COMPUTER CALCULATIONS OF PULSE FORMING NETWORK BEHAVIOR

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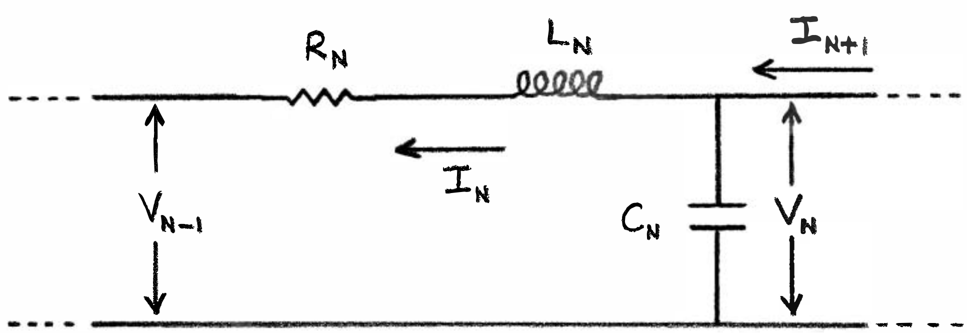
This note describes a computer code (PFNCAL) that calculates the output pulse shape of an arbitrary, voltage fed, E-section, pulse forming network. The code was written to study the feasibility of producing strange-shaped (i.e.: non-rectangular) pulses of ECRH and ICRH power for plasma heating. In particular, it appears that increasing the ECRH power as a function of time may lead to denser plasmas than the same amount of energy delivered at constant power.

A pulse forming network is basically a lumped constant transmission line each section of which can be represented as below:



The behavior of the section is determined by a voltage (loop) equation,

$$V_N - V_{N-1} = I_N R_N + L_N \frac{dI_N}{dt} ,$$

and a node (current equation),

$$I_{N+1} - I_N = C_N \frac{dV_N}{dt} .$$

With a number of sections N_S, the behavior of the line is uniquely determined by a set of $2N_S$ linear, first-order, differential equations plus a set of boundary conditions which are generally given by $V_N = V_0$ and $I_N = 0$ for all N. The first section (N = 1) which is generally terminated in a load resistor (R_L) and the last section ($N = N_S$) must be treated as special cases:

$$V_1 - I_1R_L = I_1R_1 + L_1 \frac{dI_1}{dt}$$

$$- I_{N_S} = C_{N_S} \frac{dV_{N_S}}{dt} .$$

To make the problem even more general, we allow some mutual inductance between adjacent sections, but for simplicity, we take the coupling coefficient (k) to be the same for all sections. This modifies the voltage equation as follows:

$$V_N - V_{N-1} = I_NR_N + L_N(1-2k)\frac{dI_N}{dt} + kL_N\left(\frac{dI_{N-1}}{dt} + \frac{dI_{N+1}}{dt}\right) .$$

Unfortunately, this represents a significant complication for the numerical method used, and so we simplify the problem by assuming k << 1 (as is usually the case) and keep terms only to first order in k:

$$V_N - V_{N-1} \cong (1-2k)(I_NR_N + L_N \frac{dI_N}{dt}) + k\ (V_{N+1} - V_N + V_{N-1} - V_{N-2}) .$$

For this case, the first two sections (N = 1 and 2) and the last section ($N = N_S$) have to be treated separately:

$$V_1 - I_1R_L = (1-k)(I_1R_1 + L_1 \frac{dI_1}{dt}) + k\ (V_2 - V_1)$$

$$V_2 - V_1 = (1 - 2k)(I_2R_2 + L_2 \frac{dI_2}{dt}) \pm k\ (V_3 - V_2 + V_1 - I_1R_L)$$

$$V_{N_S} - V_{N_S-1} = (1-k)(I_{N_S}R_{N_S} + L_{N_S} \frac{dI_{N_S}}{dt}) + k\ (V_{N_S-1} - V_{N_S-2})\ .$$

The computer code solves this set of $2N_S$ simultaneous, linear, differential equations (up to $N_S = 50$) using the MACC predictor-corrector subroutine DEPC. A Fortran listing of the code which includes provisions for a line-printer graph of the output is included in the appendix. The user need specify only the values of the arrays R_N, L_N, and C_N and the values of N_S, R_L, k, and the time at which the computation is to end. All voltages are normalized to the initial voltage on the line. A sample of typical output is shown for a line of 10 identical sections except for the end sections which have 25% extra inductance, terminated in its characteristic impedance ($R_L = L_N/C_N$) with a total resistance that is 20% of R_L ($R_N = 0.2\ R_L/N_S$), and a coupling coefficient of 0.15. This is typical of lines which are optimized to give a rectangular pulse with a droop of ~ 10%. The calculation takes ~ 7 seconds of 1110 computer time and costs ~ 80 cents.

A variety of cases have been run including ones in which the inductance increases with distance from the load (which steepens the rise and introduces a significant droop) and cases in which the inductance decreases with distance from the load (which produces a monotonically rising waveform). More complicated cases will be examined if there appears to be some benefit to be gained from the standpoint of plasma heating.

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*  *  *  *  UNIVAC 1110 TIME/SHARING EXEC --- MULTI-PROCESSOR SYSTEM --- VER.  MACC
NID * CW1283        FILE NAME * PR@000CW1283        PART NUMBER *  00       DATE * 082575



SPROTT,2980,4126810219
$    ,PFNCAL
N-MACC 1.14S-08/25/75-16:12:33                        PFNCAL
          C       PROGRAM PFNCAL  -  J. C. SPROTT  -  AUG 21, 1975
                  DIMENSION VI(100),VF(101),SAVE(101,100),TIME(101),AI(101)
                  COMMON NS,NN,RL,AK,AL(50),C(50),R(50)
                  EXTERNAL DERIVS
          C       NS IS THE NUMBER OF SECTIONS IN THE LINE (MAX 50)
                  NS=10
          C       RL IS THE LOAD RESISTANCE
                  RL=1.0
          C       AK IS THE COUPLING COEFFICIENT BETWEEN SECTIONS
                  AK=0.15
          C       TEND IS THE TIME AT WHICH THE COMPUTATION ENDS
                  TEND=4.0
                  DO 100 I=1,NS
                  AL(I)=1.0/FLOAT(NS)
                  C(I)=1.0/FLOAT(NS)
                  R(I)=0.2/FLOAT(NS)
                  VI(2*I-1)=0.0
                  VI(2*I)=1.0
             100  CONTINUE
                  AL(1)=1.25*AL(1)
                  AL(NS)=1.25*AL(NS)
                  NN=2*NS
                  DT=0.01*TEND
                  WRITE(6,300)
             300  FORMAT(1H1,'  STEP          TIME        CURRENT')
                  CALL DEPC(NN,0,0,VI,TEND,VF,DERIVS,1.0E-4,1.0E-4,1.0E-4,1.0,0.01,2
                 2,DT,2H..,SAVE,101,NN,NPOINT,NOTIFY,$400)
             400  CONTINUE
                  DO 600 J=1,NPOINT
                  TIME(J)=FLOAT(J-1)*DT
                  AI(J)=SAVE(J,1)
                  WRITE(6,500) J,TIME(J),AI(J)
             500  FORMAT(1H ,I6,2F13.4)
             600  CONTINUE
                  CALL GRAPH2(TIME,'R',AI,'R',NPOINT,'SMALL','AUTO','CURRENT IN PULS
                 2E FORMING NETWORK LOAD..','TIME..','CURRENT..','*')
                  CALL GRPHND
                  END

 END OF COMPILATION:          NO  DIAGNOSTICS.
$    ,DERIVS
N-MACC 1.14S-08/25/75-16:12:38                        DERIVS
                  SUBROUTINE DERIVS(TIME,V,DV,STORE,ITEST)
                  DIMENSION V(1),DV(1),STORE(1)
                  COMMON NS,NN,RL,AK,AL(50),C(50),R(50)
                  VO=V(1)*RL
                  DV(1)=(V(2)-VO+AK*(2.0*V(2)-VO-V(4))-R(1)*V(1))/AL(1)
                  DV(3)=((V(4)-V(2))*(1.0+3.0*AK)+AK*(VO-V(6))-R(2)*V(3))/AL(2)
                  DO 200 I=2,NS
             200  DV(2*I-2)=(V(2*I-1)-V(2*I-3))/C(I-1)
                  DO 250 I=4,NS
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250   DV(2*I-3)=((V(2*I-2)-V(2*I-4))*(1.0+3.0*AK)+AK*(V(2*I-6)-V(2*I))-R
     2(I-1)*V(2*I-3))/AL(I-1)
      DV(2*NS-1)=(V(2*NS)-V(2*NS-2)+AK*(V(2*NS)-2.0*V(2*NS-2)+V(2*NS-4))
     2-R(NS)*V(2*NS-1))/AL(NS)
      DV(2*NS)=-V(2*NS-1)/C(NS)
      DO 300 I=1,NN
300   STORE(I)=V(I)
      RETURN
      END

END OF COMPILATION:          NO  DIAGNOSTICS.
XN
7P-08/25-16:12
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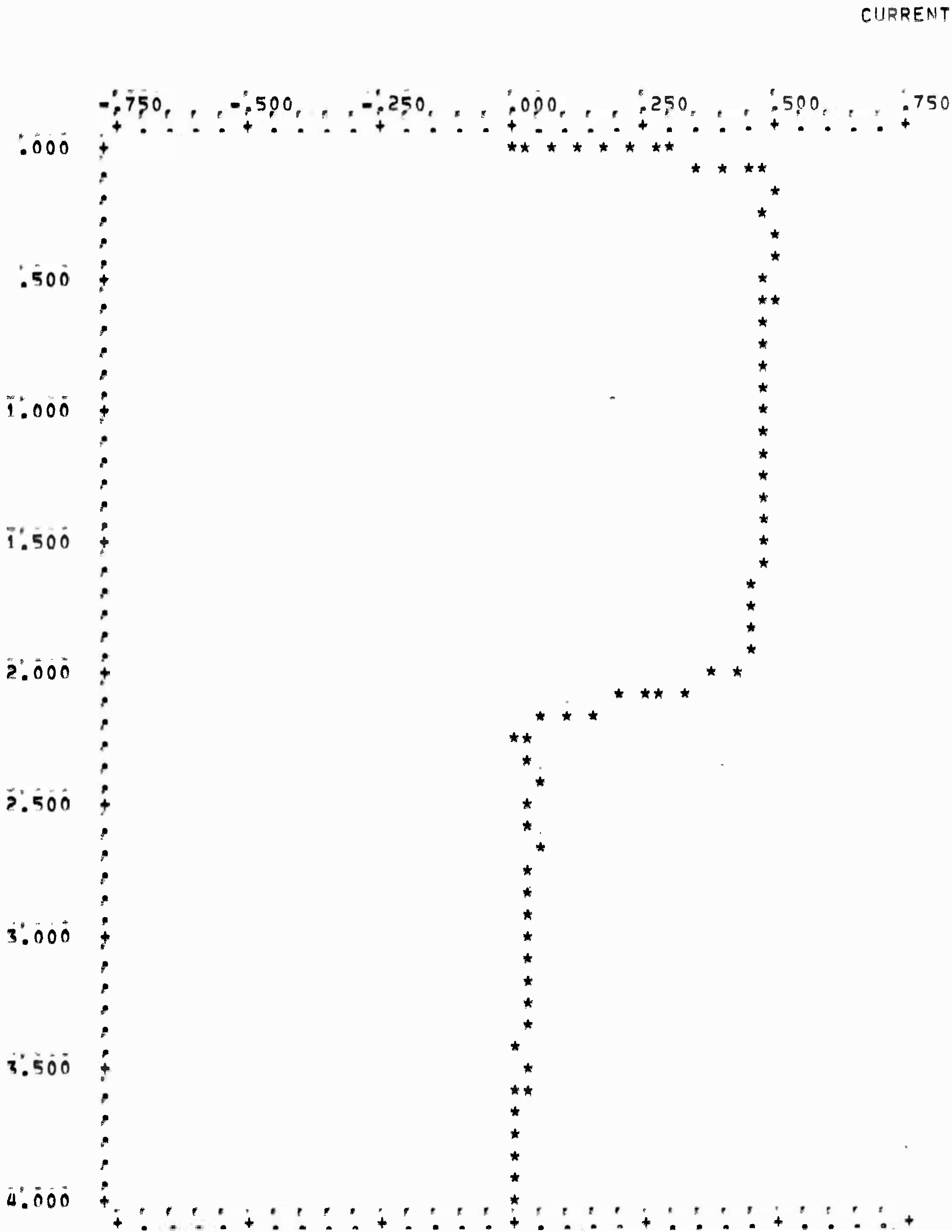
STEP	TIME	CURRENT
1	.0000	.0000
2	.0400	.2972
3	.0800	.4570
4	.1200	.5100
5	.1600	.5043
6	.2000	.4846
7	.2400	.4762
8	.2800	.4824
9	.3200	.4932
10	.3600	.4981
11	.4000	.4945
12	.4400	.4871
13	.4800	.4824
14	.5200	.4830
15	.5600	.4864
16	.6000	.4884
17	.6400	.4867
18	.6800	.4827
19	.7200	.4794
20	.7600	.4788
21	.8000	.4802
22	.8400	.4812
23	.8800	.4802
24	.9200	.4776
25	.9600	.4750
26	1.0000	.4740
27	1.0400	.4744
28	1.0800	.4749
29	1.1200	.4743
30	1.1600	.4724
31	1.2000	.4702
32	1.2400	.4689
33	1.2800	.4688
34	1.3200	.4689
35	1.3600	.4685
36	1.4000	.4673
37	1.4400	.4660
38	1.4800	.4655
39	1.5200	.4659
40	1.5600	.4662
41	1.6000	.4648
42	1.6400	.4610
43	1.6800	.4553
44	1.7200	.4496
45	1.7600	.4465
46	1.8000	.4476
47	1.8400	.4528
48	1.8800	.4589
49	1.9200	.4597
50	1.9600	.4467
51	2.0000	.4118
52	2.0400	.3514
53	2.0800	.2693
54	2.1200	.1779
55	2.1600	.0945
56	2.2000	.0352

57	2.2400	.0083
58	2.2800	.0104
59	2.3200	.0282
60	2.3600	.0456
61	2.4000	.0513
62	2.4400	.0434
63	2.4800	.0292
64	2.5200	.0188
65	2.5600	.0189
66	2.6000	.0284
67	2.6400	.0402
68	2.6800	.0460
69	2.7200	.0421
70	2.7600	.0314
71	2.8000	.0210
72	2.8400	.0171
73	2.8800	.0211
74	2.9200	.0288
75	2.9600	.0338
76	3.0000	.0321
77	3.0400	.0249
78	3.0800	.0172
79	3.1200	.0140
80	3.1600	.0167
81	3.2000	.0222
82	3.2400	.0257
83	3.2800	.0239
84	3.3200	.0180
85	3.3600	.0121
86	3.4000	.0103
87	3.4400	.0131
88	3.4800	.0177
89	3.5200	.0197
90	3.5600	.0170
91	3.6000	.0108
92	3.6400	.0049
93	3.6800	.0025
94	3.7200	.0041
95	3.7600	.0073
96	3.8000	.0095
97	3.8400	.0092
98	3.8800	.0071
99	3.9200	.0042
100	3.9600	.0000
101	4.0000	-.0075

CURRENT IN PULSE FORMING NETWORK LOAD



@FIN

RUNID: CW1283 PROJECT: 02980 USER: 4126810219

ITEM	AMOUNT	COST(DOLLARS)
CPU TIME	00:00:06.996	$0.26
FILE I/O REQUESTS	239	$0.11
FILE I/O WORDS	222142	$0.10
MEMORY USAGE	0.202	$0.12
CARDS IN	62	$0.02
PAGES PRINTED	7	$0.09
ER + CC	6	$0.06
JOB CHARGE	1	$0.05
TOTAL COST		$0.81

THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR WH'

USER BALANCE $112.11

INITIATION TIME: 16:12:32-AUG 25,1975

TERMINATION TIME: 16:13:30-AUG 25,1975

PREVIOUS RUN TIME: 13:08:18-AUG 22,1975

EOF 6 PAGES COST $0.08 USER BALANCE $112.12