NUMERICAL CALCULATIONS OF POWER BALANCE IN THE ELMO BUMPY TORUS

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This paper describes the results obtained in a numerical solution of the steady state particle and power balance equations for the Elmo Bumpy Torus device. The program is an adaptation of program SIMULT which has been remarkably successful in predicting results for the Wisconsin Toroidal Octupoles.

The program uses a UW library subroutine (ZRNEQ) to solve a set of 4 non-linear algebraic equations consisting of a particle balance equation:

$$\left.\frac{dn}{dt}\right|_{\text{ionization}} = \left.\frac{dn}{dt}\right|_{\text{diffusion}} ,$$

two energy balance equations:

$$\left.\frac{dU_e}{dt}\right|_{\mu\text{waves}} = \left.\frac{dU_e}{dt}\right|_{\text{ions}} + \left.\frac{dU_e}{dt}\right|_{\text{excitation}} + \left.\frac{dU_e}{dt}\right|_{\text{Bremsstrahlung}} + \left.\frac{dU_e}{dt}\right|_{\text{synchrotron}}$$

$$+ \left.\frac{dU_e}{dt}\right|_{\text{diffusion}}$$

$$\left.\frac{dU_i}{dt}\right|_{\text{electrons}} = \left.\frac{dU_i}{dt}\right|_{\text{charge exchange}} + \left.\frac{dU_i}{dt}\right|_{\text{diffusion}} + \left.\frac{dU_i}{dt}\right|_{\text{neutrals (elastic)}} ,$$

and a quasi-neutrality condition:

$$\left.\frac{dn_e}{dt}\right|_{\text{diffusion}} = \left.\frac{dn_i}{dt}\right|_{\text{diffusion}} .$$

These four equations are solved for the four unknowns, T_e(eV), T_i(eV), n_o (neutral density), and Φ (plasma potential). Parameters in the calculation are plasma density, n(cm^{-3}); major circumference of the torus, L(cm); and minor radius of the plasma, a(cm); and average magnetic field, B(kG). The particle confinement time, τ(sec), is also calculated.

The ionization rate is approximated by the analytic expression:

$$\left.\frac{dn}{dt}\right|_{\text{ionization}} = \frac{3.71 \times 10^{-7} n\, n_o \sqrt{T_e}\, e^{-15.6/T_e}}{T_e + 15.6} \left[\frac{T_e}{20T_e + 15.6} + \log(1.5625 + 0.1T_e)\right],$$

where T_e is the electron temperature in eV. Particle losses are assumed to be governed by neo-classical diffusion:

$$\left.\frac{dn}{dt}\right|_{\text{diffusion}} = \frac{4.2 \times 10^{13} \sqrt{T_i}\, n^2\, e^{\Phi/T_i}}{B^2 a^4 n^2/T_i^{\,3} + 1.45 \times 10^{21} (T_i + 3.75\Phi)^2} ,$$

where Φ is the potential at the center of the plasma relative to the edge. (The radial electric field is assumed to be $\vec{E}_r = \Phi\hat{r}/a$.)

The microwave heating rate is given by

$$\left.\frac{dU_e}{dt}\right|_{\mu\text{waves}} = \frac{2 \times 10^{18} p}{a^2 L} .$$

The electron-ion energy transfer rate is given by

$$\left.\frac{dU_e}{dt}\right|_{\text{ions}} = \left.\frac{dU_i}{dt}\right|_{\text{electrons}} = \frac{2.3 \times 10^{-4} n^2 (T_e - T_i)}{T_e^{\,3/2}} \log\left(\frac{5.2 \times 10^{20} T_e^{\,3}}{(40 + T_e) n}\right).$$

Excitation losses are given by

$$\left.\frac{dU_e}{dt}\right|_{\text{excitation}} = 29.1\ e^{6.98/T_e}\ \left.\frac{dn}{dt}\right|_{\text{ionization}} .$$

Bremsstrahlung losses are given by

$$\left.\frac{dU_e}{dt}\right|_{\text{Bremsstrahlung}} = 10^{-13}\ n^2\sqrt{T_e} .$$

Synchrotron radiation losses are given by

$$\left.\frac{dU_e}{dt}\right|_{\text{synchrotron}} = 3.87 \times 10^{-3}\ n\ B^2 T_e \left(1 + \frac{T_e}{2.04\text{x}10^5}\right) .$$

The energy loss through diffusion is calculated from the neo-classical model ignoring temperature gradients:

$$\left.\frac{dU_e}{dt}\right|_{\text{diffusion}} = 3.5\ T_e \left.\frac{dn}{dt}\right|_{\text{diffusion}} .$$

Similarly, the ion energy loss by diffusion is

$$\left.\frac{dU_i}{dt}\right|_{\text{diffusion}} = 3.5\ T_i\ \left.\frac{dn}{dt}\right|_{\text{diffusion}} .$$

Charge exchange losses are given by

$$\left.\frac{dU_i}{dt}\right|_{\text{charge exchange}} = 7.32\text{x}10^{-11}\ n\ n_o\ T_i^{3/2}\ (1+0.00585\ T_i^{3/2})\ e^{-0.0582\sqrt{T_e}} .$$

Finally, energy loss by elastic collisions with neutrals is given by

$$\left.\frac{dU_i}{dt}\right|_{\text{neutrals (elastic)}} = \frac{1.88\text{x}10^{-8} n\ n_o\ T_i^{1.05}}{(570 + T_i^{2.5})\ 0.29} .$$

The quasineutrality condition (equal flux of electrons and ions) is given by neo-classical theory as

$$T_e^{5/2}\, e^{-\Phi/T_e} \left[\frac{(10^{-9} B a^2 n)^2}{T_i^3} + 1370\,(T_i + 3.75\,\Phi)^2\right]$$

$$= T_i^{5/2}\, e^{\Phi/T_i} \left[\frac{86(10^{-9} B a^2 n)^2}{T_e^3} + 16(T_e - 3.75\Phi)^2\right]$$

The FORTRAN listing of the program which solves this set of equations is included in the appendix along with output for a typical data run.

A number of runs were initially made with the potential $\Phi = 0$ in order to compare with results obtained by Guest who used a more approximate form for some of the loss terms and neglected others entirely. Fig. 1 shows a case intended to be identical to a case examined by Guest, and is in remarkably good agreement with his results. A case with Bohm diffusion was also examined and is shown in Fig. 2. The experimental results appear to be more nearly consistent with neo-classical diffusion than Bohm diffusion. The effects of scaling up the microwave power density, plasma volume, and magnetic field strength are shown in Figs. 3-5 (in the $\Phi = 0$ approximation with neo-classical diffusion).

Fig. 6 shows the result of repeating the calculation of Fig. 1 with a self-consistent plasma potential. The results are very different when the potential is included, and the striking feature is the abrupt reversal of the potential from positive to negative as the density increases above $\sim 2 \times 10^{12}$ cm^{-3}. Such potential transitions have been observed experimentally. Fig. 7 shows the result of increasing the size of the torus by a factor of 10 (in volume), while maintaining the same microwave power density. Figs. 8 and 9 show the results of increasing the microwave power for the present device. (Note that V is the total machine value, 1.3×10^6 cm^3; but that the microwave power density P/V is calculated using the plasma volume, 2.9×10^5 cm^3).

The discouraging result is that the ion temperature and particle confinement time both decrease with increasing microwave power for a constant density. However, it is known experimentally that the density increases with microwave power. Furthermore, there is experimental evidence of anamolous ion heating in certain regimes. In fact, wherever $|\Phi| \gg T_i$, as is the case here, we expect a rapid $\vec{E} \times \vec{B}$ rotation of the plasma with a possible enhanced energy transfer to the ions. Finally, it must be noted that the solution obtained by the computer may not be unique, and there may be other solutions which were not found.

Discussions with G. E. Guest and R. A. Dandl are gratefully acknowledged.

```
 *  *  UNIVAC 1110 TIME/SHARING EXEC --- MULTI-PROCESSOR SYSTEM --- VER.  MACC 31.4
 C00271        FILE NAME * PR#000C00271        PART NUMBER *  00      DATE * 020575



ROTT,2980.4126810219,1M
AREТH
C 1.14S-02/05/75-15:45:00                    GARETH
      C       PROGRAM GARETH  -  JAN 9, 1975
              DIMENSION XINIT(4),XFIN(4),WORK(48),DEP(31),DNP(31),TEP(31),TIP(31
             2),PHP(31),TAP(31)
              EXTERNAL AUXFCN
              COMMON P,TWALL,A,AL,B,DEA
      C                         SPECIFY PARAMETERS
              DO 950 IP=1,7
              P=1700.0*2.0**(IP-1)
              IDENS=1
              IIMAX=100
              DEA=10000.0
              TWALL=0.025
              TEA=100.0
              TIA=100.0
              PHI=-100.0
              DNEUT=0.1
              AL=942.0
              A=10.0
              B=6.67
      C                         SPECIFY INITIAL CONDITIONS
              XINIT(1)=DNEUT
              XINIT(2)=TEA
              XINIT(3)=TIA
              XINIT(4)=PHI
              LA='ZDTI..'
              WRITE(6,400)
        400   FORMAT(1H1,'       I    IERR       DENSITY          DNEUT            TE
             2         TI              PHI            TAU')
      C                         SOLVE STEADY STATE EQUATIONS
        200   CONTINUE
              CALL ZRNEQ(XINIT,AUXFCN,4,1.0E-8,6,IIMAX,XFIN,0,LA,WORK,IERR,$900)
              XFIN(1)=ABS(XFIN(1))
              XFIN(2)=ABS(XFIN(2))
              TAU=(XFIN(2)+15.6)/371.0/XFIN(1)/SQRT(XFIN(2))/EXP(-15.6/XFIN(2))/
             2(XFIN(2)/(20.0*XFIN(2)+15.6)+ALOG(1.5625+0.1*XFIN(2)))
              WRITE(6,500) IDENS,IERR,DEA,XFIN(1),XFIN(2),XFIN(3),XFIN(4),TAU
        500   FORMAT(1H ,I6,I7,6F13.4)
              DEP(IDENS)=3.0*(ALOG10(DEA)-2.0)
              DNP(IDENS)=3.0*(ALOG10(ABS(XFIN(1)))+1.0)
              TEP(IDENS)=3.0*(ALOG10(ABS(XFIN(2)))-1.0)
              TIP(IDENS)=3.0*(ALOG10(ABS(XFIN(3)))-1.0)
              PHP(IDENS)=3.0*(ALOG10(ABS(XFIN(4)))-1.0)
              TAP(IDENS)=3.0*(ALOG10(ABS(TAU))+2.0)
              DO 700 I=1,4
        700   XINIT(I)=XFIN(I)
              DEA=DEA/1.1659
              IDENS=IDENS+1
              IF(IDENS.LE.31) GO TO 200
        900   CONTINUE
      C                         GRAPH OUTPUT
```

```
      IDENS=IDENS-1
      CALL GRAPH2(DEP,'R',DNP,'R',IDENS,'9X6','NONE','ZERO-D EBT SIMULAT
     2ION..','DENSITY..','TEMPERATURE..','N')
      CALL GRPH2V(DEP,'R',TEP,'R',IDENS,'NONE','E')
      CALL GRPH2V(DEP,'R',TIP,'R',IDENS,'NONE','I')
      CALL GRPH2V(DEP,'R',PHP,'R',IDENS,'NONE','V')
      CALL GRPH2V(DEP,'R',TAP,'R',IDENS,'NONE','T')
      CALL GRPHND
  950 CONTINUE
      STOP
      END

OF COMPILATION:          NO  DIAGNOSTICS.
UXFCN
CC 1.14S-02/05/75-15:45:05              AUXFCN
      FUNCTION AUXFCN(X,K)
      DIMENSION X(1)
      COMMON P,TWALL,A,AL,B,DEA
C                      DEFINE FUNCTIONS  -  EBT
      D1(DENS,DNEUT,TE)=371.0*DENS*DNEUT*SQRT(TE)*EXP(-15.6/TE)*(TE/(20.
     20*TE+15.6)+ALOG(1.5625+0.1*TE))/(TE+15.6)
      D2(DENS,TE)=4.2E4*SQRT(TIA)*EXP(PHI/TIA)/(B*B*A**4/TIA**3+1450.0*(
     2TIA+3.75*PHI)**2/DENS**2)
      PE1(P)=2.0E9*P/A/A/AL
      PE2(DENS,TE,TI)=2.3*DENS**2*(TE-TI)*ALOG(5.2E11*TE**3/ABS(DENS)/(4
     20.0+TE))/TE**1.5
      PE3(DENS,DNEUT,TE)=29.1*D1(DENS,DNEUT,TE)*EXP(6.98/(TE+0.1))
      PE4(DENS,TE)=1.0E-4*DENS*DENS*SQRT(TE)
      PE5(DENS,TE)=3.87E-3*DENS*B*B*TE*(1.0+TE/2.04E5)
      PE6(DENS,TE)=3.5*D2(DENS,TE)*(TE-TWALL)
      PI3(DENS,DNEUT,TI)=0.0732*DENS*DNEUT*SQRT(TI)*(TI-TWALL)*(1.0+0.00
     2585*SQRT(TI)*TI)/EXP(0.0582*SQRT(TI))
      PI6(DENS,TE,TI)=3.5*D2(DENS,TE)*(TI-TWALL)
      PI8(DENS,DNEUT,TI)=18.8*DENS*DNEUT*(TI-TWALL)**1.05/
     1                    (570.+TI**2.4)**.29
      DENS=DEA
      DNEUT=ABS(X(1))
      TEA=ABS(X(2))
      TE=TEA
      TIA=ABS(X(3))
      TI=TIA
      PHI=AMIN1(X(4),70.0*TI)
      GO TO (1,2,3,4),K
    1 CONTINUE
GNOSTIC*  DEFINE PROCEDURE ARGUMENT(S) NOT USED IN DEFINITION.
      AUXFCN=D1(DEA,DNEUT,TEA)-D2(DEA,TEA)
      RETURN
    2 CONTINUE
GNOSTIC*  DEFINE PROCEDURE ARGUMENT(S) NOT USED IN DEFINITION.
      AUXFCN=PE1(P)-PE2(DEA,TEA,TIA)-PE3(DEA,DNEUT,TEA)-PE4(DEA,TEA)-PE5
     2(DEA,TEA)-PE6(DEA,TEA)
      RETURN
    3 CONTINUE
GNOSTIC*  DEFINE PROCEDURE ARGUMENT(S) NOT USED IN DEFINITION.
      AUXFCN=PE2(DEA,TEA,TIA)-PI3(DEA,DNEUT,TIA)-PI6(DEA,TEA,TIA)-PI8(DE
     2A,DNEUT,TIA)
      RETURN
```

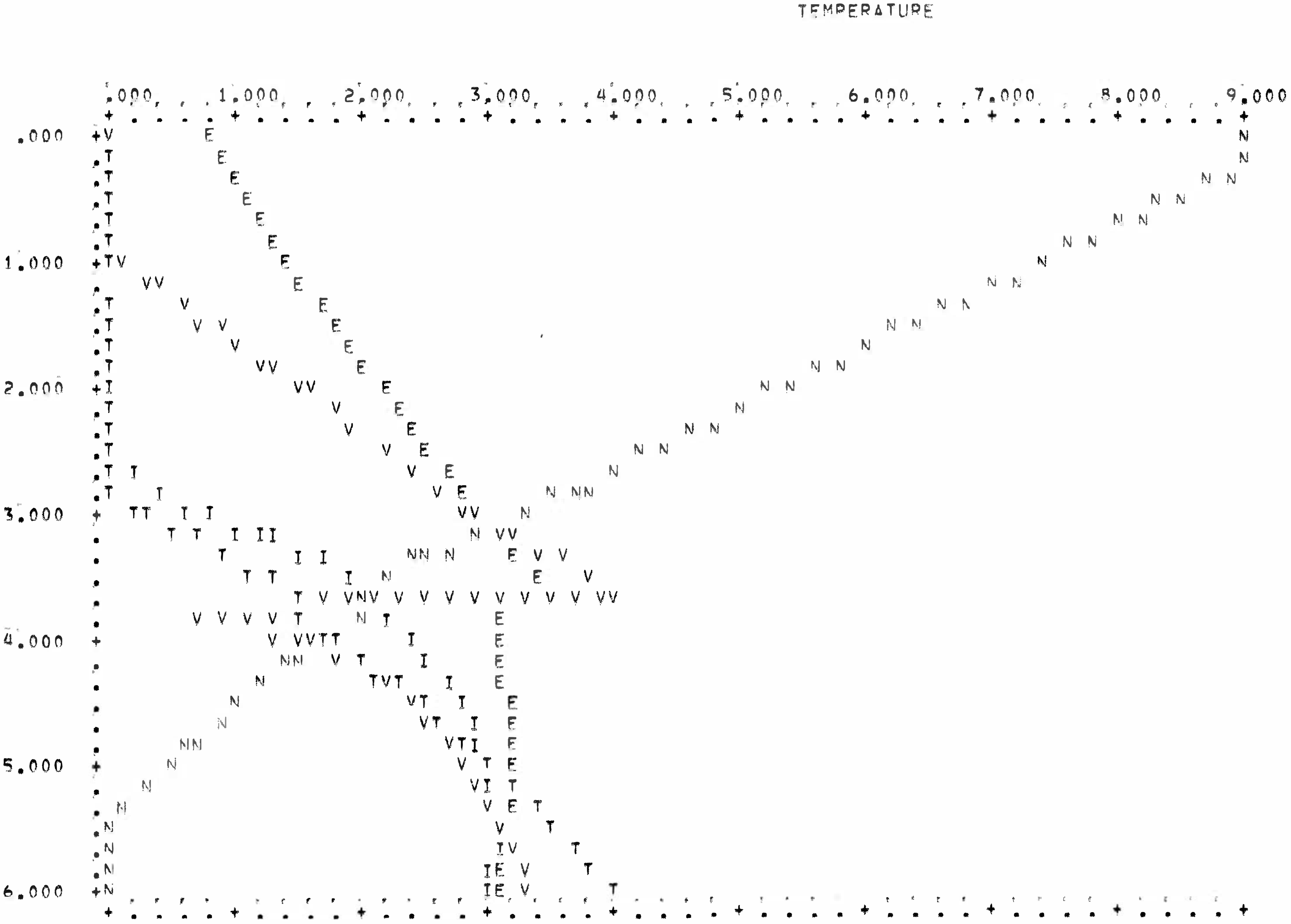
```
       4    CONTINUE
            AUXFCN=TE**2.5*EXP(-PHI/TE)*((B*A*A*DENS)**2/TI**3+1370.*(TI+3.75*
           2PHI)**2)-TI**2.5*EXP(PHI/TI)*(86.0*(B*A*A*DENS)**2/TE**3+16.0*(TE-
           33.75*PHI)**2)
            RETURN
            END

 OF COMPILATION:              3 DIAGNOSTICS.

2/05-15:45
```

```
  I    TERR      DENSITY             DNEUT          TE           TI          PHI            TAU
  1       7    10000.0000            .0674      106.0529     103.0260    -130.5989        .2146
  2       5     8577.0649            .0775      108.1724     103.9788    -122.2507        .1864
  3       5     7356.6042            .0893      110.1013     104.3225    -113.6213        .1613
  4       6     6309.8072            .1036      111.7712     103.8620    -104.7566        .1390
  5       6     5411.9626            .1208      113.1021     102.3705     -95.6752        .1190
  6       6     4641.8754            .1421      113.9972      99.5963     -86.3701        .1011
  7       6     3981.3667            .1688      114.3380      95.2872     -76.8128        .0851
  8       6     3414.8441            .2030      113.9845      89.2408     -66.9570        .0708
  9       6     2928.9339            .2477      112.7849      81.3900     -56.7274        .0581
 10       5     2512.1656            .3073      110.6124      71.9130     -45.9354        .0469
 11       6     2154.7007            .3874      107.4787      61.3351     -33.8847        .0373
 12       6     1848.1008            .4918      104.3071      50.7111     -16.3313        .0295
 13      11     1585.1281            .4526      143.2973     -50.9352     220.2752        .0313
 14       7     1359.5746            .6437      122.6559     -36.3068     157.8092        .0222
 15       7     1166.1160            .9045      106.4670     -25.7871     114.2971        .0160
 16       9     1000.1852           1.2540       93.6116     -18.2852      86.9114        .0117
 17      10      857.8654           1.7182       83.1351     -12.9584      70.5034        .0088
 18       7      735.7967           2.3393       74.1475      -9.1732      59.1255        .0066
 19       9      631.0976           3.1809       66.1359      -6.4857      49.5435        .0050
 20       9      541.2965           4.3316       58.9032      -4.5834      40.8749        .0038
 21       8      464.2735           5.9127       52.3955      -3.2418      33.1452        .0029
 22       8      398.2104           8.0900       46.5885      -2.2980      26.4958        .0023
 23       8      341.5477          11.0917       41.4464      -1.6348      20.9604        .0018
 24       8      292.9477          15.2318       36.9190      -1.1687      16.4670        .0014
 25       8      251.2631          20.9418       32.9487       -.8405      12.8869        .0011
 26       8      215.5100          28.8105       29.4770       -.6089      10.0731        .0008
 27       8      184.8443          39.6345       26.4483       -.4451       7.8837        .0007
 28       8      158.5422          54.4779       23.8117       -.3287       6.1924        .0005
 29       9      135.9827          74.7394       21.5222       -.2456       4.8927        .0004
 30      10      116.6332         102.2192       19.5398       -.1861       3.8974        .0004
 31      10      100.0371         139.1746       17.8298       -.1432       3.1370        .0003
```

ZERO-D EBT SIMULATION



```
  ITEM                                      AMOUNT     COST(DOLLARS)

 CPU TIME                         00:00:12.324          $0.46
 FILE I/O REQUESTS                         251          $0.11
 FILE I/O WORDS                         218695          $0.10
 MEMORY USAGE                            0.223          $0.14
 CARDS IN                                  112          $0.03
 PAGES PRINTED                              10          $0.15
 JOB CHARGE                                  1          $0.05

 TOTAL COST                                             $1.04
     THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR STANDARD RUNS




     INITIATION  TIME:   15:44:59-FEB  5,1975

     TERMINATION TIME:   15:45:40-FEB  5,1975

   PREVIOUS RUN TIME:   15:28:20-FEB  5,1975
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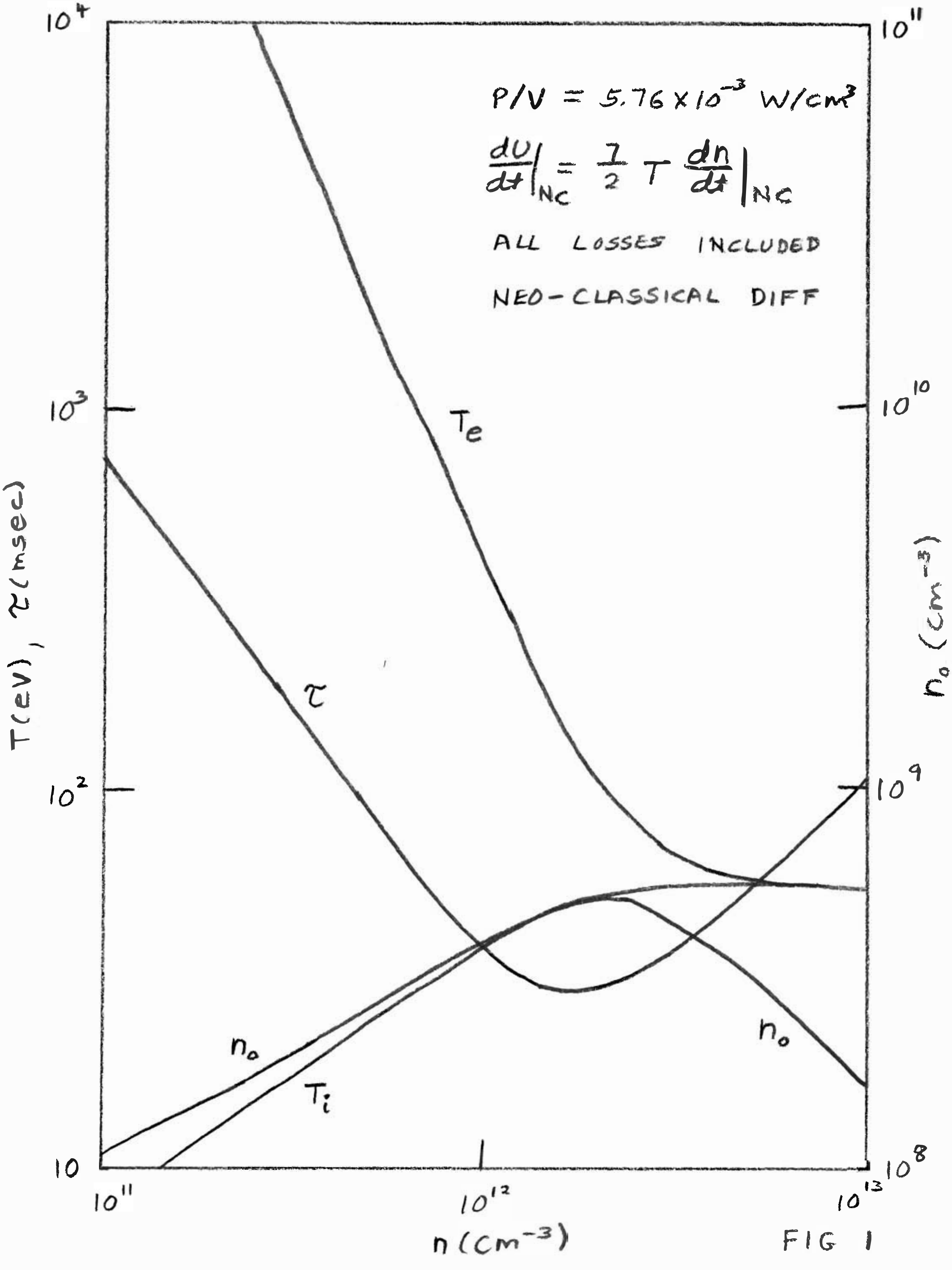
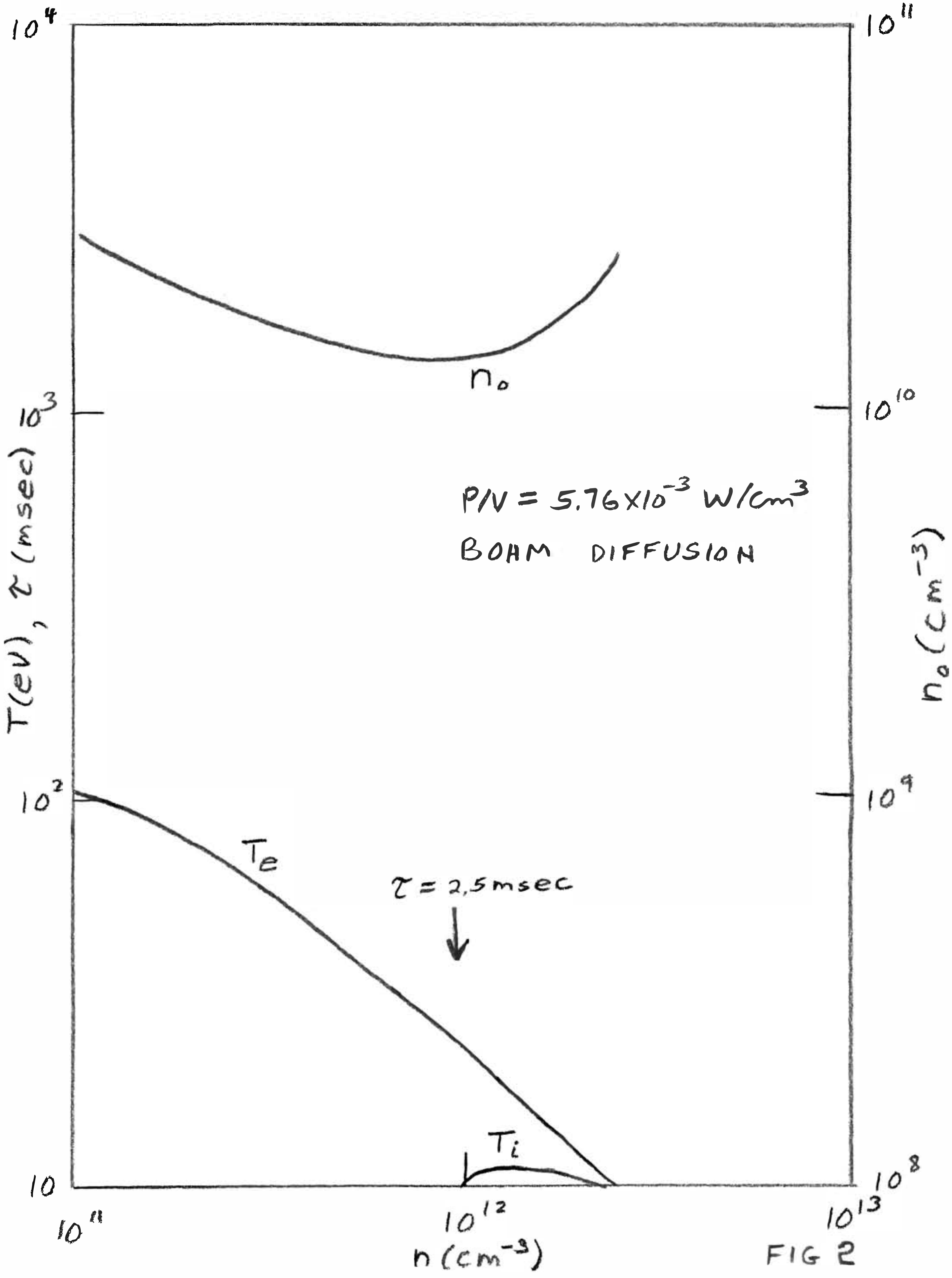


$P/V = 5.76 \times 10^{-3}\ W/cm^3$
$\left.\frac{dU}{dt}\right|_{NC} = \frac{7}{2} T \left.\frac{dn}{dt}\right|_{NC}$
ALL LOSSES INCLUDED
NEO-CLASSICAL DIFF
T_e
τ
n_o
T_i
n_o
10^4
10^3
10^2
10
$T(eV),\ \tau(msec)$
10^{11}
10^{12}
10^{13}
$n\ (cm^{-3})$
10^{11}
10^{10}
10^9
10^8
$n_o\ (cm^{-3})$

FIG 1



$P/V = 5.76 \times 10^{-3}$ W/cm^3

BOHM DIFFUSION

FIG 2

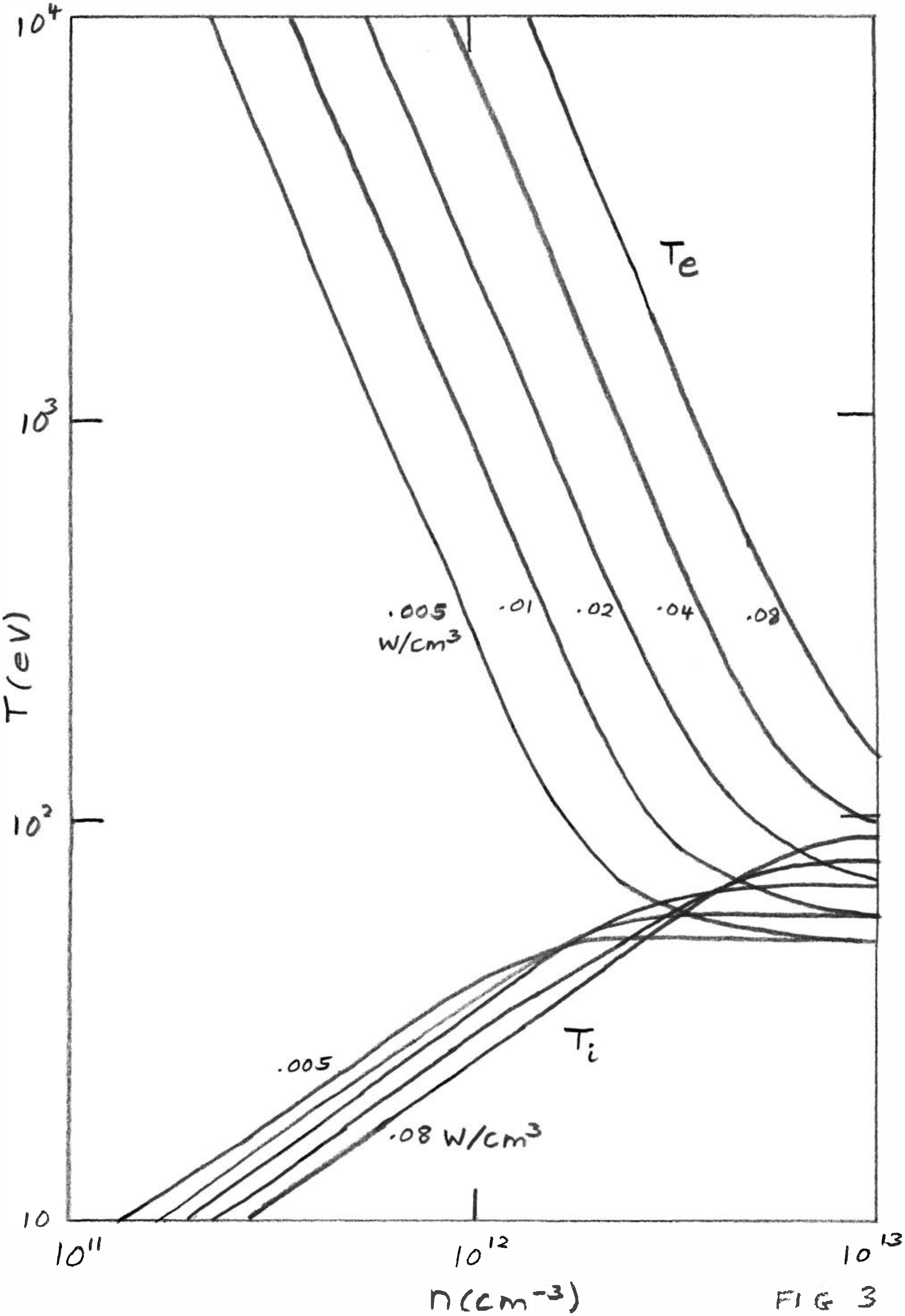

10^4
10^3
10^2
10
T (eV)
10^11
10^12
10^13
n (cm^-3)
Te
.005 W/cm^3
.01
.02
.04
.08
Ti
.005
.08 W/cm^3

FIG 3

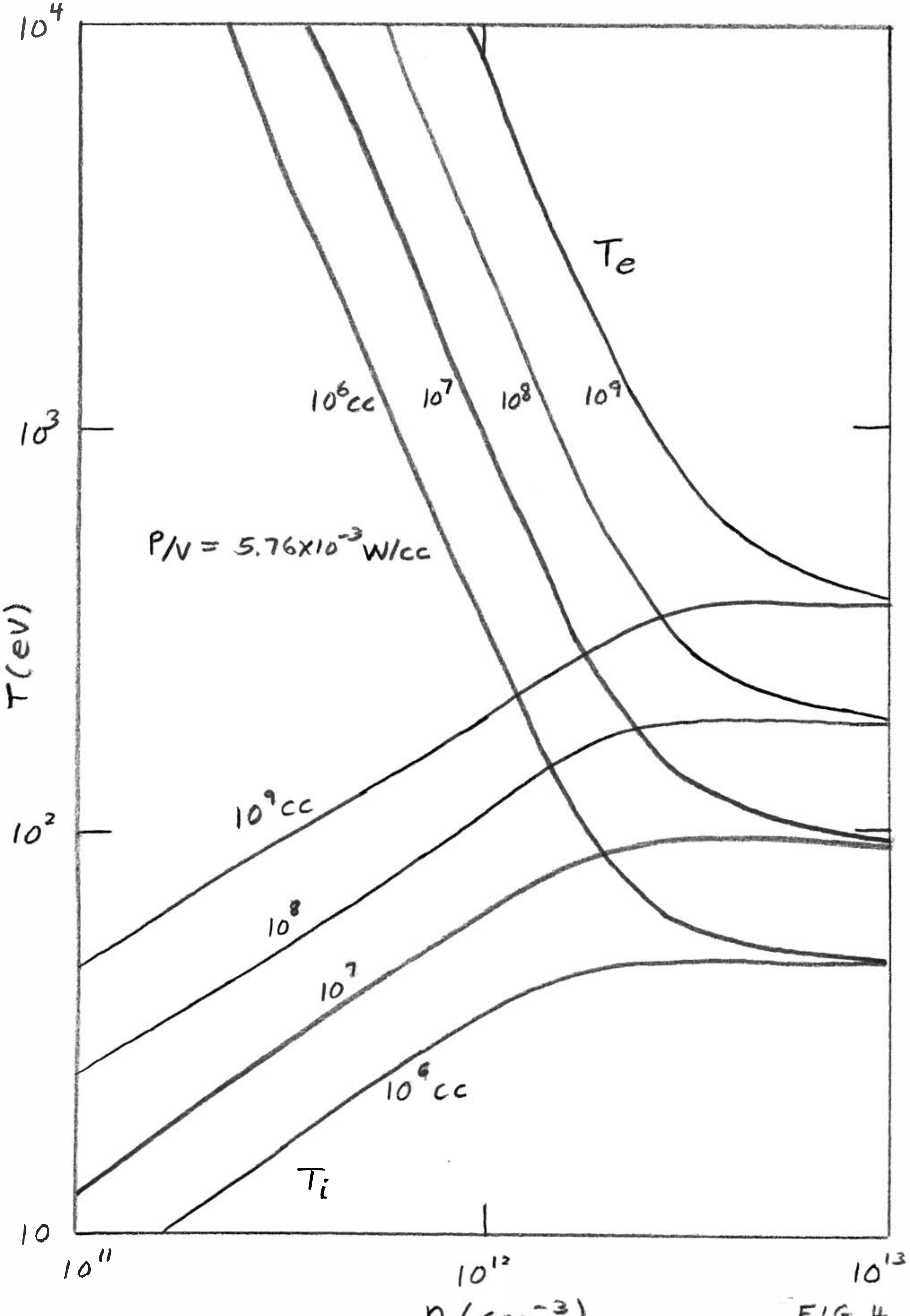

10^4
10^3
10^2
10
T(eV)
10^11
10^12
10^13
n (cm^-3)
T_e
T_i
10^6cc
10^7
10^8
10^9
P/V = 5.76x10^-3 W/cc
10^9cc
10^8
10^7
10^6cc

FIG 4

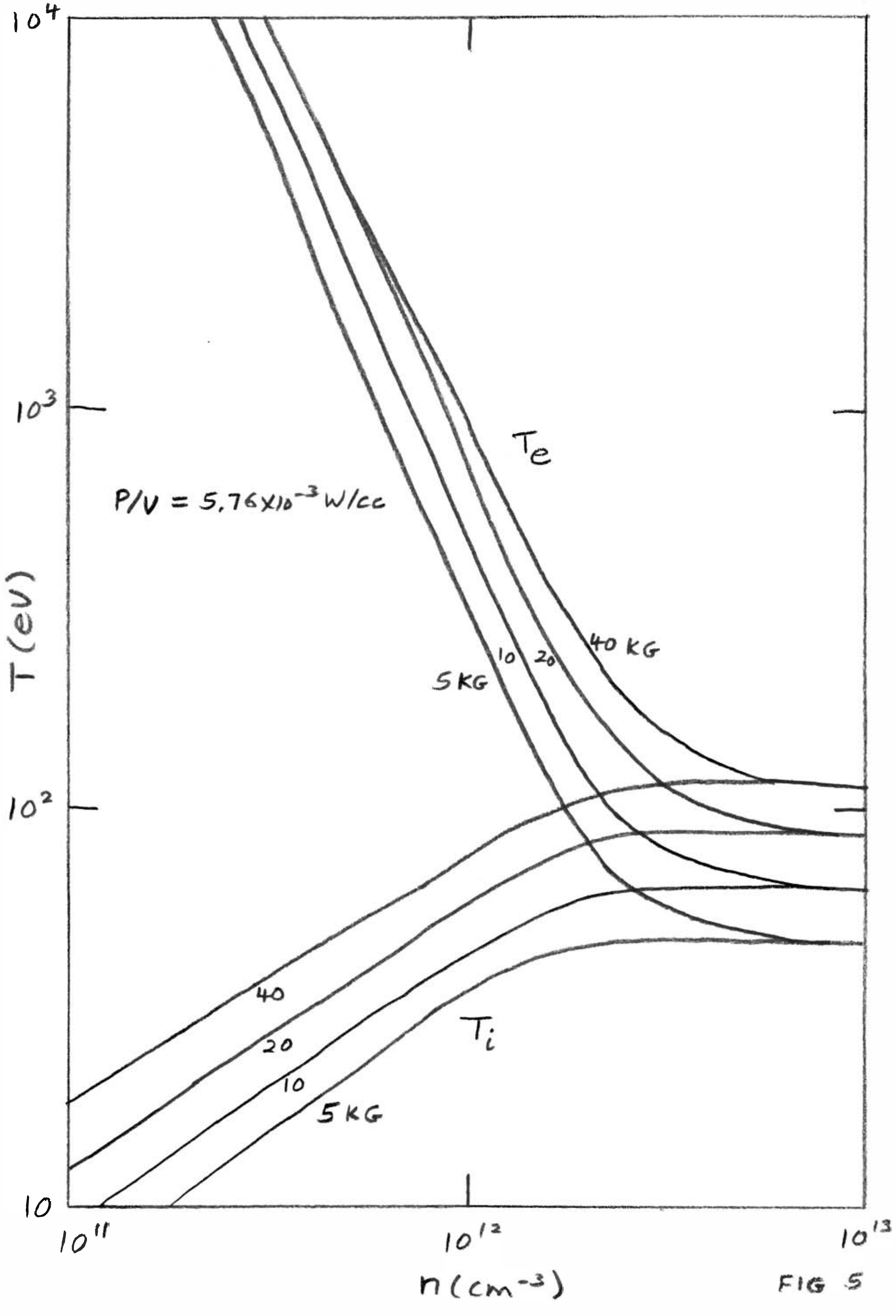

10^4
10^3
10^2
10
T (eV)
10^{11}
10^{12}
10^{13}
n (cm^{-3})
$P/V = 5.76 \times 10^{-3}$ W/cc
T_e
5 KG
10
20
40 KG
T_i
40
20
10
5 KG

FIG 5

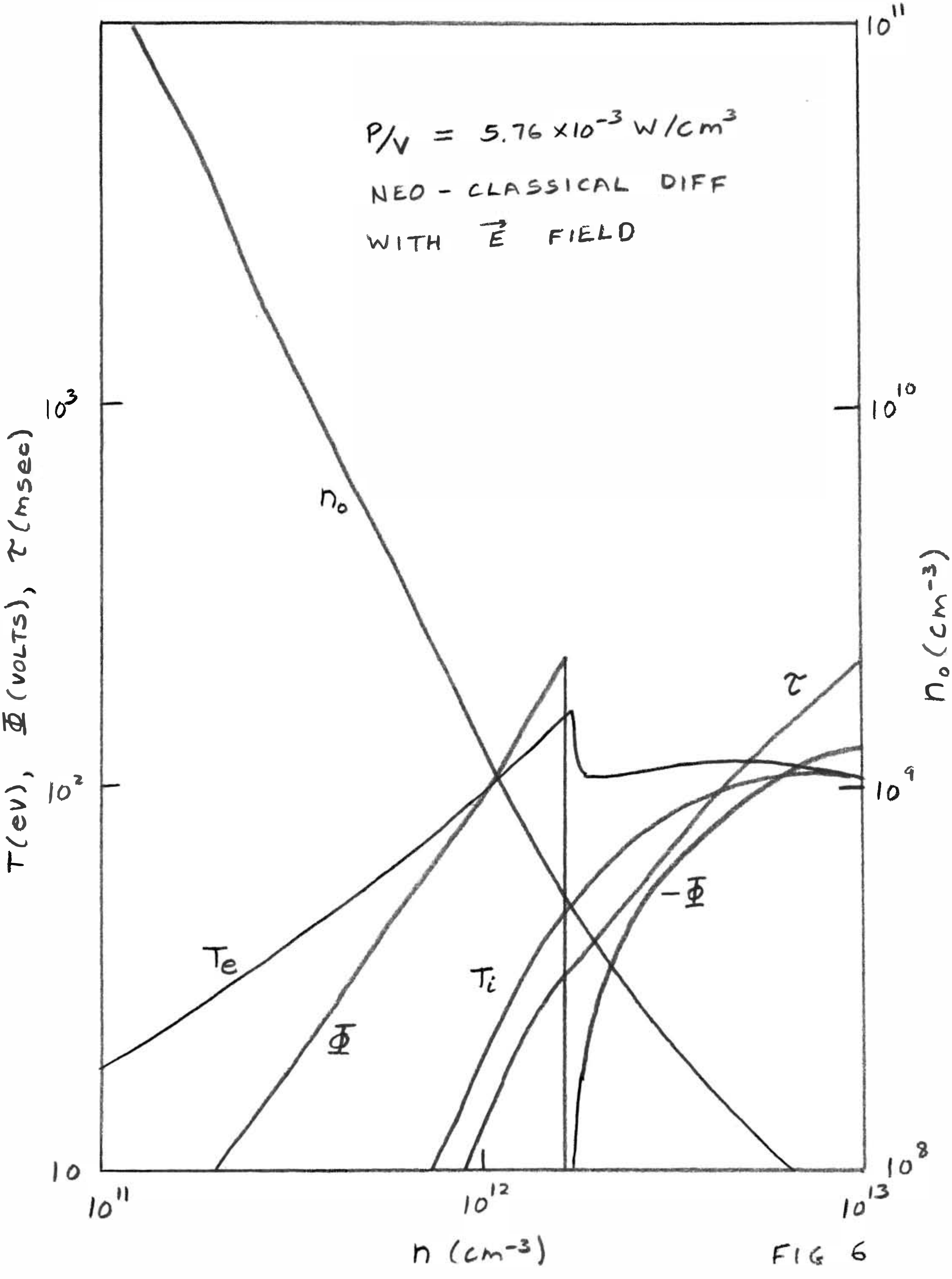
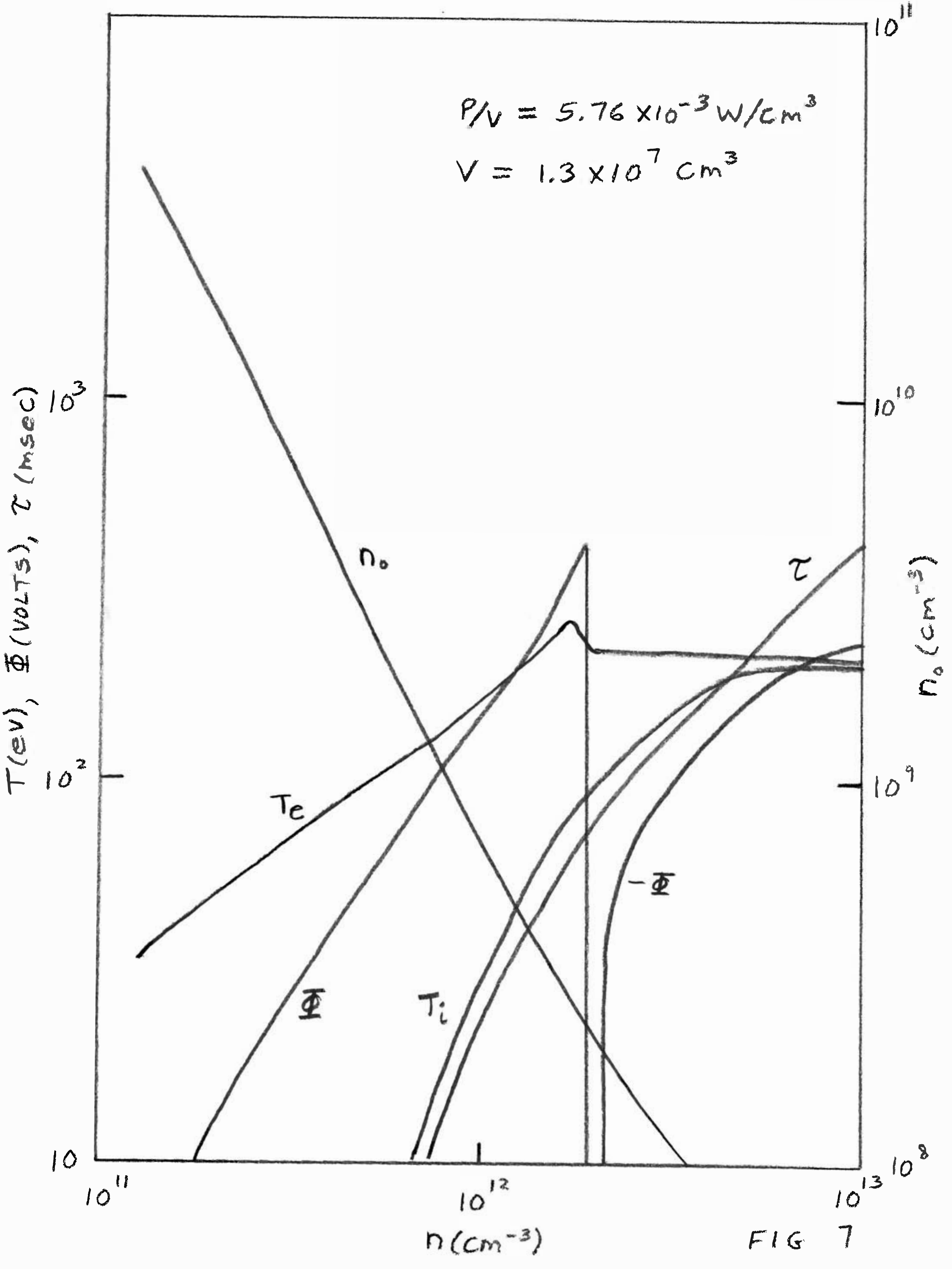

P/V = 5.76 x10^-3 W/cm^3
NEO - CLASSICAL DIFF
WITH E FIELD
T(eV), Φ (VOLTS), τ (msec)
10^3
10^2
10
10^11
10^12
10^13
n (cm^-3)
n_o (cm^-3)
10^11
10^10
10^9
10^8
n_o
T_e
Φ
T_i
τ
-Φ

FIG 6



P/V = 5.76 x10^-3 W/cm^3
V = 1.3 x10^7 cm^3
T(eV), Φ (VOLTS), τ (msec)
n₀ (cm^-3)
n(cm^-3)
10^3
10^2
10
10^11
10^10
10^9
10^8
10^11
10^12
10^13
n₀
τ
Tₑ
Tᵢ
Φ
-Φ

FIG 7

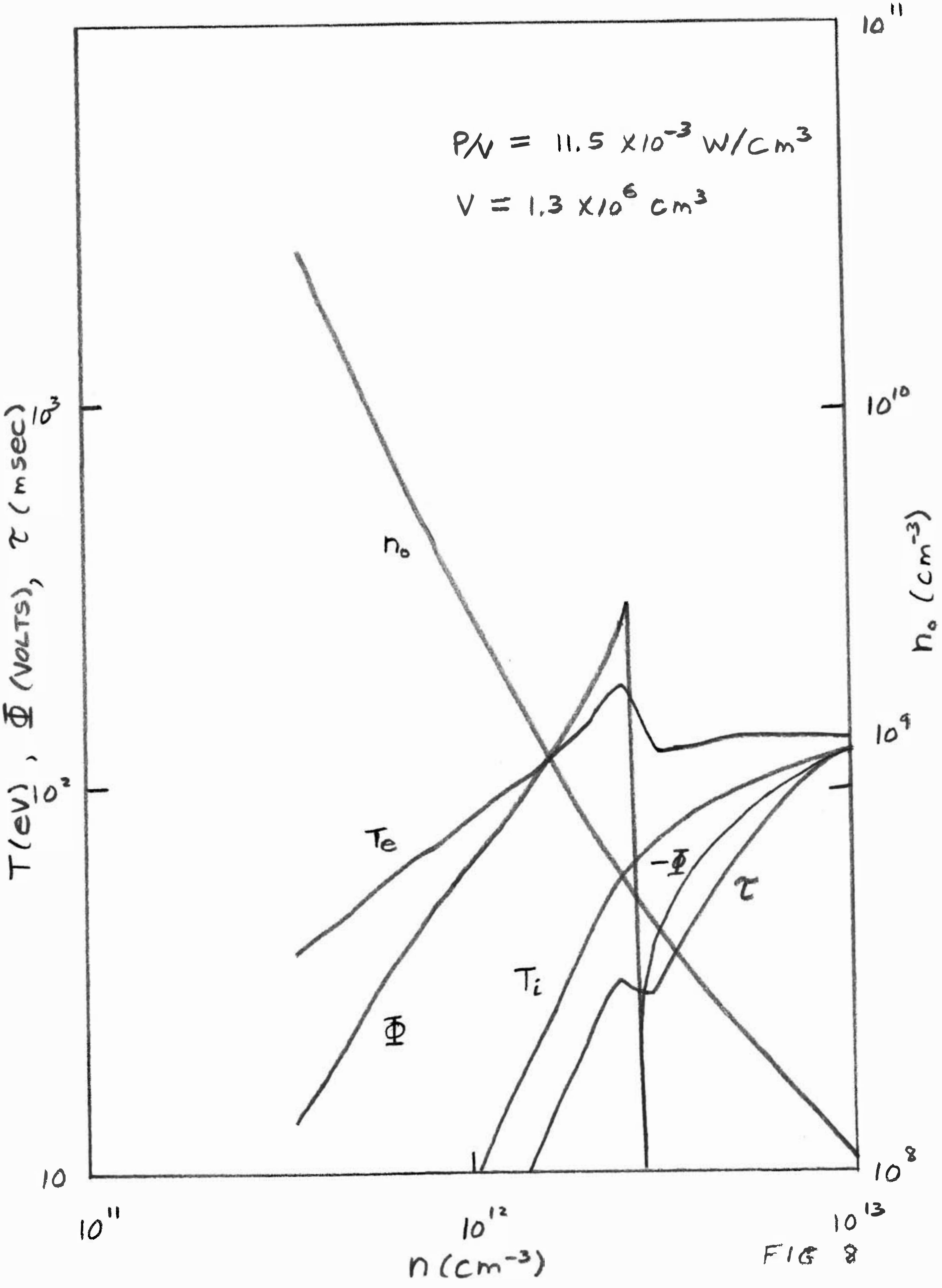

P/V = 11.5 X10^{-3} W/cm^3
V = 1.3 X10^6 cm^3
n$_o$
T$_e$
Φ
T$_i$
-Φ
τ
T(eV), Φ (VOLTS), τ (msec)
10^3
10^2
10
10^{11}
10^{10}
10^9
10^8
n$_o$ (cm^{-3})
10^{11}
10^{12}
10^{13}
n (cm^{-3})

FIG 8

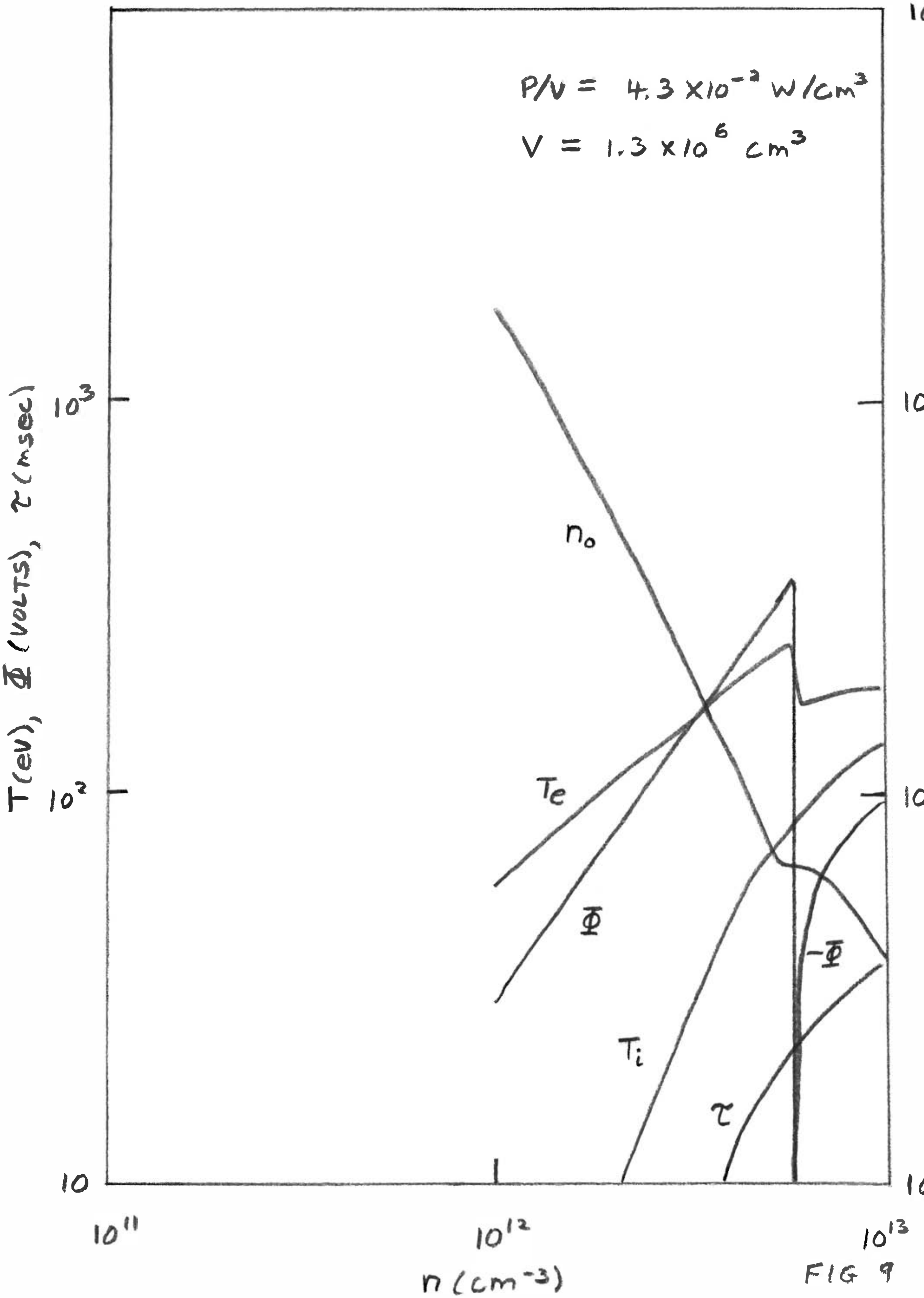

P/V = 4.3 x10^-2 W/cm^3
V = 1.3 x10^6 cm^3
T(eV), Φ (VOLTS), τ (msec)
n_o (cm^-3)
n (cm^-3)
10^11
10^12
10^13
10
10^2
10^3
10^8
10^9
10^10
10^11
n_o
T_e
Φ
T_i
τ
-Φ

FIG 9