COMMENTS ON "A STOCHASTIC MODEL OF ELECTRON CYCLOTRON HEATING"

J. C. Sprott

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In an earlier paper, GRAWE (1969) proposed a theoretical model which is often cited as the explanation of the off-resonance heating observed in microwave produced, mirror confined plasmas such as described by DANDL et al (1971). This note will show that 1) Grawe's result depends crucially on a hypothesis that is without physical basis, 2) an error in his mathematics led to an erroneous result, and 3) even when corrected the theory predicts a heating rate orders of magnitude less than is observed experimentally.

A collection of charged particles can be heated only if in the particles' frame of reference the electric field has a fourier component at zero frequency. All other components lead to a periodic oscillation of the particles' energy. A zero frequency component is present if the particles cross a region of gyroresonance ($\omega=\omega_c$) or if some collision mechanism is present to broaden the frequency spectrum of the electric field as seen by the particles. Grawe provided such a mechanism when he hypothesized that the bounce time of an electron between the mirrors is the characteristic time for phase randomization to occur.

If phase randomization does occur in a bounce time, the disturbing feature of Grawe's calculation is not that off-resonance heating occurs, but that it is so small at low energies. The collisional heating rate is easily calculated from the conductivity, and is given by

$$\frac{dW_\perp}{dt} = \frac{e^2 E_\perp{}^2(\omega^2+\omega_c{}^2)\nu}{m[(\omega^2-\omega_c{}^2)^2+4\omega^2\nu^2]}$$

For $\nu = \omega_B/2\pi$, and $\omega >> \omega_c$,

$$\frac{dW_\perp}{dt} = \frac{e^2 E_\perp^2 \omega_B}{2\pi m \omega^2} \quad .$$

This rate depends linearly on velocity ($\omega_B \alpha v$) and is very much larger than that calculated by Grawe at low energies.

The discrepancy arises because Grawe in deriving his equation (15) neglected a term (T2) which for $\delta<0$ (off-resonance) and for low energies can be written in his notation as $\frac{1}{2\alpha(1-\delta)}$. For $\omega>>\omega_c$, this additional term gives a heating rate identical to that derived above from the conductivity. For resonance cases and for certain off-resonance cases, the neglected term is unimportant, and Grawe's heating rate agrees well with that calculated in other ways.

Grawe's published results show only cases for which the particles turn very close to the resonance. In the ELMO experiment (DANDL et al, 1971) using 3 cm resonance plus 8 or 5 mm upper off resonance, the particles turn very far from the resonance. If we consider a 500 keV electron that turns at Z_t = 10 cm where the field is 1.5 times the midplane field, and if we assume that B is adjusted so that at a frequency of 10 GHz the electron turns at the resonance surface, then for a typical electric field strength of 10V/cm virtually no heating is expected above about 15 GHz according to Grawe's result, and even if corrected as suggested above, the heating rate at 35 GHz is about 1 keV/sec which is several orders of magnitude smaller than is observed in the Oak Ridge experiments.

A more plausible explanation of the observed heating is that it is a cyclotron resonance phenomenon involving the fundamental and harmonics appropriately shifted by the relativistic mass increase and the doppler effect. Such a theory has been worked out (SPROTT 1972) and gives results more nearly in agreement with experiment.

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J. C. Sprott

Department of Physics
University of Wisconsin
Madison, Wisconsin 53706
U.S.A.

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