HARMONIC CYCLOTRON HEATING IN THE TOROIDAL OCTUPOLE

J. C. Sprott

October 1976

Plasma Studies PLP 701

University of Wisconsin

Previous calculations of cyclotron resonance heating (both electron and ion) in the toroidal octupole have considered only the effect of resonance at the fundamental. This was justified by noting that harmonic absorption scales like some positive power of $k_{\perp}\rho$ which is a small quantity for most of our experiments. The purpose of this note is to examine more carefully the conditions under which the above approximation is valid and to derive formulas for calculating the harmonic cyclotron heating of both electrons and ions.

It is well known (see, for example, O. Eldridge, Phys. Fluids 15, 676 (1972)) that the heating rate for a group of non-relativistic, monoenergetic, charged particles (charge e) in a uniform magnetic field B with a perpendicular rf electric field $E_{\perp}$ (frequency ω, wavenumber k) is given by

$$\frac{dW}{dt} = \frac{\pi}{2} \; e \, E_{\perp}^{2} \sum_{N=1}^{\infty} \int J_{N-1}^{2} \left(\frac{Nk_{\perp}v_{\perp}}{\omega}\right) \delta(NB - B_o),$$

where $B_o = M\omega/e$. For a non-uniform field, the heating rate can be averaged over the plasma volume assuming the particle density to be constant in space, giving the result

$$\frac{d\overline{W}}{dt} = \frac{\pi}{2V} \int \sum_{N=1}^{\infty} J_{N-1}^{2} \left(\frac{Nk_{\perp}v_{\perp}}{\omega}\right) e \, E_{\perp}^{2} \; \delta(NB-B_o) \; dV.$$

If we further assume an isotropic Maxwellian distribution and average the above equation over all velocities, we obtain the result

$$\frac{d\overline{\overline{W}}}{dt} = \frac{\pi}{2V} \int \sum_{N=1}^{\infty} e^{-x} \; I_{N-1} \; (x) \; eE_{\perp}^{2} \, \delta(NB-B_o) \; dV,$$

where $X = NKT \; k_{\perp}^{2} / M\omega^{2}$ and $I_{N-1}(x)$ is a hyperbolic Bessel function of order N - 1.

To illustrate the magnitude of the finite temperature effect, we can expand the above expression in powers of the quantity x keeping only terms of first order:

$$\frac{d\overline{W}}{dt} \simeq \frac{\pi}{2V} \int \sum_{N=1}^{2} (1-X) \left(\frac{X}{2}\right)^{N-1} eE_\perp^2 \, \delta(NB-B_o) \, dV.$$

This expression can be rewritten in a more familiar form as

$$\frac{d\overline{W}}{dt} = \frac{eE_{\perp o}^{\;2}}{B_o} (G_1 + G_2)$$

where $E_{\perp o}$ is the electric field at some reference position and G_1, G_2 are dimensionless, temperature-dependent heating efficiencies for the fundamental and second harmonic respectively:

$$G_1(B_o) \simeq \frac{\pi}{2} \frac{B_o}{V} \int (1-X) \frac{E_\perp^2}{E_{\perp o}^2} \delta(B - B_o) \, dV$$

$$G_2(B_o) \simeq \frac{\pi}{2} \frac{B_o}{V} \int X \frac{E_\perp^2}{E_{\perp o}^2} \delta(B - B_o/2) \, dV.$$

Note that in the cold plasma ($X = 0$) limit, $G_2 = 0$ and G_1 reduces to the ordinary G that we have previously used. The effect of non-zero temperature is in lowest order to reduce slightly the fundamental heating and to add some heating at the second harmonic.

It can also be shown that the dominant heating term for the N-th harmonic can be written as

$$G_N(B_o) \cong \frac{\pi}{2} \frac{B_o}{V} \int \frac{N}{\Gamma(N)} \left(\frac{X}{2}\right)^{N-1} \frac{E_\perp^2}{E_{\perp o}^{\;2}} \delta(B - B_o/N) \, dV,$$

so that

$$G_3(B_o) \cong \frac{3\pi}{16} \frac{B_o}{V} \int X^2 \frac{E_\perp^2}{E_{\perp o}^{\;2}} \delta(B - B_o/3) \, dV, \text{ etc.}$$

<u>Electron Cyclotron Heating</u>

Consider first the low density ECRH case in which the electric fields are homogeneous and isotropic and have a wavelength equal to that in free space. Then the largest value X may have is NKT/mc^2. Since $mc^2 = 511$ keV, it is clear that for our plasmas G_1 never departs significantly from the usual G, and G_2 is important only in cases where G_1 is zero (no fundamental resonance present).

In order to evaluate G_2, we average over the spectrum of $k_\perp$ to obtain

$$G_2(B_o) \cong \frac{2\pi}{3} \frac{B_o}{V} \int \frac{KT}{mc^2} \delta(B - B_o/2)\, dV.$$

For temperature constant in space, G_2 can be written

$$G_2(B_o) \cong \frac{4}{3} \left(\frac{KT}{mc^2}\right) G(B_o/2)$$

Similarly, for G_3 we obtain

$$G_3(B_o) \cong \frac{27}{20} \left(\frac{KT}{mc^2}\right)^2 G(B_o/3), \text{ etc.}$$

Therefore, if $G(B_o)$ is known, no additional calculation is required to get G_2, G_3, etc.

<u>Ion Cyclotron Heating</u>

For the ICRH case, no obvious simplification is possible since both $E_\perp$ and $k_\perp$ vary greatly over the volume. However, if we take X small, T constant in space, and $k_\perp = \nabla_\perp E_\perp / E_\perp$, the second harmonic term can be written as

$$G_2(B_o) \cong \pi \frac{B_o}{V} \left(\frac{KT}{M\omega^2}\right) \int \left(\frac{\nabla_\perp E_\perp}{E_{\perp o}}\right)^2 \delta(B - B_o/2)\, dV.$$

Once G_2 is known, the first order correction to G_1 can be calculated from

$$G_1(B_o) \cong G(B_o) - \frac{1}{2} G_2(2B_o).$$

In order to calculate G_2 an existing computer code (GICALC) which calculates G for protons in the small toroidal octupole with a fifth hoop in the midcylinder near the floor was modified. Figure 1 shows $G(B_o)$, $G_2(B_o)$, and $G_1 + G_2 = G(B_o) + G_2(B_o) - 1/2\ G_2(2\ B_o)$ for $KT/f^2 = 100$ eV/MHz^2, where $B_o = 0.615\ f\ (MHz)/B(kG)$ and B is measured at the intersection of the midplane with the outer wall. It is clear that for the temperatures now being achieved in the octupole, the harmonic corrections are not negligible. The general effect of non-zero temperature is to broaden the resonance that occurs when the resonance zone is just above the fifth hoop.

In order to proceed further, we calculate numerically a $\Delta G(B_o)$ which is to be added to the zero temperature $G(B_o)$ as a first order temperature correction:

$$\Delta G(B_o) \cong G_2(B_o) - \frac{1}{2} G_2\ (2\ B_o),$$

and fit the results from GICALC to an analytic function as follows:

$$\Delta G = \begin{cases} -1.75 \times 10^{-5}\ B_o^{3.8}\ \dfrac{KT}{f^2} & B_o < 0.9082 \\[2ex] (3.95 \times 10^{-5} - 4.8 \times 10^{-5}\ B_o)\ \dfrac{KT}{f^2} & 0.9082 \leq B_o \leq 1.9204 \\[2ex] 9.5 \times 10^{-5}\ B_o^{-1.88}\ \dfrac{KT}{f^2} & B_o > 1.9204 \end{cases}$$

where KT is in eV and f is in MHz.

This correction was then added to program SIMULT which calculates the spatially-averaged ion temperature and other plasma properties as a function of time in the small octupole with ICRH. (The second and third harmonic terms for ECRH were also added to SIMULT, but that has little effect for normal operation as shown previously.) Figure 2 shows the predicted peak ion temperature vs ICRH frequency for a typical case without (o) and with (x) the finite temperature correction. In all cases, the result is to decrease KT_i by ~ 20 - 30 eV. Figure 3 shows the peak ion temperature vs the poloidal bank voltage for fixed ICRH frequency (2 MHz) again without (o) and with (x) the finite temperature correction. At low bank voltages, the ion heating is somewhat enhanced by the second harmonic effect, but for the higher bank voltages more typically used, the heating is uniformly reduced.

In summary, we have shown that the finite temperature correction for ICRH, though not negligible, is not of great importance. Other finite temperature effects such as the relativistic mass increase, doppler effects, and non-circular gyromotion due to ∇B, have been neglected. Relativity is completely unimportant for ions (MC^2 = 938 MeV), and the doppler effects mainly shift the position of the resonance, but don't greatly effect the heating rate. The ∇B effect may be important since $\nabla B/B \sim \nabla E/E$, but the calculation of the resulting heating is beyond the scope of this paper, and might better be a part of someone's Ph.D. thesis.

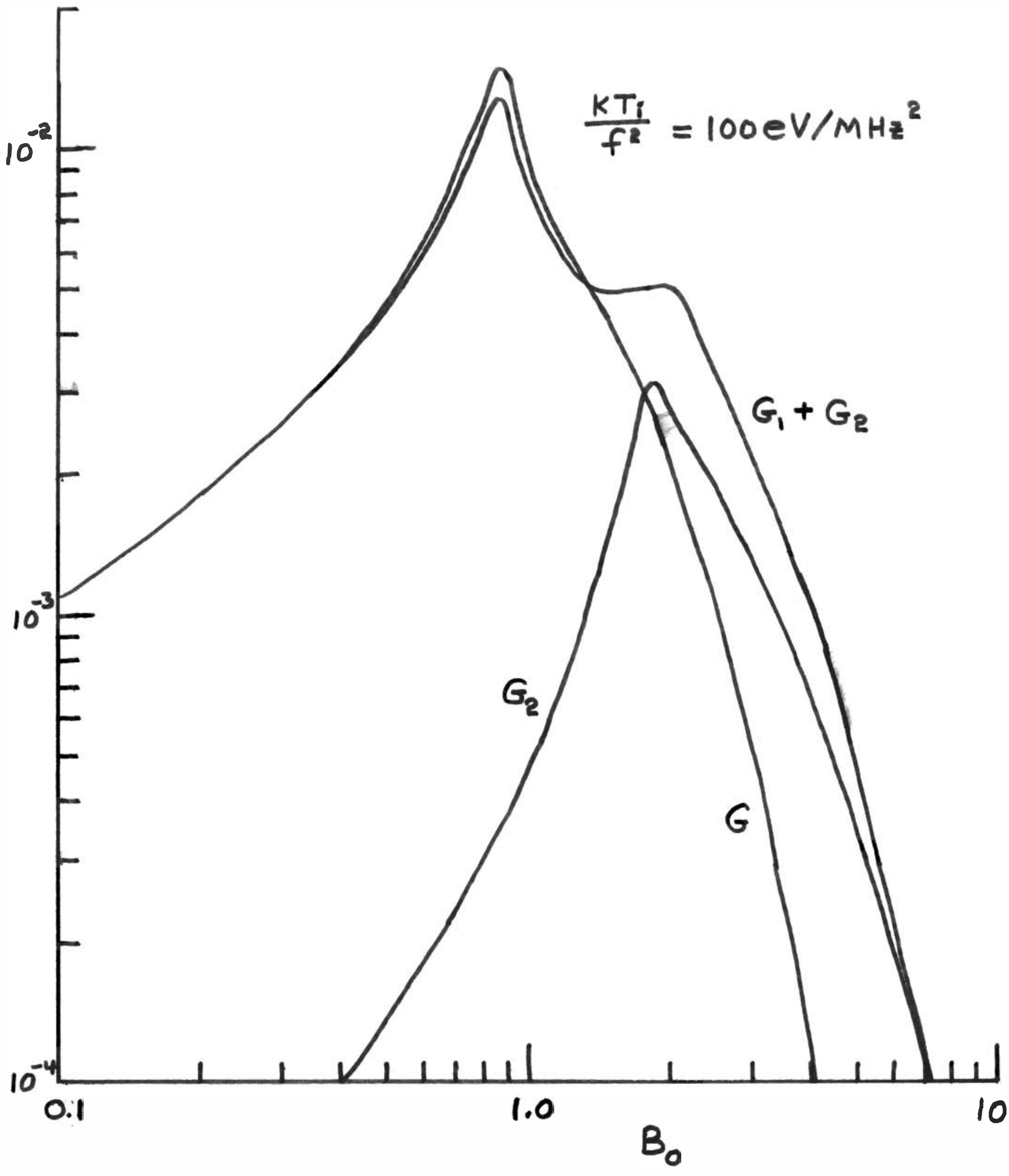

FIG 1

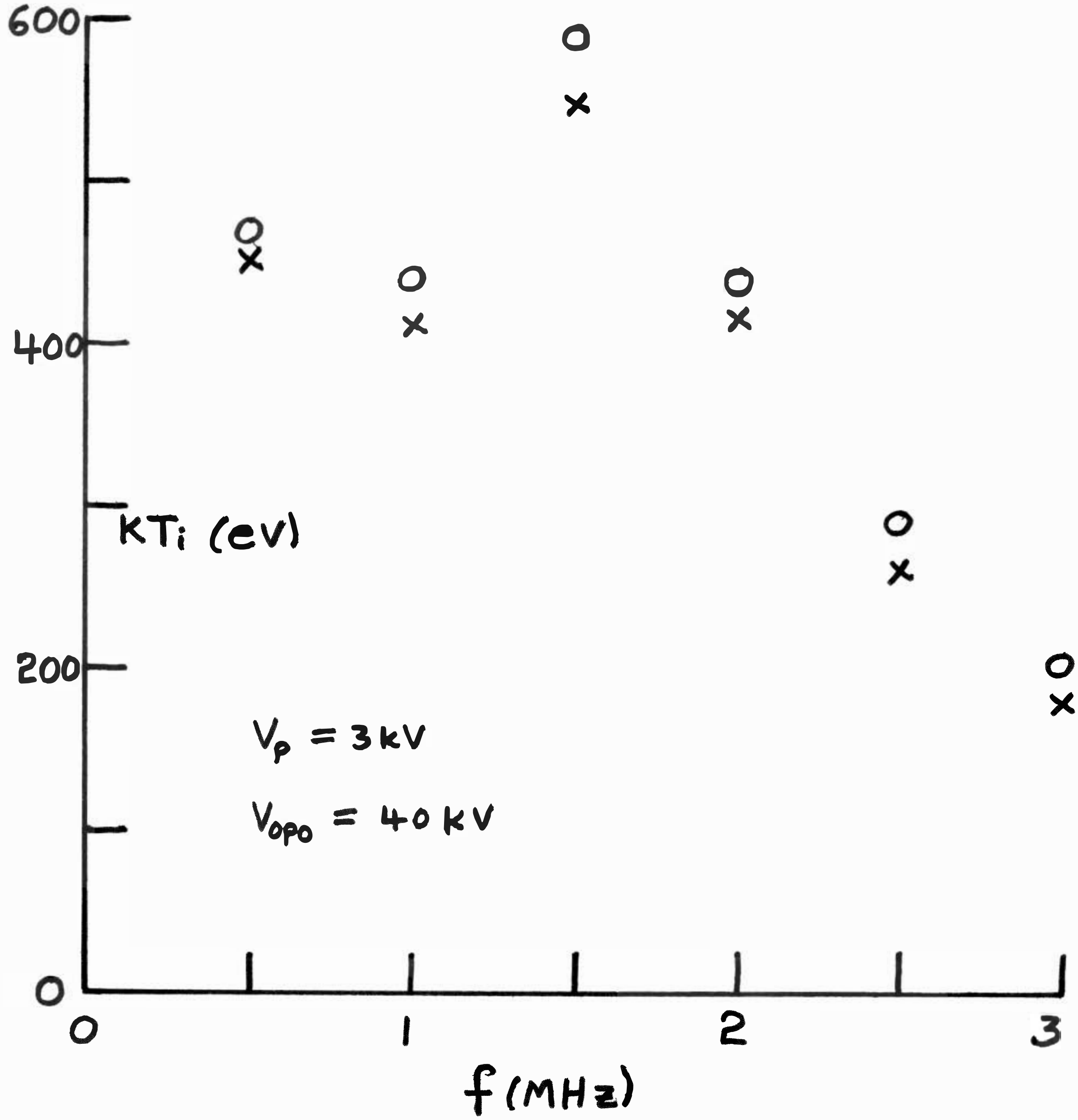

FIG 2

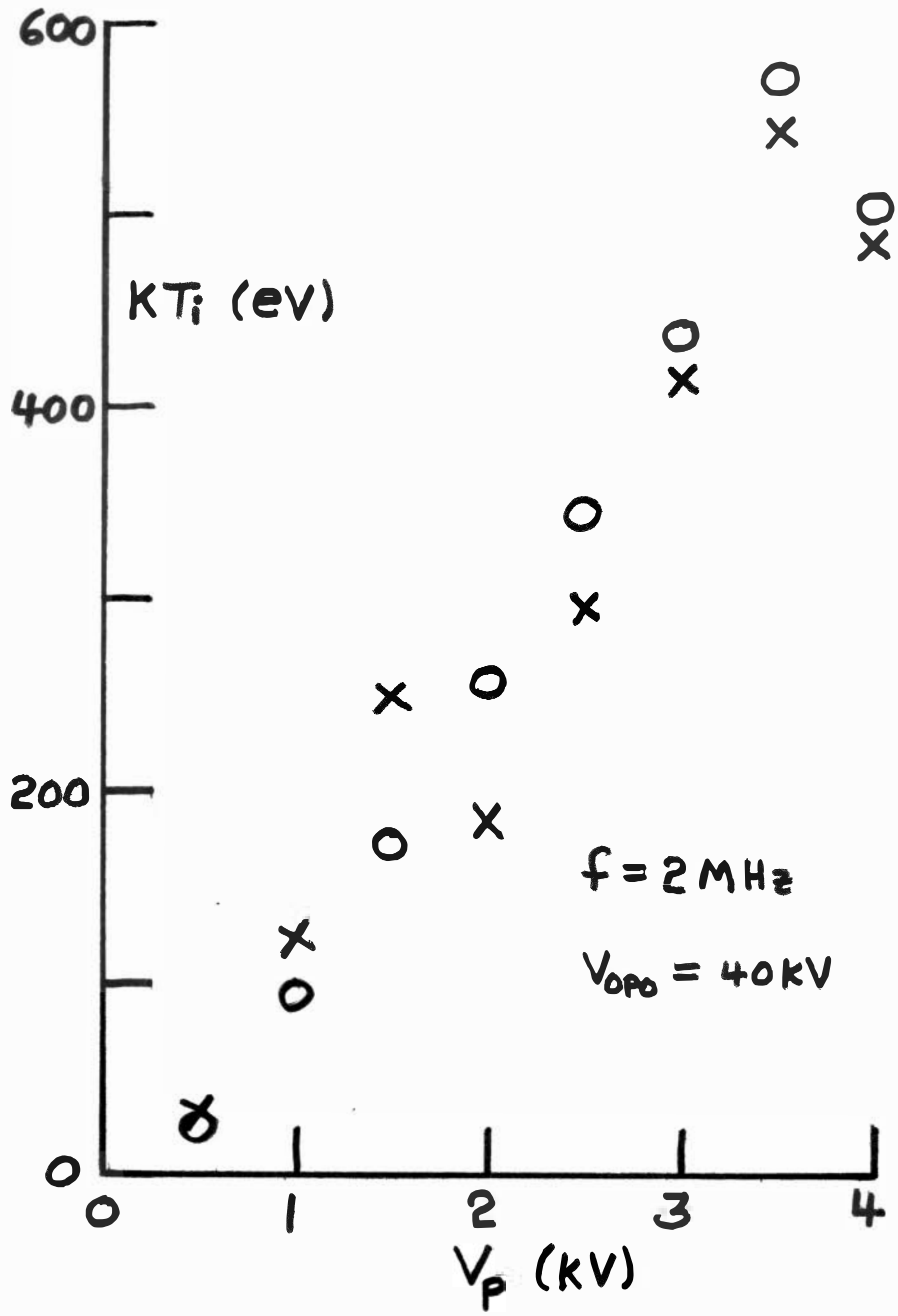

FIG 3