CALCULATION OF ECRH REDUCTION AT HIGH DENSITIES

by

J. C. Sprott

and

R. B. Campbell

September, 1977

PLP 735

Plasma Studies

University of Wisconsin

CALCULATION OF ECRH REDUCTION AT HIGH DENSITIES

R. B. Campbell and J. C. Sprott

No ECRH experiment in a plasma device large compared with the free space wavelength at the heating frequency has achieved densities greater than a few tenths of the critical density defined by $\omega_p = \omega$. Experiments in the small Toroidal Octupole at Wisconsin[1] showed that the heating rate drops precipitously for $\omega_p^2 \gtrsim 0.1\ \omega^2$. Wong in PLP 560 provided a quantitative explanation of this phenomenon in terms of the evanescence of the right circularly polarized cyclotron wave in a narrow zone on the low field side of the resonance. However, his calculation assumed a solution of the form $e^{i(k(x)x-\omega t)}$ in a region where $\frac{1}{k^2}\frac{dk}{dx}$ is much greater than unity, and so it is subject to question. Furthermore, the predicted density limit was about half of what was observed experimentally. Since the scaling of density with microwave frequency and magnetic field configuration is critical to understanding the behavior of cyclotron heated bumpy tori (EBT), it is appropriate to attempt a more exact calculation.

The starting point for such a calculation is the one-dimensional wave equation:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega(\omega-\omega_c)}\right]E = 0 .$$

Taylor expanding $\omega_c(x)$ about the resonance at $\omega_c = \omega$ gives

$$\frac{d^2E}{dx^2} + \frac{\omega^2}{c^2}\left(\frac{\Delta x}{x} + 1\right)E = 0$$

where Δx is the width of the evanescent zone:

$$\Delta x = \frac{\omega_p^2}{\omega^2}\left[\frac{\omega_c dx}{d\omega_c}\right]_{\omega_c = \omega} .$$

The resulting differential equation is highly non-linear, and so it was solved numerically in the evanescent region $\Delta x < x < 0$ where $E(x)$ is real. The boundary conditions were taken as

$$E\ (x = -\Delta x) = E_o$$

and

$$\frac{dE}{dx}\ (x = -\Delta x) = 0\ .$$

The second condition results from the fact that $k = 0$ at $x = -\Delta x$. The numerical solution gives $E(x = 0)$, and since the heating rate scales as $E^2(o)$, the normalized heating rate is $E^2(o)/E_o{}^2$, which turns out to be a function only of the quantity $\omega_p{}^2L/\omega^2\lambda$ where

$$L = \omega_c \Big/ \left.\frac{d\omega_c}{dx}\right|_{\omega_c = \omega} \qquad \text{(scale length of field gradient)}$$

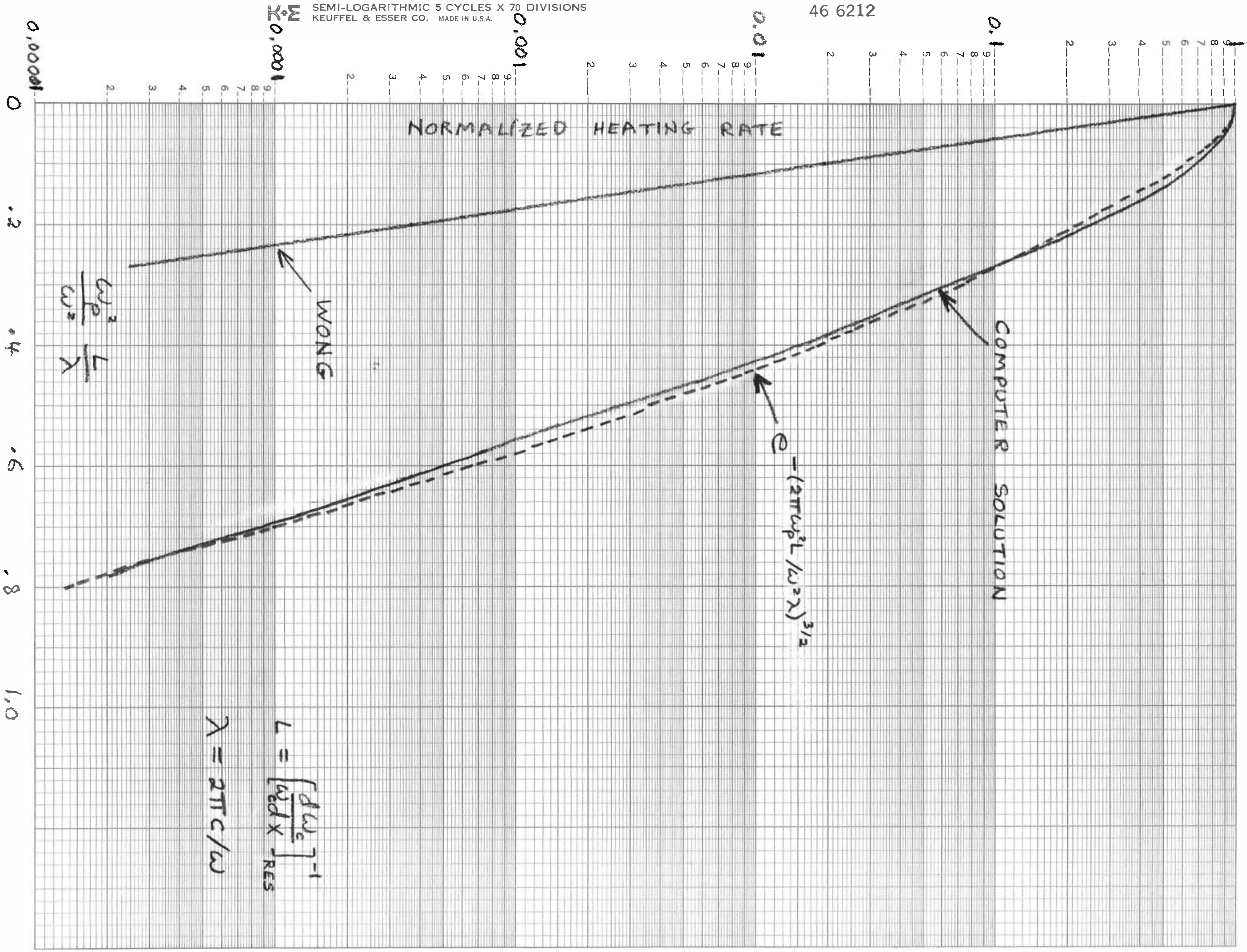
and

$$\lambda = 2\pi c/\omega \qquad \text{(free space wavelength of microwaves).}$$

The result is shown in the figure along with the calculation by Wong for comparison. The two results are seen to differ significantly. A useful analytical expression that models the result to within about 20% over the range shown is given by $e^{-(2\pi\omega_p{}^2L/\omega^2\lambda)^{3/2}}$. Roughly speaking, the ECRH becomes ineffective whenever the normalized heating rate falls below $1/Q$ where Q is the quality factor of the microwave cavity in the absence of plasma (typically 10^4). This result is believed to account for the density limit in the Octupole, EBT, and other ECRH devices. It is not an optimistic scaling for large devices with high frequency microwaves ($L/\lambda >> 1$), however, since the density in such devices is predicted to fall considerably short of the critical density.

---

[1] J. D. Barter, J. C. Sprott, and K. L. Wong, Phys. Fluids 17, 810 (1974).



SEMI-LOGARITHMIC 5 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.
46 6212
NORMALIZED HEATING RATE
0.00001
0.0001
0.001
0.01
0.1
0
.2
.4
.6
.8
1.0
WONG
COMPUTER SOLUTION
$e^{-(2\pi\omega_p^2 L/\omega^2\lambda)^{3/2}}$
$\frac{\omega_p^2}{\omega^2}\frac{L}{\lambda}$
$L = \left[\frac{d\omega_c}{\omega_c dx}\right]^{-1}_{RES}$
$\lambda = 2\pi c/\omega$