
10
T_H (ev)
10
10^2
10^3
10^4
10^5
10^6
P_{ICRH} (WATTS)
$n_o = 0$
$n_o = 10^9$
$n_o = 10^{10}$
$n_o = 10^{11}$ cm^{-3}
EBT-S
$P_{ECRH} = 50$ kW
$\tau_e = 5$ msec
$\tau_i = 15$ msec
$n = 5 \times 10^{12}$ cm^{-3}
99 % $^2D^+$

FIG 7