HOW TO BUILD A BETTER BUMPY TORUS

J. C. SPROTT

Plasma Studies

University of Wisconsin

September 1978

PLP 760

HOW TO BUILD A BETTER BUMPY TORUS

J. C. Sprott

INTRODUCTION

This note will summarize the results of experiments on the Oak Ridge Elmo Bumpy Torus (EBT) device, outline the steps that Oak Ridge is proposing to scale up to an eventual EBT reactor, and discuss an alternate approach for reaching breakeven conditions in a much more expeditious manner. It will be shown that a very modest level of ion cyclotron resonance heating power applied to a small EBT device has the potential of realizing quite spectacular temperatures and confinement times.

THE EBT-I DEVICE

The first (and so far the only) EBT device has been operating at Oak Ridge for about five years. The characteristics of the device are as follows:

Major radius: R = 150 cm

Minor radius (coil throat): a = 10 cm

Plasma volume: $V = 5 \times 10^5\ cm^3$

Number of bumps: N = 24

Mirror ratio (on axis): M = 2.0

Magnetic field strength (throat): B = 10 kG

Bulk ECRH heating: P_B = 60 kW @ 18 GHz

Profile ECRH heating: P_P = 20 kW @ 10.6 GHz

The vacuum chamber is also a microwave cavity with aluminum walls and oil diffusion pumps capable of producing a base vacuum of $\sim 1 \times 10^{-7}$ torr. The magnetic fields and microwave sources are continuous.

THE EBT-I PLASMA

Over a narrow interval of hydrogen gas pressure in the 10^{-6} torr range, high beta (∿50%), mirror confined, hot electron (∿100 keV) annuli are observed to form near the mirror midplanes where the second harmonic of the cyclotron frequency coincides with the bulk ECRH heating frequency (i.e., at B = 3.2 kG). Coincident with the formation of the annuli, the bulk toroidally confined plasma enters a quiescent condition (T-mode) in which the diamagnetic field of the annuli apparently provides

MHD stability for the bulk plasma.

Typical parameters which characterize the plasma in this stable regime are listed below:

Density: $n = 2 \times 10^{12}\ \text{cm}^{-3}$

Plasma volume: $V = 5 \times 10^{5}\ \text{cm}^{3}$

Electron temperature: $T_e = 150$ eV

Ion temperature: $T_i = 50$ eV

Confinement time: $\tau = 0.02$ sec

THEORY OF NEOCLASSICAL DIFFUSION IN A BUMPY TORUS

Much theoretical effort has gone into refinements of the neoclassical transport theory for a bumpy torus. The theory predicts two distinct regimes in which the transport scaling is very different. In the collisional regime, particles collide before undergoing an entire drift orbit, and the confinement time is predicted to scale according to

$$\tau = C_1 a^2 B^2 R^2 n\, T_i^{-7/2} . \qquad (1)$$

In the collisionless regime, particles undergo many drift orbits before colliding, and the confinement time is predicted to scale according to

$$\tau = C_2 R^2 T_i^{3/2} / a^2 n . \qquad (2)$$

The above equations assume a constant mirror ratio. A mirror ratio of about

2.0 is believed to be optimum for formation of the annuli, although there is no direct experimental evidence for this in a torus. Determination of the constants C_1 and C_2 above is the subject of much of the EBT theory effort. However, the claim is that EBT-I with the parameters listed earlier is in reasonable agreement with the best collisional regime neoclassical predictions. Rather than rely on the theoretical estimates of C_1, we will assume the theoretical scaling to be correct, and use the empirical values of the plasma parameters to determine C_1 with the result:

$$C_1 \; = \; 3.9 \times 10^{-23} \text{ sec -eV}^{7/2} - \text{cm}^3/\text{G}^2 \quad . \qquad (3)$$

It is also claimed that EBT-I is close to the collisionless regime, although just how close is difficult to determine. Suppose we assume that at $n = 2 \times 10^{12}$ cm^{-3}, another factor of two in ion temperature is required for the ions to become collisionless (i.e., T_i = 100 eV). This is about the value predicted by theory. We can then estimate C_2 by equating equations (1) and (2) with T_i = 100 eV to get

$$C_2 \; = \; 1.56 \times 10^4 \text{ sec/cm}^3/\text{eV}^{3/2} \quad . \qquad (4)$$

Although the values above are not very precise, they do provide a basis for judging how a second generation device is likely to perform. The crucial question, of course, is whether the transition to the much more favorable collisionless scaling will in fact occur, and whether it will hold up to reactor conditions. The goal of the next generation device, then, should be to push as far as possible into the collisionless regime.

ELECTRON CYCLOTRON RESONANCE HEATING

The basic EBT relies on ECRH to heat the electrons, and thus indirectly the ions, to reach the collisionless regime. If the electrons are heated to a temperature T_e, the ion temperature can be determined from the ion power balance equation:

$$\frac{T_i}{\tau} = \frac{T_e - T_i}{\tau_{ei}} \tag{5}$$

where τ_{ei} is the electron-ion equilibration time,

$$\tau_{ei} = 2 \times 10^7 \, T_e^{3/2}/n \ . \tag{6}$$

Examination of equation (5) shows that there is an optimum electron temperature which will produce the maximum ion temperature. This optimum can be determined by using τ from equation (1) and differentiating equation (5) with respect to T_e. Setting $dT_i/dT_e = 0$ gives the following relation:

$$T_e = 3T_i \ . \tag{7}$$

It is interesting to note that this is about the ratio of T_e/T_i observed in EBT-I. Now we can solve equation (5) for T_i using the optimum value of T_e, with the result:

$$T_i = 7 \times 10^{-6} (aBn)^{2/5} \tag{8}$$

The prediction for EBT-I is T_i = 58 eV, in remarkable agreement with the observed value. The ECRH power required to sustain the plasma at this temperature is

$$P = \frac{en(T_i + T_e)V}{\tau} = \frac{4eT_i^{9/2}V}{C_1 a^2 B^2 R^2} \quad . \tag{9}$$

For EBT-I in the collisional regime, the required power is 3.1 kW which is about the minimum power required in the experiment to achieve the advertised parameters when the coupling efficiency (∿20%) and other non-diffusive losses are taken into account. Higher ECRH power is expected to increase T_e, but T_i actually decreases slightly as the microwave power is increased.

EBT-S

The next EBT experiment to be done by Oak Ridge is one in which the magnetic field is increased to 13 kG and a new 200 kW, 28 GHz bulk heating source is used with the present EBT-I device. We can use the above scaling laws to predict the parameters that would be expected in such a device. First, we expect the density to rise proportionately with the square of the bulk ECRH frequency, so that the expected density is $5 \times 10^{12} cm^{-3}$. The ion temperature required to reach the collisionless regime is

$$T_i = \left[\frac{C_1 a^4 B^2 n^2}{C_2}\right]^{1/5} \quad , \tag{10}$$

which has a value of 160 eV. On the other hand, the largest ion temperature that can be expected from ECRH as given by ion equation (8) is 93 eV. The power required to get there as given by equation (9) is 15 kW. Hence the EBT-S should nearly double the ion temperature, but it gets no closer to the collisionless regime because of the increase in density.

EBT-SA

A second experiment planned by Oak Ridge is the addition of aspect ratio enhancement (ARE) coils to EBT-S to increase the effective aspect ratio. It is hard to predict what improvement in the confinement will be obtained, but we can turn the question around and ask what effective aspect ratio is required to reach the collisionless regime. Equating equations (1) and (2) gives

$$\frac{R}{A} = aBRn \sqrt{C_1 / C_2 \, T_i^5} \quad , \qquad (11)$$

which for T_i = 93 eV implies an aspect ratio of 58. It is questionable whether an effective aspect ratio of such a value can be obtained, but it is certainly a worthy experiment.

EBT-II

The next major step proposed by Oak Ridge in the EBT program is a larger, higher field, superconducting device called EBT-II with the following characteristics:

Major radius: $R = 488$ cm

Minor radius (coil throat): $a = 10$ cm

Plasma volume: $V = 1.6 \times 10^6$ cm^3

Number of bumps: $N = 48$

Mirror ratio (on axis): $M = 2.25$

Magnetic field strength (throat): $B = 60$ kG

Bulk ECRH heating: P_B = 2 MW @ 120 GHz

The density consistent with the ECRH frequency is $\sim 1 \times 10^{14}$ cm^{-3}. If the neoclassical scaling laws are applied to this device, the ion temperature required to reach the collisionless regime is 980 eV. From equation (8), the largest ion temperature that can be expected from ECRH is 570 eV. The power required to get there as given by equation (9) is 770 kW. So despite having built a considerably more expensive device, one is still a factor of two in temperature below that required to reach the collisionless regime with ECRH alone. However, the density is sufficiently high that neutral beams could be used to heat the ions to the required 980 eV. We can calculate the beam power required to reach the collisionless regime from

$$P = \frac{neT_iV}{\tau} = \frac{7a^2n^2eV}{10C_2R^2\sqrt{T_i}} \quad , \qquad (12)$$

which for EBT-II is 1.5 MW. A beam that delivers 1.5 MW to the ions will probably not push the state of the art in beams by the time EBT-II would be operational, but it won't be cheap either. Any other mechanism (such as ICRH) that is capable of delivering 1.5 MW to the ions ought to achieve the same result.

ION CYCLOTRON RESONANCE HEATING

Although EBT-II has a good chance of reaching the collisionless regime with multimegawatts of 120 GHz microwaves plus multimegawatts of neutral beams, a better strategy appears to be the application of a modest level of ICRH to a small EBT device.

As an example, suppose we take the existing EBT-I and ask how much ICRH power delivered to the ions would be required to reach the collisionless regime. From equation (12) the answer is a mere 6.4 kW. The ion temperature obtained as a function of ICRH power can be calculated from

$$P = \frac{neT_iV}{\tau} = \frac{neV}{C_1a^2B^2R^2nT_i^{-9/2} + C_2R^2\sqrt{T_i}/a^2n} , \qquad (13)$$

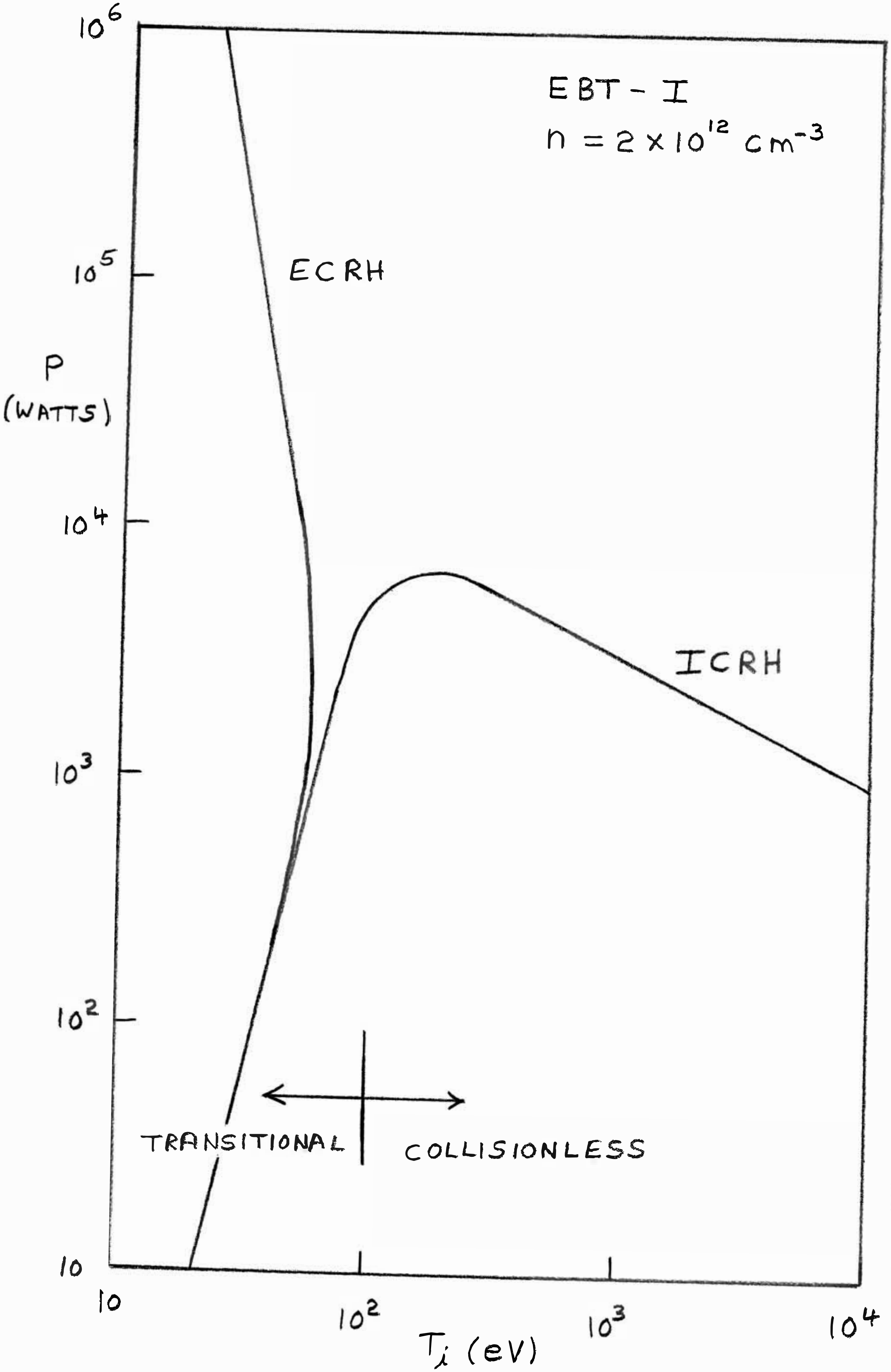
which for EBT-I parameters gives the curve labeled "ICRH" in figure 1. The interesting feature of this curve is its double-valuedness. What this means is that once the ions get into the collisionless regime, they thermally run away until stopped by some mechanism not considered here. One such mechanism is the finite gyroradius modification of the drift orbits. It is certainly reasonable to assume that neo-classical scaling will hold up to the temperature at which there are about 10 gyro-diameters across the plasma, which corresponds to an ion temperature of 10 keV for protons. By comparison, the ion temperature that can be obtained using ECRH alone can be calculated from the following transcendental equation:

$$\frac{T_i}{\tau} = \frac{T_e - T_i}{\tau_{ei}} , \qquad (14)$$

where T_e is given by

$$T_e = \frac{P\tau}{neV} . \qquad (15)$$

The solution of equation (14) for EBT-I parameters is shown in the curve labelled "ECRH" in figure 1. As mentioned before, the curve shows that one can never reach the collisionless regime with ECRH alone, and the maximum obtainable ion temperature is 58 eV. The electron temperature, however, continues to rise with increasing ECRH


EBT - I
n = 2 x 10^12 cm^-3
ECRH
ICRH
P
(WATTS)
10^6
10^5
10^4
10^3
10^2
10
10
10^2
10^3
10^4
T_i (eV)
TRANSITIONAL
COLLISIONLESS

power. If the ECRH is removed and ICRH is used to sustain the bulk plasma, the electron temperature stays very close to the ion temperature in the collisional regime, but in the collisionless regime, the ratio $F = T_e/T_i$ depends only on the aspect ratio:

$$F^{5/2} = \frac{(1 - F)R^2}{1282a^2} \quad . \tag{16}$$

For, EBT-I the solution is F = 0.4, and so the electrons will have a temperature ∿40% of the ions. The electron temperature will be even lower if neoclassical diffusion is not the dominant electron energy loss mechanism. In an ion cyclotron heated EBT reactor, one might want to avoid very high aspect ratios so that ignition conditions could be achieved at the lowest possible electron temperature in order to reduce losses due to impurity radiation, Bremsstrahlung, and synchrotron radiation. This would involve an economic tradeoff, since the lower aspect ratio would degrade the confinement and require higher ICRH power levels to reach the collisionless regime.

COMPARISON WITH TOKAMAKS

The results of the previous section admittedly sound too good to be true. Not only can one reach the collisionless regime with very modest ICRH power, but once reached, the ion temperature should continue to rise to reactor levels. It is instructive to digress for a moment to see why an EBT behaves so favorably compared with a tokamak. First, a tokamak does not work well at low densities because the electrons tend to run away in temperature and the ion heating is poor. The high densities mean that the plasma remains collisional to a higher temperature than in a low density bumpy torus. This means that much larger powers are required to reach the collisionless regime in a tokamak because of the higher densities and higher required temperatures. Second, ohmic heating in a tokamak, unlike ECRH, becomes progressively less effective as the plasma gets hot, since the resistivity goes down, but the current cannot be increased without violating the Kruskal-Shrafranov limit. Like the EBT, the tokamak requires auxiliary heating to reach ignition temperatures, but in an EBT the auxiliary heating does not degrade the effectiveness of the ECRH. Third, the scaling of confinement time in the neoclassical collisionless regime is much less favorable in a tokamak than an EBT. For a tokamak,

$$\tau \propto \frac{a^2 B^2 \sqrt{T_i}}{n} \left(\frac{a}{R}\right)^{3/2} \qquad (17)$$

This should be compared with equation (2) for the EBT. Note that the tokamak optimizes at low aspect ratio (R/a) in contrast with the EBT. It's easy to increase the aspect ratio of a torus, but there is a definite lower limit of order unity. Note also that a tokamak needs a high magnetic field for good confinement, but the confinement in an EBT is field independent. Finally, and most important, in the collisionless regime, the power required to achieve a given ion temperature in a tokamak is proportional to $\sqrt{T_i}$, and so there is no thermal runaway. One must not only supply enough power to reach the collisionless regime, but must continue to increase the power to raise the temperature. In the collisionless regime, the tokamak, like the EBT, has $n\tau$ independent of density, but since it operates at higher density, the electrons and ions are nearly equilibrated, and the electron energy loss, which tends to be anaomalous anyway, is higher. Figure 2 shows a plot of $n\tau$ vs. T_i for the EBT devices discussed, along with a state-of-the-art tokamak (PLT). Also shown are the predicted results of adding 1.5 MW of neutral beams (or ICRH) to EBT-II and 6 kW of ICRH to EBT-I assuming neoclassical scaling.

MAGNETIC FIELD ERRORS

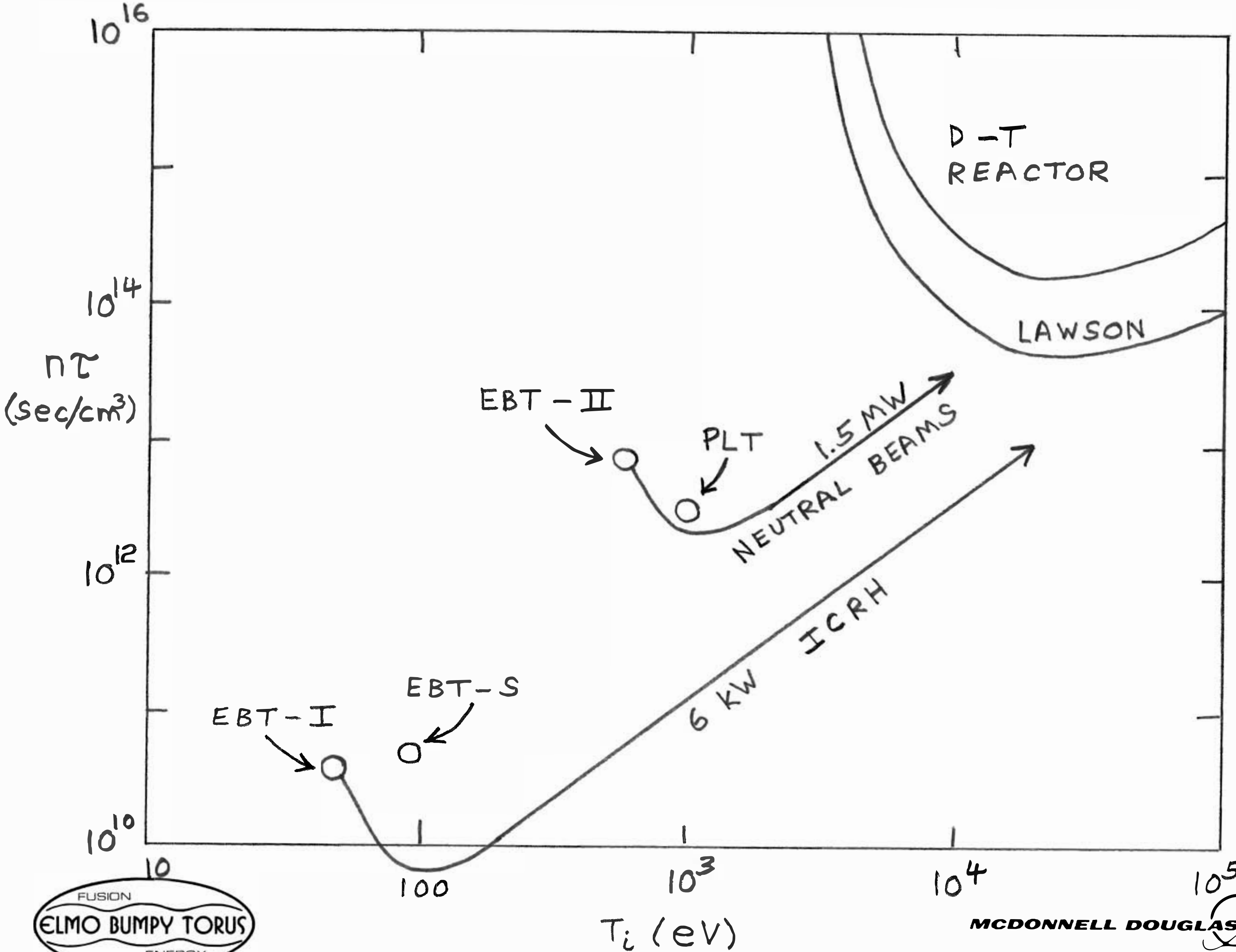
The effects of magnetic field errors on the drift trajectories in a bumpy torus have been extensively studied. The maximum tolerable field error is given by

$$\frac{\Delta B}{B} \sim \frac{\bar{\rho}}{R} \quad , \tag{18}$$

where ρ is the ion gyroradius. Measurements on EBT-I are consistent with the above estimate. It is clear from equation (18) that field errors become less of a concern as the ion temperature increases since

$$\rho = \frac{102\sqrt{T_i}}{B} \quad . \tag{19}$$

For EBT-I at 50 eV, the maximum allowable field error is $\Delta B/B \sim 5 \times 10^{-4}$. In EBT-II, on the other hand, at 570 eV, one requires $\Delta B/B \sim 8 \times 10^{-5}$. Hence field errors are expected to be less of a problem in an ICRH-heated EBT-I than in an EBT-II. If the field errors exceed the values above, it does not mean that the device will not work, but only that additional ion heating power will be required to reach the collisionless regime.



10^{16}
10^{14}
10^{12}
10^{10}
$n\tau$ (sec/cm³)
D - T
REACTOR
LAWSON
1.5 MW
NEUTRAL BEAMS
EBT - II
PLT
6 KW ICRH
EBT - S
EBT - I
10
100
10^3
10^4
10^5
T_i (eV)

BETA LIMITS

It is predicted that the β of the toroidal plasma in an EBT can approach the beta of the annulus which is typically ∿50%. What temperature limit does this impose on an ICRH-heated EBT-I? The β is given by

$$\beta = 4.02 \times 10^{-11} n(T_i + T_e)/B^2 \quad . \tag{20}$$

For $\beta = 0.5$, $n = 2 \times 10^{12}$ cm^{-3}, $T_i = 2.5\ T_e$, and B = 3200 gauss (at the position of the rings), the limiting T_i is ∿45 keV. The beta does not appear to pose a serious limitation on an ICRH experiment in an EBT-I. Since one would like to operate a reactor at high beta, it would be good to test the beta limits anyway. For an ECRH plasma with $\omega_{pe} = \omega_{Ce}$, the β is given by

$$\beta = 3.9 \times 10^{-6}(T_i + T_e) \quad , \tag{21}$$

or $T_i + T_e = 128$ keV at $\beta = 0.5$, independent of density and field strength.

CHARGE EXCHANGE

The preceding calculations have assumed that neoclassical diffusion is the only ion energy loss mechanism. This is clearly an over-simplification. Probably the most serious non-diffusive ion energy loss is charge exchange. We will now estimate the power lost by charge exchange for EBT-I as a function of ion temperature. By integrating the charge exchange cross section over a Maxwellian distribution, the charge exchange time τ_{CX} can be approximated by

$$\frac{1}{\tau_{CX}} = 4.9 \times 10^{-11} n_0 \sqrt{T_i}\,(1 + 0.00585\, T_i^{3/2})\, e^{-.0582\sqrt{T_i}} \quad (22)$$

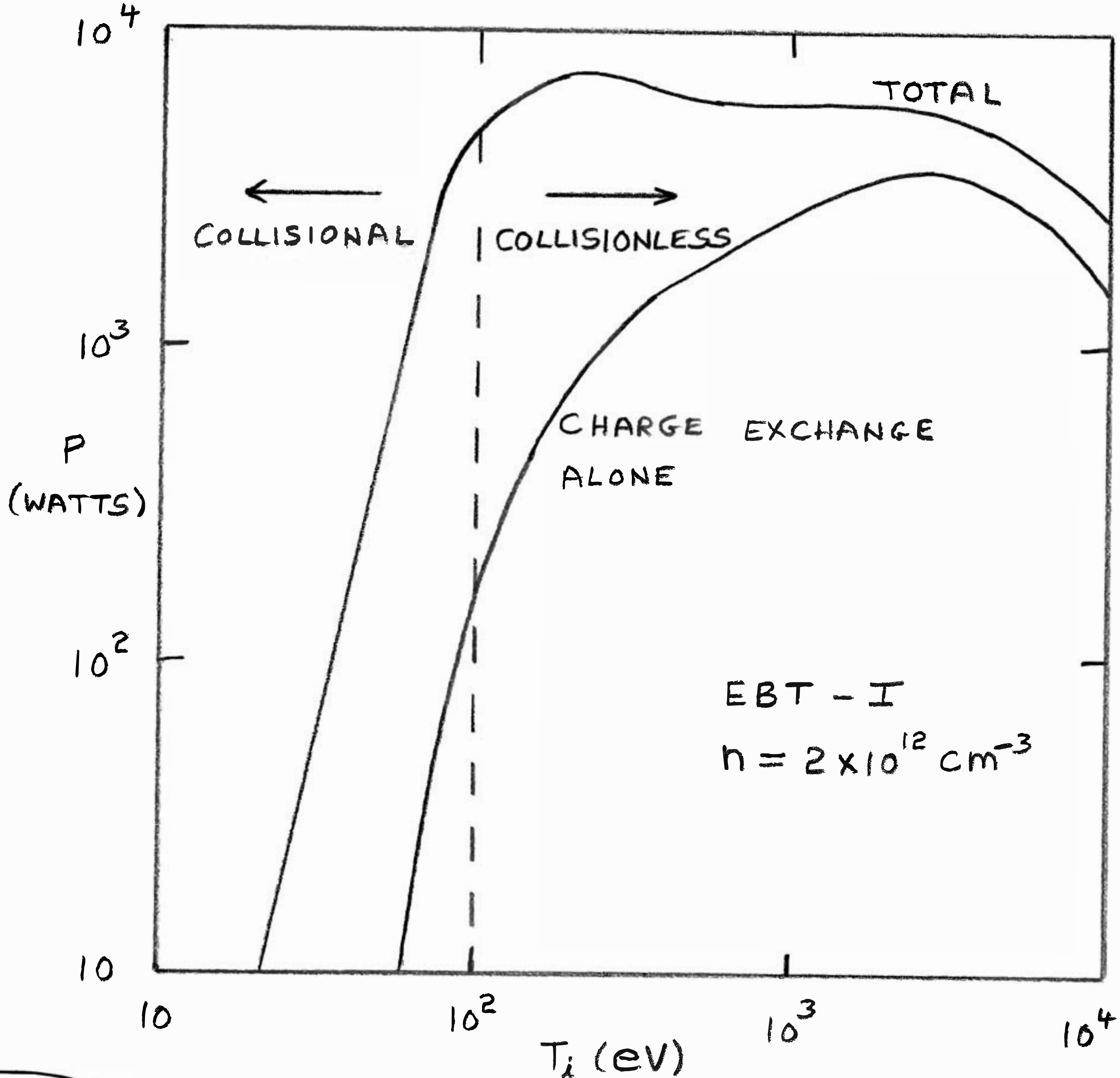
where n_0 is the neutral hydrogen density in cm^{-3}. We know that the neutral density must be such that the ionization rate is equal to the particle loss rate by diffusion. For T_e between about 50 eV and 10^4 eV, the ionization rate integrated over a Maxwellian distribution is nearly independent of T_e and the ionization time τ_i is given approximately by

$$\tau_i = 2 \times 10^7 / n_0 \quad . \quad (23)$$

Setting τ_i equal to τ, the neoclassical confinement time then enables one to determine n_0 as a function of T_i from equation (23) and then τ_{CX} from equation (22). Having found τ_{CX}, one can then calculate the ICRH power required to overcome charge exchange losses from

$$P = \frac{enT_i V}{\tau_{CX}} \quad . \quad (24)$$

If one then adds this power to the power required to overcome losses by neoclassical diffusion (equation 13), one arrives at the result of figure 3. Although charge exchange losses are not completely negligible, neither do they change the general conclusion that the ion temperature should run away once the plasma becomes collisionless. The only effect of charge exchange is to raise slightly the power required (from 6.4 kW to 7.4 kW).



FUSION ELMO BUMPY TORUS ENERGY

MCDONNELL DOUGLAS CORPORATION

ELECTRON DRAG

So far, the only ion energy loss mechanisms considered have been neoclassical diffusion and charge exchange. The ions can also lose energy by collisions with the electrons which then dissipate their energy by a variety of mechanisms. The calculation of this electron drag is complicated because of the way in which the electron and ion power balance equations couple with each other and with the particle balance equation. Consequently, a computer code (EBTICH) was written to numerically solve this coupled set of equations. The input parameters to the code are P_{ECRH}, a, R, B, C_1, and C_2 (all as defined previously). The plasma volume is assumed to be given by

$$V = 33.333\ a^2 R, \tag{25}$$

and the density is assumed to be given by

$$n = 2 \times 10^4\ B^2\ , \tag{26}$$

as required for ECRH. The code scans through a range of T_i (10 eV to 10^4 eV) and calculates T_e, n_0, P_{ICRH}, and τ, as a function of T_i.

For a given value of T_i, the required ICRH power may be negative, implying that in the absence of ICRH, the ions would be at a higher temperature than that specified. The equations that are solved iteratively are

$$\tau_i = \tau\ , \tag{27}$$

where τ_i is given by equation (23) and τ is given by the sum of equations 1 and 2, and

$$P_{ECRH} = 1.2 \times 10^{-26}\, n^2 V (T_e - T_i)\, T_e^{-3/2} + 2.4 \times 10^{-19}\, nVT_e/\tau . \qquad (28)$$

The ICRH power required is then calculated from

$$P_{ICRH} = 2.4 \times 10^{-19}\, nVT_i/\tau + 1.2 \times 10^{-26}\, n^2 V (T_i - T_e) T_e^{-3/2} + 1.8 \times 10^{-29} n n_0 V T_i^{3/2} (1 + 0.00585\, T_i^{3/2}) e^{-0.0582 \sqrt{T_i}} . \qquad (29)$$

The result of solving the above equations for EBT-I parameters with P_{ECRH} = 3 kW, is virtually identical to the results of figure 3 except that the ICRH power required to reach the runaway regime is ∿13 kW. This is because the electrons dissipate about the same power by diffusion as the ions since the confinement times of electrons and ions are equal and the temperatures are comparable. Next, a more realistic electron temperature dependent ionization rate, including ionization by energetic ions, was substituted for τ_i:

$$\frac{1}{\tau_i} = \frac{371 n_0 \sqrt{T_e}}{T_e + 15.6}\, e^{-15.6/T_e} \left(\frac{T_e}{20T_e + 15.6}\right) \ln(1.5625 + 0.1T_e) + \frac{0.22 n_0 (T_i/280)^3}{[1 + (T_i/280)^5]^{1/3}} . \qquad (30)$$

The result was practically indistinguishable from the case in which equation (23) was used to determine the ionization rate. Then an excitation loss term was added to the electron power balance:

$$P_{ex} = 4.7 \times 10^{-18} \frac{nV}{\tau_i} . \tag{31}$$

This term corresponds to a constant energy loss of 29.1 eV per electron-ion pair produced. The result is to raise the ICRH power requirement to ∿14 kW. Next a term was included to account for Bremsstrahlung losses (with z = 1):

$$P_{BR} = 1.7 \times 10^{-32} n^2 V \sqrt{T_e} . \tag{32}$$

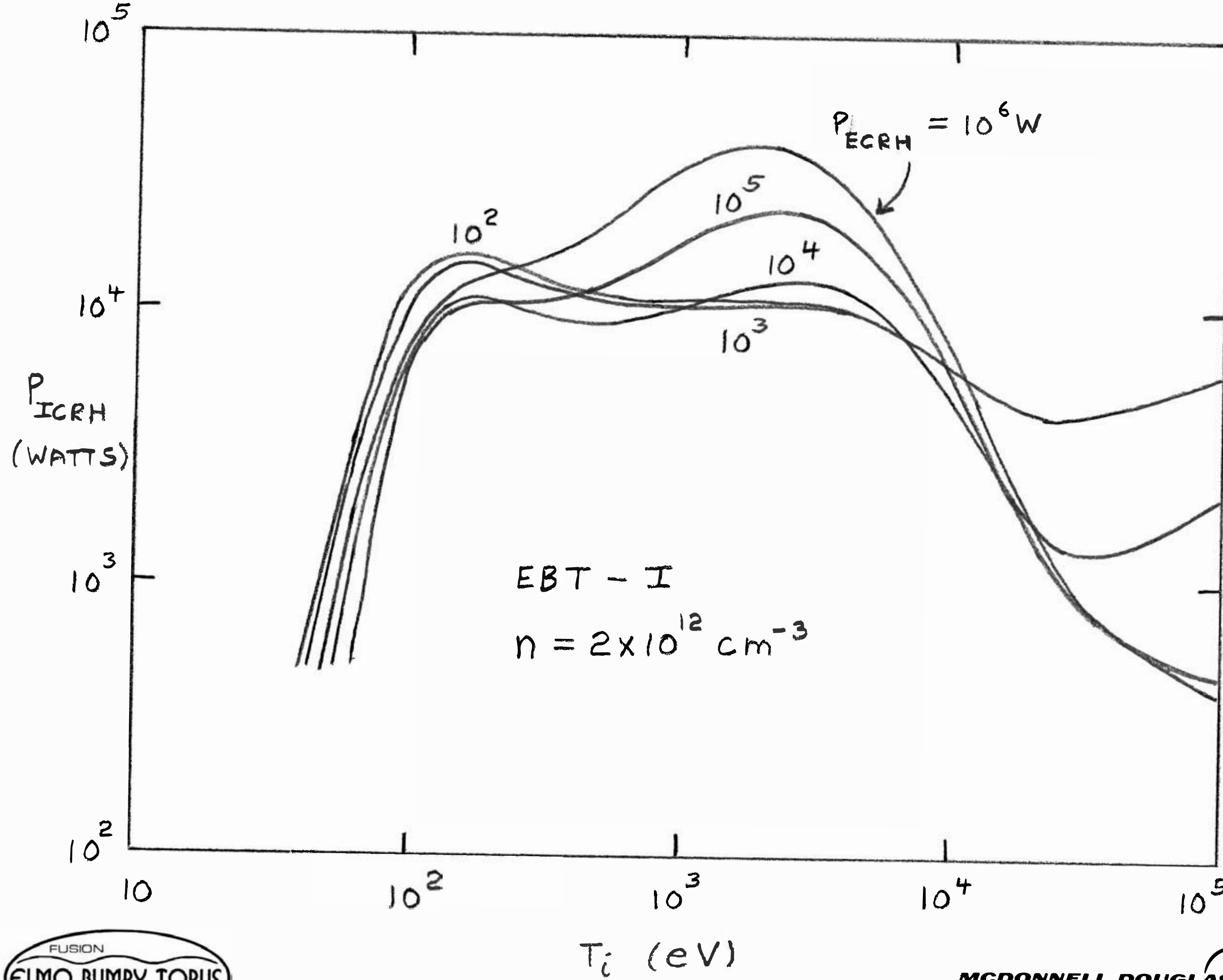
For EBT-I parameters with $T_e = 10^4$ eV, because of the low density, this term only amounts to 3 watts. Similarly, a synchrotron radiation term was added:

$$P_{SR} = 1.0 \times 10^{-26} B^2 n V T_e [1 + T_e/204400]. \tag{33}$$

Some averaging over the spatially non-uniform field was done to arrive at the above estimate, but it is pessimistic in the sense that finite beta and reabsorption of the radiation by the plasma have been ignored. For EBT-I parameters with $T_e = 10^4$ eV, the synchrotron radiation from the core plasma is 10 kW. Including these terms lowers the electron temperature somewhat, but it does not change the amount of ICRH power required to reach the collisionless regime.

POINT MODEL PREDICTIONS

The point model numerical code of the previous section can be used to predict the scaling of EBT devices with ICRH. Figure 4 shows the ICRH power required to achieve



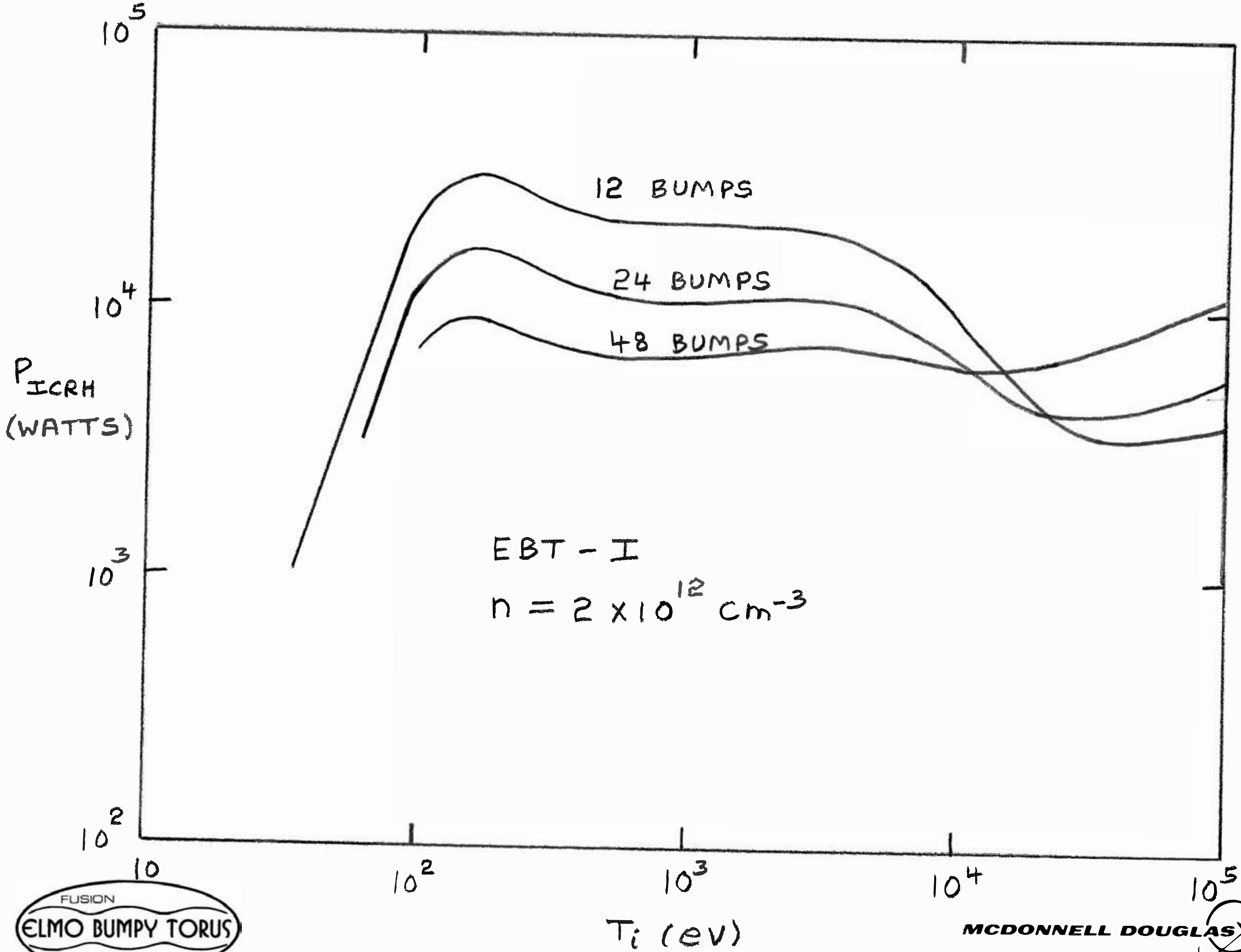
FUSION ELMO BUMPY TORUS ENERGY

MCDONNELL DOUGLAS CORPORATION

a given ion temperature for various levels of ECRH power. The maximum ICRH power required varies only by a factor of about three for ECRH powers of 10^2 to 10^6 watts. Furthermore, too much ECRH power is undesirable since the electron temperature rises to such a high value that ionization is inefficient and the neutral density and hence the charge exchange losses increase. Consequently, it is probably best to use the minimum ECRH power that is required to sustain the annulus. In what follows, we assume that the ECRH power into the toroidal core is zero, and that the only power input is ICRH. Under these conditions, $\sim$16 kW of ICRH is required to reach the collisionless regime. Figure 5 shows the ICRH power required to achieve a given ion temperature in a device like EBT-I except with different numbers of bumps (and hence different major radii). The ICRH power required to reach the collisionless regime scales as $1/N$ (since the power scales as N but the confinement time scales as $R^2/a^2 \propto N^2$). The confinement time, and hence $n\tau$, rises as N^2. For the 48-bump torus, $n\tau = 10^{14}$ sec/cm^3 at T_i = 35 keV. One can also increase the magnetic field for an EBT-I device (as with EBT-S) and ask how the various quantities scale with B. Figure 6 shows the result. The ICRH power required to reach the collisionless regime rises sharply with B, primarily because of the higher density, but the value of $n\tau$ is only a function of temperature, for a fixed aspect ratio.

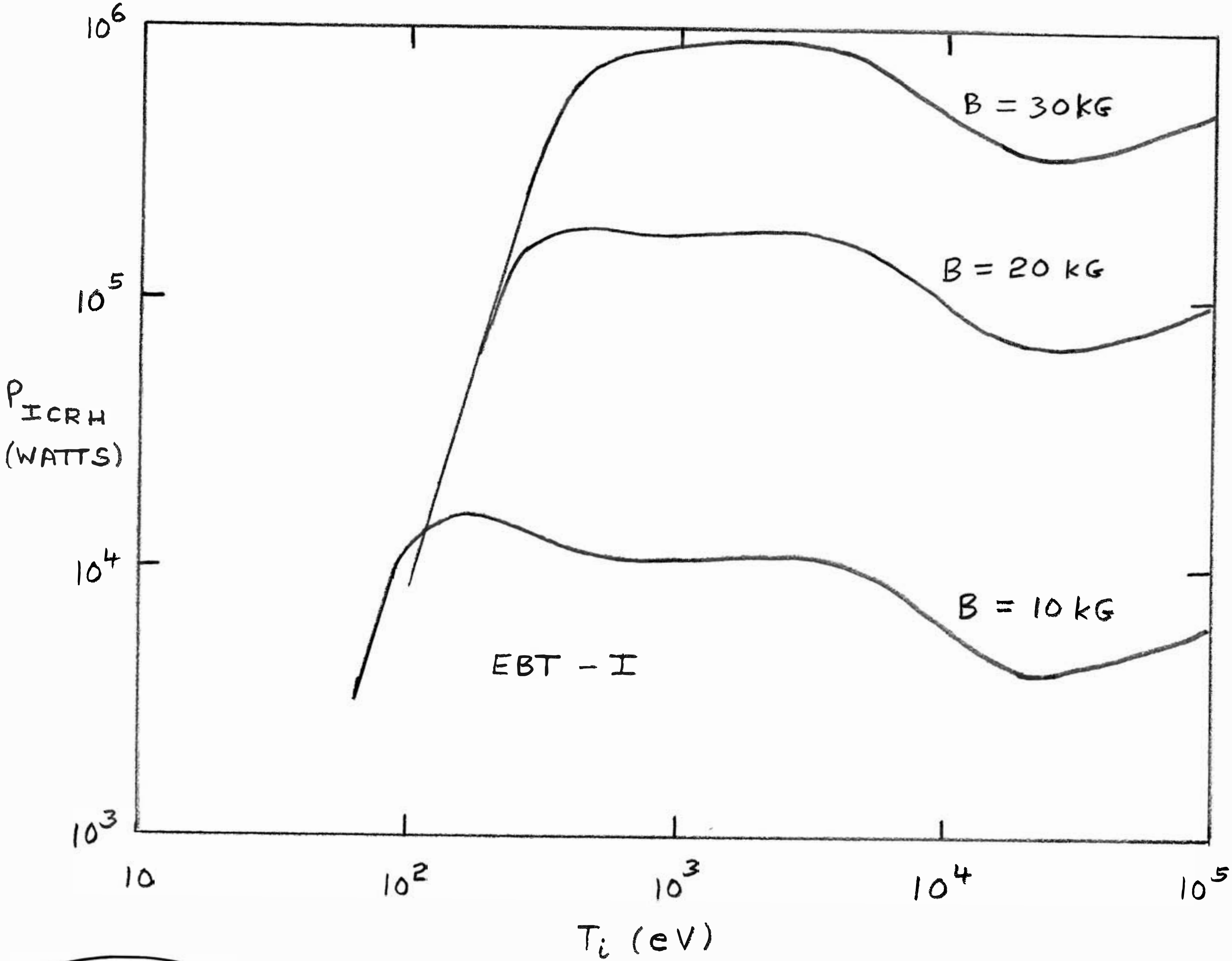
EBT IGNITION REQUIREMENTS

From the preceding discussion, it appears relatively simple to satisfy the Lawson criterion in a very modest sized device with low field and low ICRH power. For example, an EBT sized device with 10 kG fields and 48 bumps achieves $n\tau = 10^{14}$ sec/cm^3 at T_i = 35 keV with only 9 kW of ICRH and no ECRH. Under such conditions, the electron temperature is only 3.5 keV. Such a device would appear to be a very



FUSION ELMO BUMPY TORUS ENERGY

MCDONNELL DOUGLAS CORPORATION



FUSION ELMO BUMPY TORUS ENERGY

MCDONNELL DOUGLAS CORPORATION

attractive reactor. Unfortunately, the Lawson criterion is a necessary, but by no means a sufficient, condition for ignition. It was derived on the assumption that Bremsstrahlung is the dominant energy loss and that $T_e = T_i$. The fusion energy produced by a 50/50 mixture of deuterium and tritium is given by

$$P_n = 9 \times 10^{-13} n^2 V \langle\sigma v\rangle \tag{34}$$

where $\langle\sigma v\rangle$ is the fusion reaction rate given approximately by

$$\langle\sigma v\rangle = exp[-111.86 + 13.85 \ln T_i - 0.621(\ln T_i)^2] . \tag{35}$$

The 48 bump torus described above produces about 2.5 kwatts of fusion power, even though the Lawson criterion is satisfied. We can now ask what conditions must be satisfied to do a true breakeven experiment in an EBT. This can be determined by equating the fusion power produced as given by equation (34) with the ICRH power required to heat the plasma as given by equation (12):

$$9 \times 10^{-13} n^2 V \langle\sigma v\rangle = \frac{7a^2 n^2 eV}{10 C_2 R^2} \left[\frac{C_2}{C_1 a^4 B^2 n^2} \right]^{1/10} \tag{36}$$

Using the largest value of $\langle\sigma v\rangle$ (9×10^{-16} cm^3/sec at 70 keV), breakeven is reached when the aspect ratio exceeds

$$\frac{R}{a} = \frac{750}{[a^2 B^3]^{1/10}} \tag{37}$$

For an EBT-I device (a = 10 cm and B = 10 kG), a radius of 3 meters is required. The size could probably be further reduced by increasing the mirror ratio and incorporating other aspect ratio enhancement techniques.

EDGE PRESSURE

EBT-I operates in the stable T mode only over a narrow range of (gauge) pressures in the high 10^{-6} torr range. This is presumed to be a result of the requirement that the cold, poorly confined plasma at the edge must have a density sufficient to stabilize the outer edge of the annulus by some poorly understood line-tying mechanism. If the toroidal plasma is pushed far into the collisionless regime by ICRH, the particle confinement time becomes very long, and the edge pressure must be reduced in order to maintain an ionization rate in the bulk plasma that balances the diffusion losses. At the other extreme, in a larger, higher density device, the edge pressure may have to be unacceptably high in order to refuel the plasma core despite the long confinement time. It is difficult to know precisely the edge neutral pressure that would be required and the edge neutral pressure that might be expected as one departs significantly from EBT-I parameters. It is nevertheless useful to have an estimate of the edge pressure required to balance the particle loss. Assuming an exponential decay of neutral density inside the plasma, the edge pressure p in torr (gauge) can be expressed in terms of the density n_0 on the axis by

$$p = 1.35 \times 10^{-17}\, n_0\, e^{7.2 \times 10^{-14} a n / \sqrt{T_n}} , \qquad (38)$$

where T_n is the neutral temperature in eV. It is tempting to assume T_n = 0.026 eV (room temperature), but a better approach is probably to determine the effective value of T_n from EBT-I data. Taking $n = 2 \times 10^{12}\ cm^{-3}$, $p = 9 \times 10^{-6}$ torr, and $n_0 = 1 \times 10^9\ cm^{-3}$ (from equation (23) with $\tau = 20$ msec) gives $T_n = .049$ eV, a quite reasonable number. Equation (38) was incorporated into program EBTICH in order to monitor the edge neutral pressure required for the various configurations. The more the neutral pressure departs from the nominal EBT-I value (9×10^{-6} torr) the more worry one should have as to whether stable T-mode operation is possible.

EBTR-48

Oak Ridge has proposed (ORNL/TM 5669) an EBT reactor design (EBTR-48) with the following characteristics:

Major radius: $R = 6000$ cm
Minor radius (coil throat): $a = 77.5$ cm
Plasma volume: $V = 1.2 \times 10^9\ cm^3$
Number of bumps: $N = 48$
Mirror ratio: $M = 1.78$
Magnetic field strength (throat): $B = 45$ kG
Density: $n = 1.2 \times 10^{14}\ cm^{-3}$
Ion temperature: $T_i = 15$ keV
Confinement time: $\tau = 7.5$ sec
Thermonuclear output power: $P_n = 4000$ MW

The parameters above imply a confinement time as given by equation (2) with $C_2 = 8.17 \times 10^4\ sec/cm^3\ /eV^{3/2}$, about a factor five more optimistic than equation (4). Furthermore, the density is given by

$$n = 6 \times 10^4 B^2 , \qquad (39)$$

which is a factor of three higher than the conservative value assumed in equation (26). Finally, the synchrotron radiation was assumed to be given by

$$P_{SR} \simeq 4.7 \times 10^{-31} B^2 V \sqrt{nBT_e^{5}/a}\,(1 + T_e/204400) \quad (40)$$

which is more conservative than equation (33) by a factor of 5000. These values were put into the point model (EBTICH) to verify the agreement with the EBTR-48 design. The results at T_i = 15 keV are as follows:

T_e = 14.1 keV

n_0 = 1.8×10^6 cm^{-3}

P_{ICRH} = 250 MW (or neutral beams)

$n\tau$ = 9×10^{14} sec/cm^{-3}

P_n = 3700 MW

The results are consistent with the Oak Ridge predictions,and inspire confidence in the validity of the point model for reactor scaling.

A COMPACT EBT REACTOR

The use of ICRH offers considerable flexibility in the design of an EBT reactor, since the coupling of the ICRH to the plasma is uniformly good over a wide range of sizes and densities. As a design example we will consider an EBT reactor with the following characteristics:

Major radius: $R = 750$ cm

Minor radius (coil throat): $a = 20$ cm

Plasma volume: $V = 1.0 \times 10^7$ cm^3

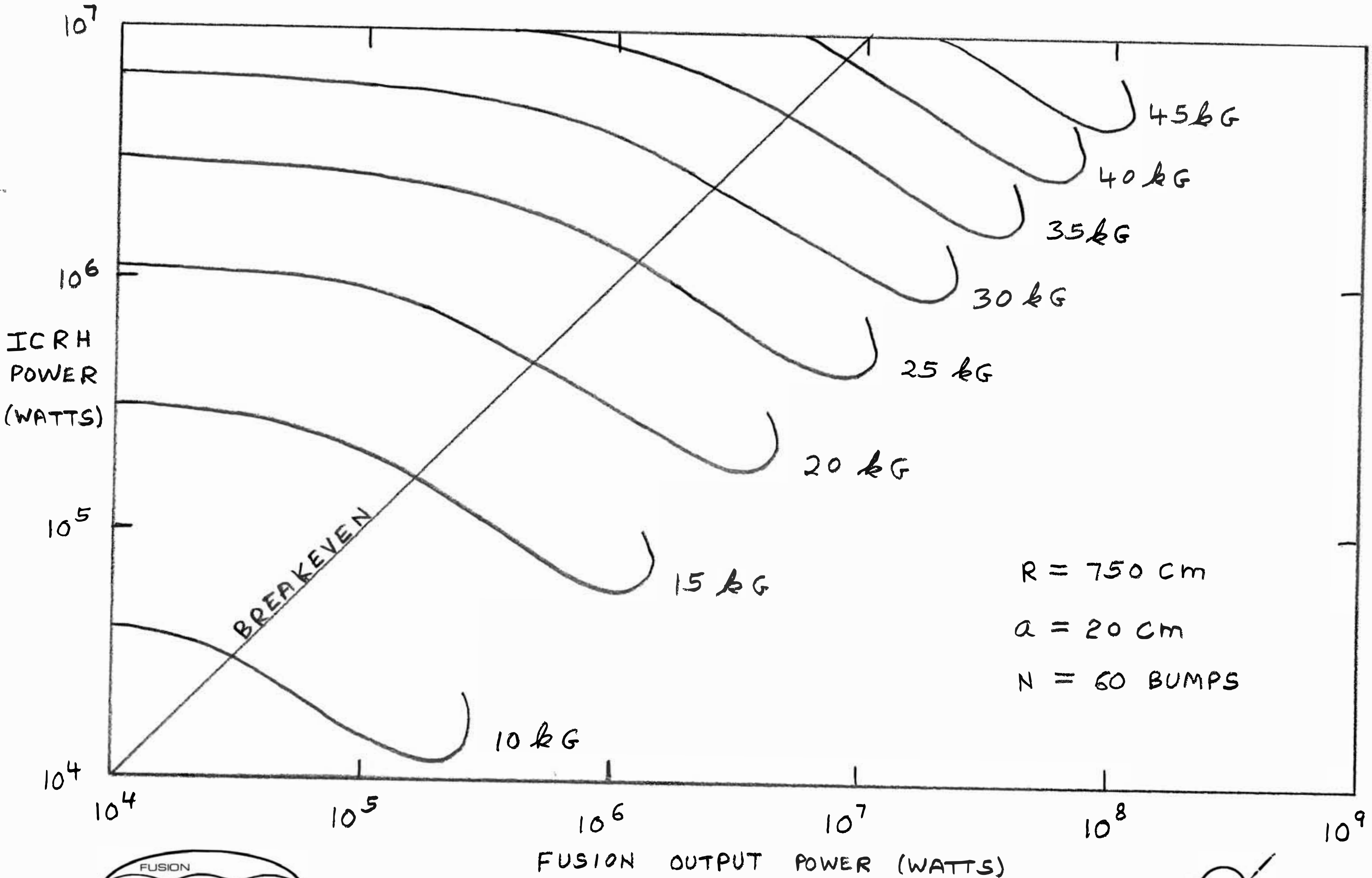
Number of bumps: $N = 60$

Mirror ratio (on axis): $M = 2.0$

This case was run for a variety of ion temperatures and magnetic field strengths using the EBTICH code with the same scalings as for EBTR-48. The results are shown in figure 7. The maximum power multiplication for all cases is about 20, and it occurs at an ion temperature of about 40 keV and an electron temperature of about 32 keV. For all cases the edge neutral pressure is much higher than in EBT-I, and so an alternate fueling method (such as cluster injection) is assumed unless an anomalous neutral penetration is obtained as is the case with tokamaks.

AMBIPOLAR POTENTIAL

The unequal diffusion of electrons and ions in a bumpy torus gives rise to a radial ambipolar electric field. This electric field in turn modifies the diffusion in two ways: (1) in the presence of collisions, it causes the electron and ion fluids to move in the direction of $-\vec{E}$ and $+\vec{E}$ respectively, producing an effect akin to electrostatic confinement; (2) in the absence of collisions, it produces an $\vec{E} \times \vec{B}$ drift which combines with the ∇B drifts to alter the particle drift trajectories. These effects can be included in the expression for the ion confinement time in the following approximate manner:



FUSION ELMO BUMPY TORUS ENERGY

MCDONNELL DOUGLAS CORPORATION

$$\tau = \left[\frac{C_1 a^2 B^2 R^2 n}{T_i^{7/2}} + \frac{C_2 R^2 T_i^{3/2}}{a^2 n}\left(1 + \frac{\Phi^2}{T_i^2}\right)\right] e^{-\Phi/T_i} \quad , \tag{41}$$

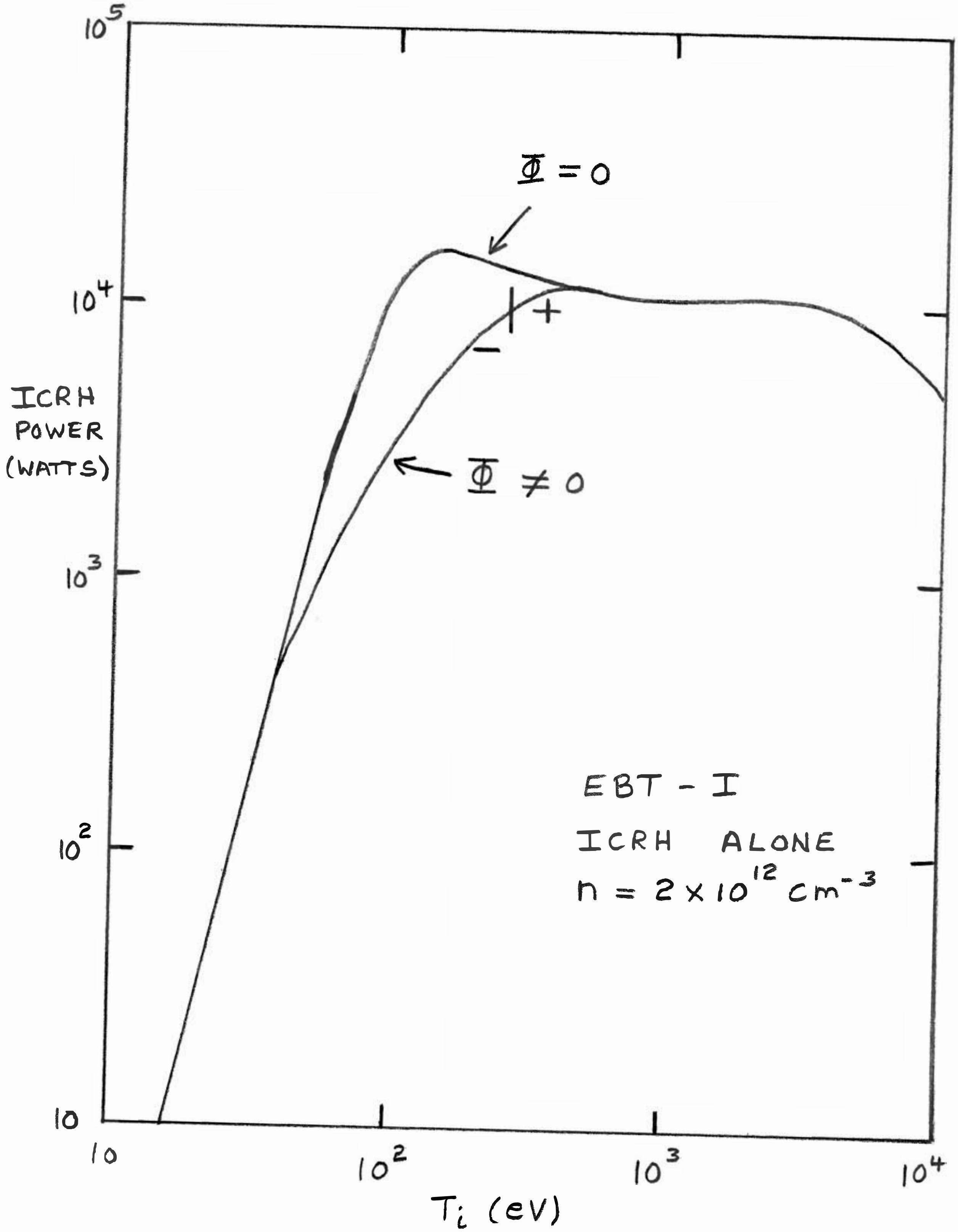
where Φ is the plasma potential, which is assumed to be given by $\Phi = Ea$. The potential Φ must be determined by equating the ion confinement time τ given above with the electron confinement time given by

$$\tau_e = \left[\frac{96 C_1 a^2 B^2 R^2 n}{T_e^{7/2}} + \frac{C_2 R^2 T_e^{3/2}}{96 a^2 n}\left(1 + \frac{\Phi^2}{T_e^2}\right)\right] e^{\Phi/T_e} \quad , \tag{42}$$

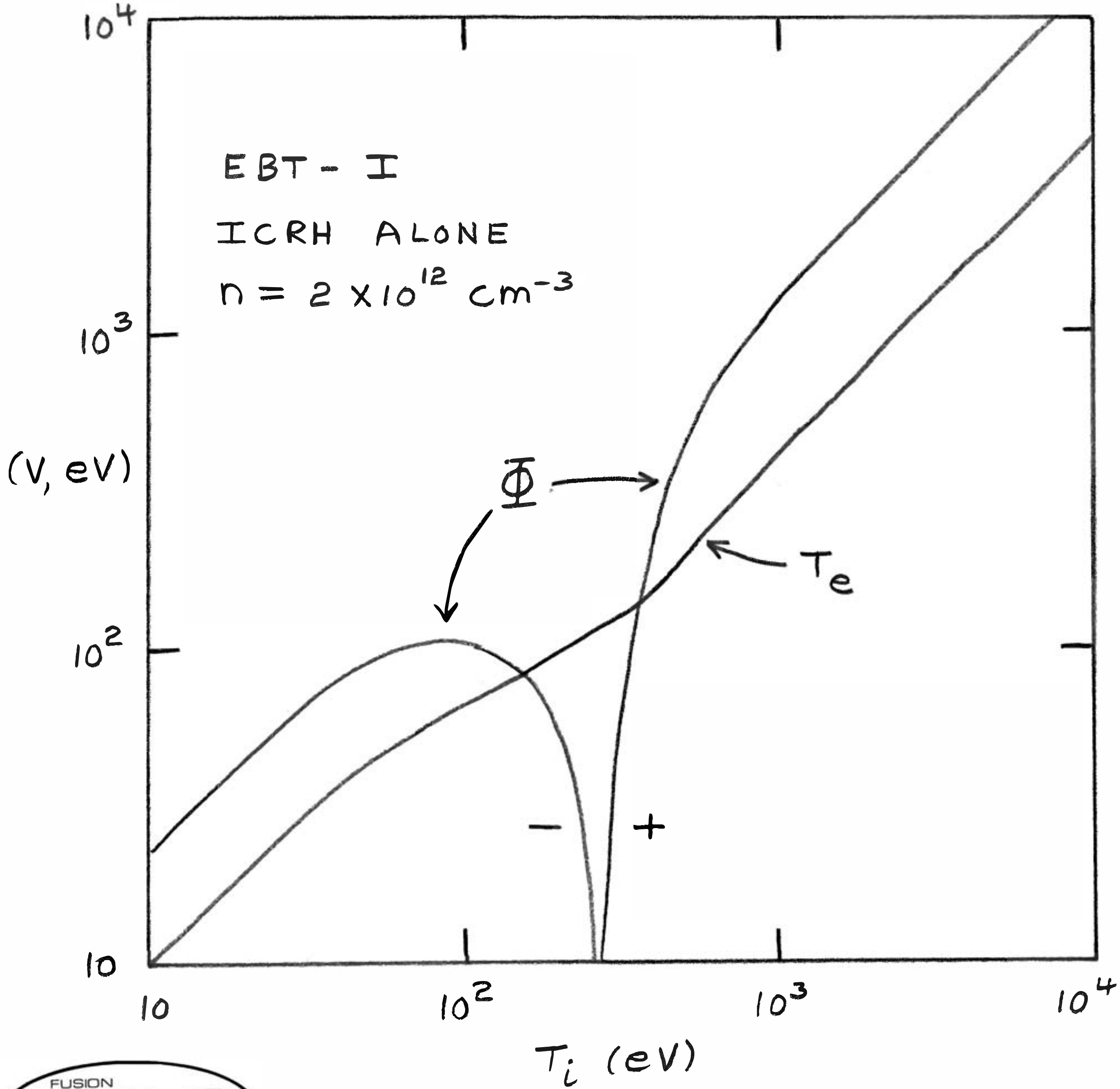
where the factor of 96 comes from the ratio of electron to ion collision frequencies, which is given by $2\sqrt{m_i/m_e}$. In order to assess the importance of this ambipolar potential, a computer code, EBTPHI, which is a modification of EBTICH, was written. Because the coefficients C_1 and C_2 were previously adjusted to fit experimental data, we adopt the same tact here, and adjust them so that the results with the ambipolar potential agree with the results without the ambipolar potential in the limit of low and high T_i. The required values are:

$$\begin{aligned} C_1 &= 3.98 \times 10^{-24} \\ C_2 &= 21197.5 \end{aligned} \tag{43}$$

A comparison of the results with and without the ambipolar potential is shown in figure 8. The effect of the potential is to smooth out the transition from collisional to collisionless behavior, and to reduce somewhat the ICRH power required to achieve a given ion temperature. Since the difference is rather small, it seems hardly worth the effort to include it. In effect, the act of choosing C_1 and C_2 empirically for the $\Phi = 0$ case automatically accounts approximately for the presence of any electric field. Figure 9 shows the electron temperature T_e and potential Φ



$\Phi = 0$
$\Phi \neq 0$
+
−
ICRH POWER (WATTS)
10^5
10^4
10^3
10^2
10
10
10^2
10^3
10^4
T_i (eV)
EBT - I
ICRH ALONE
$n = 2 \times 10^{12}$ cm^{-3}



FUSION
ELMO BUMPY TORUS
ENERGY

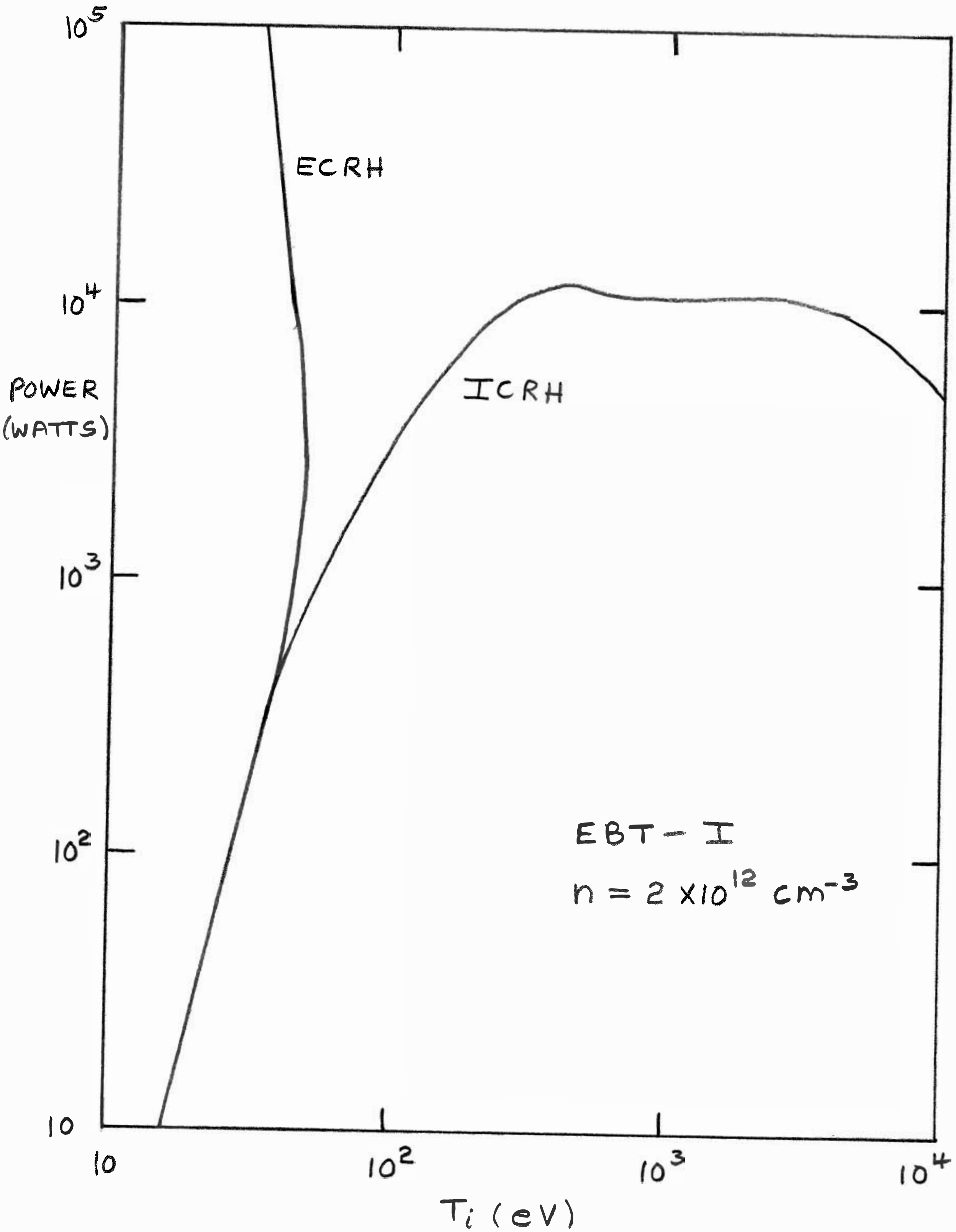
MCDONNELL DOUGLAS
CORPORATION

as a function of T_i. Figure 10 compares the ion temperature that can be obtained using either ICRH or ECRH alone, including the effect of the ambipolar field.

B-SCALING IN EBT-I

One possible strategy for upgrading an EBT-I type device with ICRH to reactor conditions is to increase the magnetic field without changing the size. This is an especially attractive proposal if the first generation device demonstrates that the toroidal plasma can be produced and sustained with ICRH alone. Then the only ECRH required is the lower frequency profile heating. In order to assess the possibilities of such a scaling, the EBTPHI code was run for various values of B corresponding to profile heating at 10.6, 18.0, 28.0, and 55.0 GHz respectively. The density was assumed to scale as B^2 in order to insure penetration of the profile heating waves. In the table below are listed the results at the point at which the potential reverses from negative to positive. This roughly corresponds to the transition from collisional to collisionless behavior, and it is about the point at which the ICRH power is largest.

$f_{\mu w}$ (GHz)	B_{THROAT} (kG)	n (cm^{-3})	T_i (eV)	T_e (eV)	P_{ICRH} (kW)	τ (ms)	n_0 (cm^{-3})
10.6	10	2×10^{12}	267	114	10	10.7	1.3×10^9
18.0	17	5.8×10^{12}	503	218	71	9.3	1.5×10^9
28.0	26.4	1.4×10^{13}	840	370	386	8.5	1.8×10^9
55.0	51.9	5.4×10^{13}	1880	835	5900	7.4	2.3×10^9


10^5
10^4
10^3
10^2
10
POWER (WATTS)
ECRH
ICRH
EBT - I
n = 2 X10^12 cm^-3
10
10^2
10^3
10^4
Ti (eV)

The conclusion from the above is that one can achieve higher temperatures in a higher field device, but the higher densities cause the confinement time to remain low so long as one insists on operating with negative potentials. In the collisionless regime, the potential is strongly positive ($\Phi \sim T_i$) and the $n\tau$ product is independent of field strength , and so an extrapolation to higher fields appears to lack merit.

COIL POSITION CONSIDERATIONS

A bumpy cylinder has axisymmetry, and hence concentric drift surfaces. When bent into a bumpy torus, the drift surfaces overlap and give rise to an enhanced neoclassical transport of particles and energy across the field. This enhancement can be minimized by an intelligent choice of coil shape and location. As an example, we will consider a 24-bump torus with a mirror ratio of $M = 2.0$. The magnetic field on the axis of a circular current loop of radius A, carrying current I, a distance Z from the plane of the loop is given by

$$B(Z) = \frac{\mu_0 I A^2}{2(A^2 + Z^2)^{3/2}} \qquad (44)$$

For two such axisymmetric loops separated a distance L, the mirror ratio on axis is

$$M = \frac{1}{2}(1 + L^2/4A^2)^{3/2} \qquad (45)$$

For $M = 2$, we require $A = 0.406L$. In a large aspect ratio torus with major radius R_0 and N bumps, this translates into

$$A = 0.406\left(\frac{2\pi R_0}{N}\right) = 0.106R_0 \quad . \tag{46}$$

For a 150 cm radius torus, a coil radius of 15.9 cm is required. Such a configuration is shown in figure 11(a) (not to scale). The lowest order convection required is an outward displacement of the coils by an amount Δr in order to compensate for the $1/R$ decrease of the field due to the toroidal curvature, as shown in figure 11(b). The magnetic field along the minor axis can be Fourier analyzed with a lowest order term given by

$$B(Z) = \frac{1}{2}B(o)\left[(M+1) - (M-1)\cos\frac{2\pi Z}{L}\right], \tag{47}$$

where Z is measured from the mirror midplane along the curved $R = R_0$ axis. Since $\nabla\cdot\vec{B} = 0$, there must also be a radial field given by

$$B_r = -r\frac{\partial B}{\partial Z} = \frac{\pi r(1 - M)B(o)}{L}\sin\frac{2\pi Z}{L} \quad . \tag{48}$$

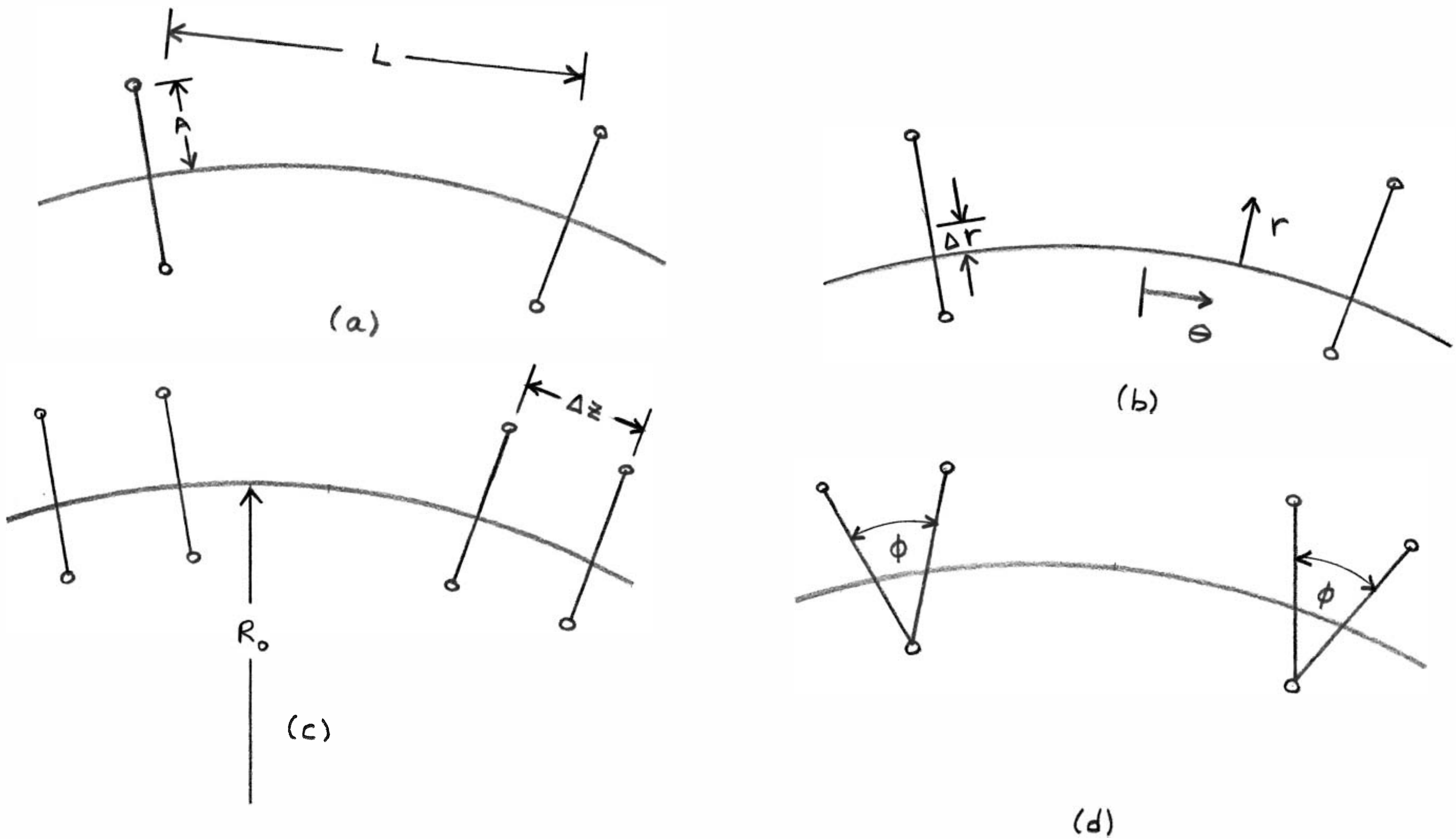
This radial field, in turn, requires that the toroidal component of the field fall off with minor radius. Using $\nabla \times \vec{B} = 0$,

$$B(r, Z) = B(Z) + \int_0^r \frac{\partial B_r}{\partial Z}dr$$

$$= \frac{1}{2}B(o)\left[(M+1) - (M-1)\left(1 + \frac{\pi^2 r^2}{L^2}\right)\cos\frac{2\pi Z}{L}\right] \quad . \tag{49}$$

With toroidal curvature, the field is

$$B(r, \theta) = \frac{R_0 B(o)}{2(R_0 + r)}\left[(M+1) - (M-1)\left(1 + \frac{N^2 r^2}{4R_0^2}\right)\cos N\theta\right] . \tag{50}$$


L
A
(a)
Δr
r
θ
(b)
Δz
R_o
(c)
φ
φ
(d)

The radial shift in position of the magnetic axis in the mirror midplane can be calculated from

$$\frac{\partial B(r, o)}{\partial r} = 0 \ , \tag{51}$$

which gives the result

$$\Delta r = \frac{4R_0}{(M - 1)N^2} \quad . \tag{52}$$

For R_0 = 150 cm, M = 2.0, and N = 24, the coils should be displaced outward by an amount Δr = 1.04 cm in order to recenter the drift surfaces on the $R = R_0$ axis. If each coil is split into two pieces and separated by an amount ΔZ, as shown in figure 11(c), the mirror ratio will decrease by an amount

$$\Delta M = -40.2(\Delta Z)^2/R_0{}^2 \quad . \tag{53}$$

In order to restore the mirror ratio to a value of 2.0, the major radius of the torus should be increased by an amount

$$\Delta R_0 = 11.1(\Delta Z)^2/R_0 \ . \tag{54}$$

For example, if ΔZ = 10 cm, the torus radius should be increased from 150 cm to 157.4 cm. Rather than separate the coils by a constant ΔZ, it is probably better to cant them so as to spread out the currents in azimuth on the large radius side of the torus and compress them on the small radius side, as shown in figure 11(d). This will increase the field line length on the inside and decrease it on the outside, resulting in a centering of the §dl surfaces.

THERMAL STABILIZATION

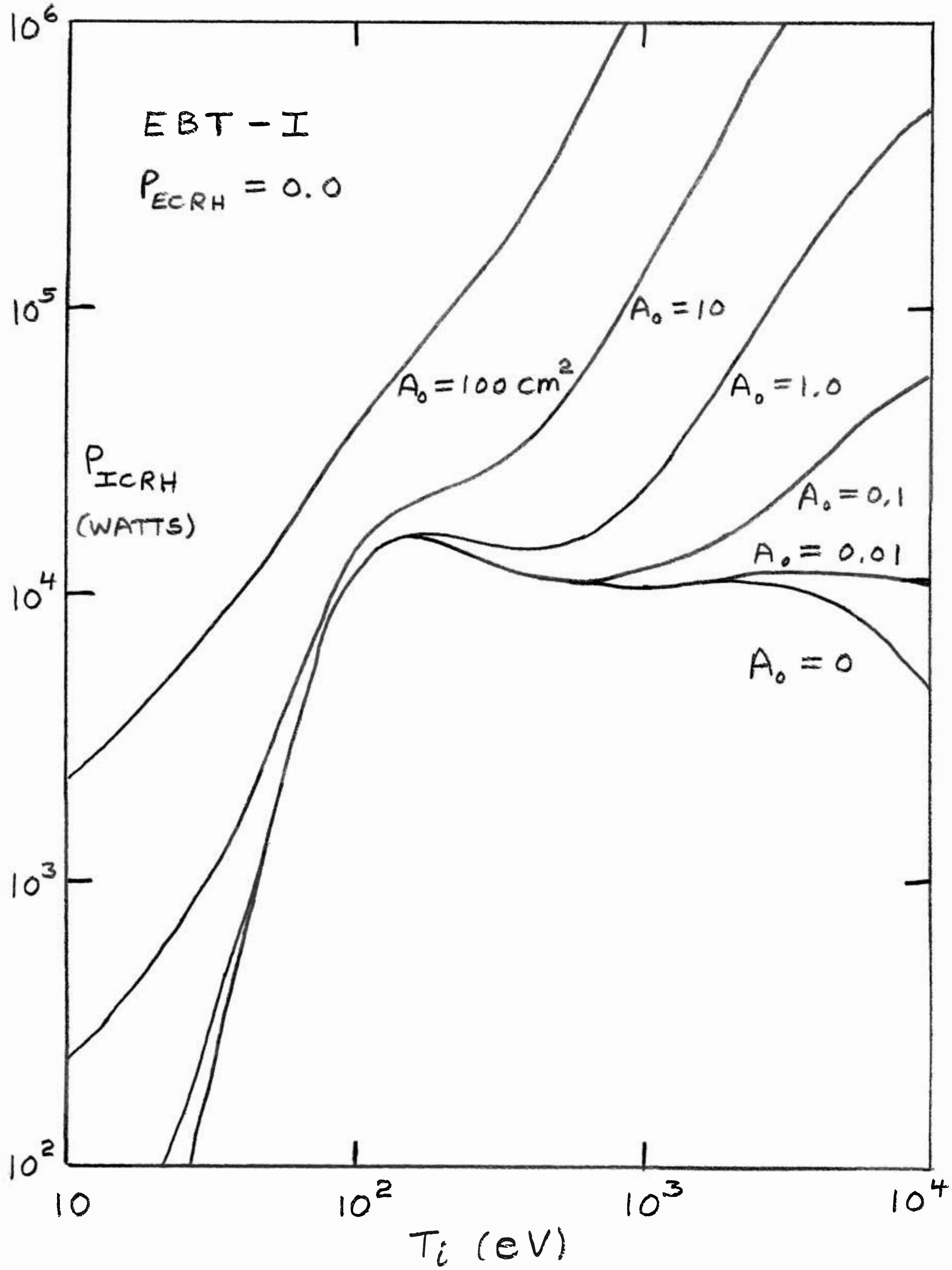
If indeed the ion temperature is thermally unstable as predicted by collisionless neoclassical theory, one might wish to devise techniques whereby the temperature could be stabilized at an arbitrary value. Three such techniques are discussed below.

1. An obstacle of area A_o (in cm^2) inserted into the plasma will limit the particle lifetime to a value given by

$$\tau = \frac{2.56 \times 10^{-6} V}{A_o \, T_i^{\frac{1}{2}}} \quad . \tag{55}$$

This loss was added to the neoclassical loss (assuming Φ = o, since the obstacle would likely short - circuit any radial ambipolar field), and the power required to achieve a given ion temperature was calculated for various value of A_o using the EBTICH point model code. The result is shown in figure 12. Note that only a very small obstacle area is required. This poses some difficulty since the obstacle is required to dissipate most of the input ICRH power. However, the fact that the obstacle short - circuits the radial electric field could be an advantage if large ambipolar potentials are somehow detrimental to the stability or confinement. In fact, the obstacle could be biased to keep any desired space potential. Furthermore, if the obstacle were a thin rod traversing the torus diameter, its cross-sectional area could be varied to control the plasma pressure profile.

2. A second possibility is the controlled introduction of impurities. For example, methane has a charge exchange cross-section considerably higher than hydrogen. A 10% concentration of methane would probably suffice to insure thermal stability at all temperatures of interest.


EBT - I
$P_{ECRH} = 0.0$
P_{ICRH} (WATTS)
$A_0 = 100\ cm^2$
$A_0 = 10$
$A_0 = 1.0$
$A_0 = 0.1$
$A_0 = 0.01$
$A_0 = 0$
10^2
10^3
10^4
10^5
10^6
10
T_i (eV)

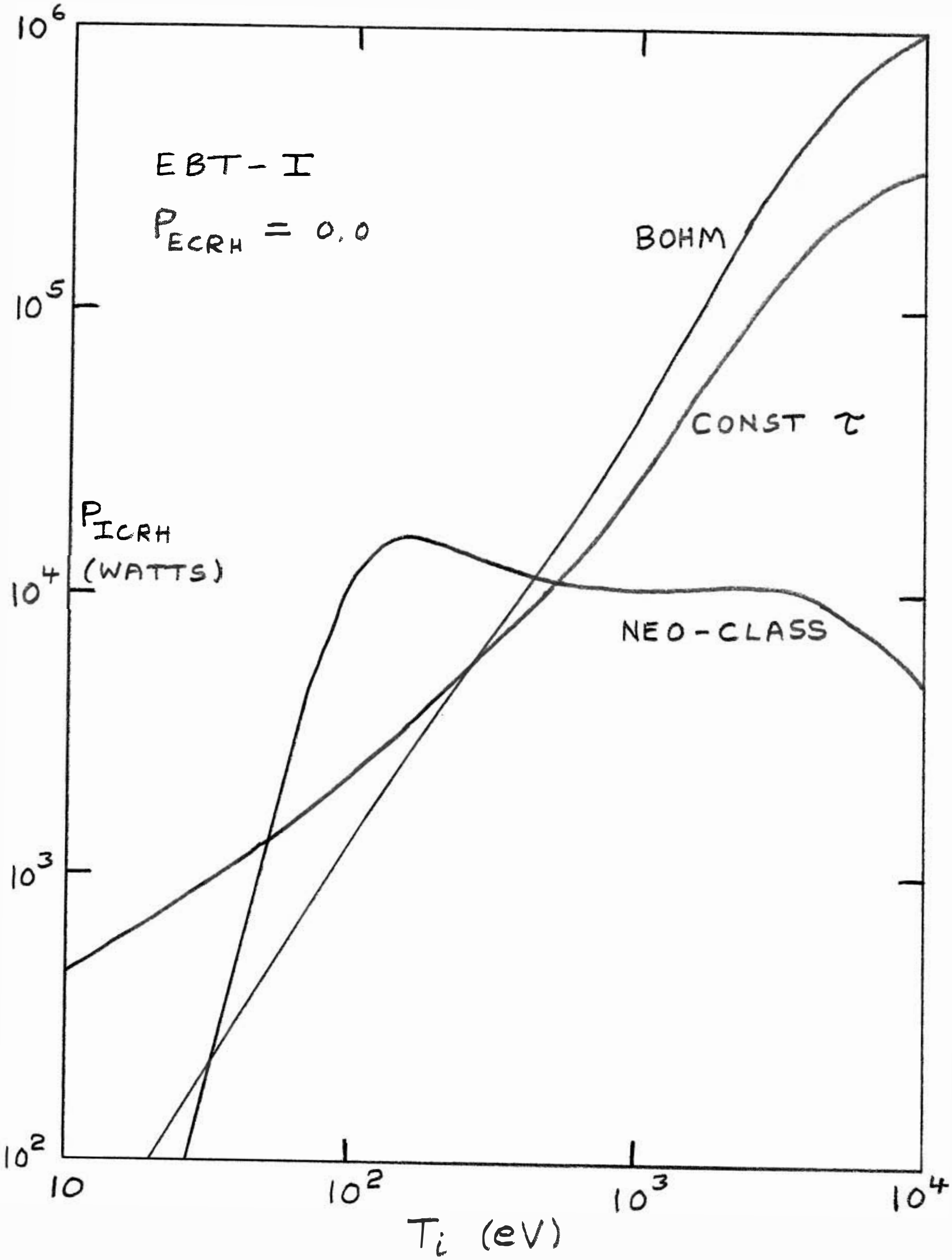
3. Probably the most appealing technique is to monitor the ion temperature and apply feedback control to the ICRH source. Since the thermal instability is a very gentle one ($d P_{ICRH}/dT_i$ is small), and since the growth rate of the temperature is slow ($\tau \sim 0.25$ sec for a 10% excess of power), it should be a rather simple matter to construct an appropriate feedback loop. It may also be possible to sense a different parameter, such as space potential, or feedback regulate something other than the ICRH power, such as edge pressure.

Thermal instability is not likely to pose a fundamental limitation to an ion cyclotron heated EBT, even if it does occur.

LIMITS OF EBT PERFORMANCE

The neoclassical theory previously described is the most optimistic transport scaling that one could suppose for a EBT. In this section, two considerably more pessimistic scalings will be considered. (1). The first is to assume that the particle confinement time remains constant at 0.02 sec independent of temperature. This is the scaling observed in tokamaks, and it also reasonably well fits the most recent ORNL EBT-I experimental data. The point model prediction of the ICRH power required for this case is shown in figure 13. (2). The second is the very pessimistic Bohm scaling with a coefficient adjusted to give $\tau = 0.02$ sec for $T_e = 150$ eV and $B = 10$ kG:

$$\tau = 0.0003\ B/T_e. \tag{56}$$



MCDONNELL DOUGLAS CORPORATION

FUSION ELMO BUMPY TORUS ENERGY

The results are shown in figure 13. It should be emphasized that there is no reason to suppose a Bohm scaling, but it encompasses a variety of eventualities such as loss of equilibrium, field errors, and some types of instabilities. The true behavior will undoubtedly lie somewhere between the Bohm and neoclassical scaling, barring some catastrophic instability outside the scope of transport theory. Summarized below are the results of the three scaling laws for an assumed input ICRH power of 100 kW for EBT-I with $n = 2 \times 10^{12}$ cm^{-3} and $P_{ECRH} = 0.0$:

	T_i	Te	n_o	τ
Neoclassical	$\gtrsim$10,000 eV	$\gtrsim$4,000 eV	$\lesssim 9 \times 10^6$cm^{-3}	$\gtrsim$1.8 sec
τ = 0.02 sec	2,500 eV	440 eV	7.4×10^8cm^{-3}	0.02 sec
Bohm	1,600 eV	280 eV	1.3×10^9cm^{-3}	0.01 sec