ZERO-DIMENSIONAL STEADY STATE PLASMA SIMULATION COMPUTER CODE

by

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This note describes a computer code (ZEDTID) that calculates the spatially averaged electron (and ion) density, neutral density, electron temperature, and ion temperature in a steady state cylindrical plasma. It is a simplification of a time-dependent code (SIMULT) described in PLP 505 and is much faster, more accurate, and quite adequate for a variety of quasi-steady state situations such as a cw microwave plasma in a toroidal octupole. It uses a UW library subroutine (ZRNEQ) that seeks a solution of N simultaneous non-linear algebraic equations in N unknowns. In this case, it is used to solve particle balance equations for electrons (or ions) and neutrals and energy balance equations for electrons and ions. The various particle and energy loss terms are a refined version of those presented in PLP 505 and will described in detail in a forthcoming PLP by Patau. The physical processes considered are listed below:

1. ionization
2. classical radial diffusion (e-i and e-n collisions)
3. obstacle losses
4. microwave heating (including finite cavity Q)
5. electron-ion energy equipartition
6. neutral collisions (excitation)
7. bremsstrahling
8. synchrotron radiation (ignoring reabsorption)
9. radial energy transport (ignoring ∇T)
10. ion charge exchange
11. neutral shielding (thermals and Franck-Condons)
12. finite beta

A large number of cases have been investigated, and a representative sample are described below. For each case considered, the microwave power is varied from 1 watt to 10^6 watts. Convergence is obtained only if a reasonably close initial trial solution is supplied. This was usually done by using SIMULT with constant field and low power to approach a steady state. Initial trial solutions for successively higher power levels are determined by fitting a quadratic curve to the preceeding three solutions for each unknown. In spite of this, convergence usually fails before the power reaches 10^6 watts. It should also be pointed out that other solutions may exist depending on the time history of the approach to a steady state.

Small octupole: The terms used to describe confinement in the small octupole are identical to those used in the refined version of SIMULT. (In fact, the function statements can be directly transferred from one program to the other.) The parameters used are as follows: T_{wall} = 0.025 eV (room temperature), L = 270 cm (major circumference of toroid), a = 18 cm (adjusted so $\pi a^2 L$ = total volume), A_o (obstacle area) = 90 cm^2 (emperically determined from lifetime vs energy for ions and electrons), p = 1 x 10^{-5} torr (at plasma boundary assumed H_2), B = 1 kG (crude volume average), f = 2.45 GHz, and Q = 2000 (without plasma). The results shown in Fig. 1 agree remarkably well with both experimental results and with the peak values of the time-dependent program SIMULT. The results agree within 1 part in 10^6 with the steady state (B = constant) infinite time limit of SIMULT. The electron temperature stays nearly constant at ~ 5 eV while the density increases linearly with microwave power. The computer printout for this case is included in the appendix.

Large octupole: For the large octupole, the parameters were changed to the following: $L = 800$ cm, $a = 50$ cm, $A_o = 700\ cm^2$ (actual geometric surface area of levators), $p = 1 \times 10^{-6}$ torr, $B = 1$ kG, $f = 2.45$ GHz, and $Q = 20,000$. The results shown in Fig. 2 are similar to those for the small octupole and agree almost perfectly with published values (Fig. 5 of Phys. Fluids 14, 1795 (1971)). With levitation ($A_o = 0$), Fig. 3 shows that the electron temperature is lower and the density is higher, but this case is probably unrealistic because with classical radial diffusion as the only particle loss mechanism, the time required to reach a steady state is much longer than the duration of the magnetic field pulse. For this case, the time dependent program SIMULT would be more appropriate. One use for this program would be to try various loss terms with different parametric dependences, seeking a good fit between the density vs microwave power curve and experimental measurements. In this way some information about the nature of the anomolous losses in levitated multipoles can hopefully be obtained.

Toroidal quadrupole: The Wisconsin toroidal quadrupole (without ohmic heating) was investigated using the following parameters: $T_{wall} = 0.025$ eV, $L = 160$ cm, $a = 6$ cm, $A_o = 3\ cm^2$, $p = 1 \times 10^{-4}$ torr, $B = 1$ kG, $f = 3$ GHz, and $Q = 500$. The results are shown in Fig. 4. The electron temperature is nearly constant at 3 eV and the density increases linearly with microwave power. The actual density in the experiment is considerably below the predicted value at 10 kW input power, and this is evidence either of anomalous losses (perhaps instabilities) or of the absence of a steady state (unlikely), or of a high reflected microwave power (very likely).

UWECE mirror: The electron cyclotron heated mirror device in B442 Engineering was studied by adding a loss cone term as described in PLP 518. The term includes scattering of electrons and ions on one another as well as on neutrals. Ambipolar potentials are also considered. The parameters are as Follows: T_{wall} = 0.025 eV, L = 60 cm, a = 6 cm (limiter radius), $A_o = 0$, $p = 1 \times 10^{-4}$ torr, B = 1 kG, f = 2.45 GHz, and Q = 500. Figure 5 shows that T_e stays nearly constant at ~ 30 eV while the density increases linearly with power in reasonable agreement with experiment. In the experiment there is also a runaway component of electrons (> 10 keV) not treated in the calculation. These energetic electrons have only a small effect on the particle and power balance, however.

ELMO mirror: The Oak Ridge ELMO mirror device was treated in the same way as above using the following parameters: T_{wall} = 0.025 eV, L = 25 cm, a = 10 cm, $A_o = 0$, $p = 5 \times 10^{-5}$ torr, B = 3 kG, f = 10.6 GHz, and Q = 10,000. The results shown in Fig. 6 are very similar to those in Fig. 5, but differ by as much as an order of magnitude from those predicted by the less refined Oak Ridge program SIMULEBT (PLP 489).

ELMO Bumpy Torus: The Oak Ridge ELMO Bumpy Torus was studied using the following parameters: T_{wall} = 0.025 eV, a = 10 cm, L = 175 cm, $A_o = 0$, $p = 1 \times 10^{-5}$ torr, B = 5 kG, f = 18 GHz, and Q = 10,000.

A neoclassical radial diffusion term as proposed by Guest (ORNL-TM-3694) was used. Ambipolar potentials were neglected, and ions were assumed to diffuse at the same rate as electrons (as suggested by Guest). Electron-neutral collisions were added, however, and the radial energy transport corresponding to this particle diffusion (ignoring ∇T) was taken from Kovrizhnykh (Sov. Phys. - JETP 29, 475 (1969)). The results shown in Fig. 7 give a somewhat higher density and lower temperature than was previously calculated using SIMULEBT (PLP 489). It has not been possible to get numerical convergence for powers above 1 kW, and so the interesting collisionless regime which the experiment will hopefully reach (with powers $\sim$ 30 kW) has not been investigated. The EBT case was also run with Bohm diffusion, and the result is shown in Fig. 8. There is not a great difference from the neoclassical case, as has been pointed out before (ORNL-TM-3694). Experimental measurements should be forthcoming.

```
1
.105-10/12/75-09:06:46                ZEDTID
C       PROGRAM ZEDTID - SEPT 2, 1975
        DIMENSION XINIT(4),XFIN(4),WORK(48),XOLD(4),XOLD2(4)
        DIMENSION PP(31),DEP(31),TEP(31),TIP(31),DNP(31)
        EXTERNAL AUXFCN
        COMMON / /TWALL,A,AL,AO,PRES,BO,F,Q

C       SPECIFY PARAMETERS
        IPWR=1
        PFAC=1.5848932
C       IIMAX IS NUMBER OF ITERATIONS
C       PO IS PEAK MICROWAVE POWER IN WATTS
C       DEA IS INITIAL DENSITY IN 10**9/CC
C       TWALL IS WALL TEMPERATURE IN EV
C       TEA IS INITIAL ELECTRON TEMPERATURE IN EV
C       TIA IS INITIAL ION TEMPERATURE IN EV
C       AL IS LENGTH IN CM
C       A IS RADIUS IN CM
C       AO IS OBSTACLE AREA IN SQ CM
C       PRES IS NEUTRAL PRESSURE IN 10**-5 TORR
C       BO IS FIELD AT OUTER WALL IN KGAUSS
C       F IS MICROWAVE FREQUENCY IN GHZ
C       Q IS THE MICROWAVE CAVITY Q
        IIMAX=100
        PO=1.0
        DEA=0.2
        TWALL=0.025
        TEA=5.0
        TIA=0.025
        AL=270.0
        A=18.0
        AO=90.0
        PRES=1.0
        BO=1.0
        F=2.45
        Q=2000.0

C       SPECIFY INITIAL CONDITIONS
        XINIT(1)=DEA
        XINIT(2)=TEA
        XINIT(3)=TIA
        XINIT(4)=330.00*PRES
        LA='ZDTI..'
        WRITE (6,400)
  400   FORMAT(1H1,'  IPWR   ITER          POWER        DENSITY           TE
     2          TI             DNEUT          FIELD          JSAT')

C       SOLVE STEADY STATE EQUATIONS
  200   CONTINUE
        P=PO*PFAC**(IPWR-1)
        CALL ZRNEQ(XINIT,AUXFCN,4,1.0E-4,6,IIMAX,XFIN,0,LA,WORK,IERR,&900)
        B=SQRT(ABS(BO*BO-4.05E-3*XFIN(1)*(XFIN(2)+XFIN(3))))
        SJ1=0.124*XFIN(1)*SQRT(AMAX1(XFIN(2),XFIN(3)))
        WRITE(6,500) IPWR,IERR,P,XFIN(1),XFIN(2),XFIN(3),XFIN(4),B,SJ1
  500   FORMAT(1H ,I6,I7,7F13.4)
        PP(IPWR)=ALOG10(P)
```

```
      DEP(IPWR)=ALOG10(XFIN(1))+1.0
      TEP(IPWR)=ALOG10(XFIN(2))+1.0
      TIP(IPWR)=ALOG10(XFIN(3))+1.0
      DNP(IPWR)=ALOG10(XFIN(4))+1.0
      DO 700 I=1,4
      IF (IPWR.EQ.1) XINIT(I)=XFIN(I)
      IF (IPWR.EQ.2) XINIT(I)=XFIN(I)**2/XOLD(I)
      IF (IPWR.GE.3) XINIT(I)=SQRT(XOLD2(I)*XFIN(I)**3)/XOLD(I)**2
      IF (IPWR.GE.2) XOLD2(I)=XOLD(I)
  700 XOLD(I)=XFIN(I)
      IPWR=IPWR+1
      IF (IPWR.LE.31) GO TO 200
  900 CONTINUE
C     GRAPH OUTPUT
      IPWR=IPWR-1
      CALL GRAPH2(PP,'R',DEP,'R',IPWR,'6X6','NONE','ZERO-D TIME INDEPEND
     .ENT SIMULATION..','MICROWAVE POWER..','DENSITY AND TEMP..','D')
      CALL GRPH2V(PP,'R',TEP,'R',IPWR,'NONE','E')
      CALL GRPH2V(PP,'R',TIP,'R',IPWR,'NONE','I')
      CALL GRPH2V(PP,'R',DNP,'R',IPWR,'NONE','N')
      CALL GRPHND
      STOP
      END

COMPILATION:         NO DIAGNOSTICS.
```

```
C
.105-10/12/75-09:06:50          AUXFCN
      FUNCTION AUXFCN(X,K)
      DIMENSION X(1)
      COMMON P,TWALL, ,AL,A0, RES,B0,F,G

C     DEFINE FUNCTIONS  -  OCTUPOLE
C     D1 IS DN/DT DUE TO IONIZATION
      D1(DENS,DNEUT,TE)=371.0*DENS*DNEUT*SQRT(TE)*EXP(-15.6/TE)*(TE/(20.
     20*TE+15.6)+ALOG(1.5625+0.1*TE))/(TE+15.6)
C     D2 IS DN/DT DUE TO DIFFUSION
      D2(DENS,TE)=DENS*(0.33*DENS/SQRT(TE)+0.001*DNEUT*TE)/B/B/A/A
C     D3 IS DN/DT DUE TO OBSTACLE LOSSES
      D3(DENS,TE)=2.0 5*DENS*A0*SQRT(TE+TIA)/A/A/AL
C     D4 IS DN/DT DUE TO FIELD DECAY
      D4(DENS)=0.5*DENS*AMAX1(0.0,BOLD-B)/BOLD/DT
C     PE1 IS DUE/DT DUE TO MICROWAVES
      PE1(P)= .0E9*P*DEA/(DEA+DEO)/A/A/AL
C     PE2 IS DUE/DT DUE TO ION COLLISIONS
      PE2(DENS,TE,TI)=2.5*DENS**2*(TE-TI)*ALOG(5.0E11*TE**3/ABS(DENS)
     2/(40.0+TE))/TE**1.5
C     PE3 IS DUE/DT DUE TO EXCITATION
      PE3(DENS,DNEUT,TE)=29.1*D1(DENS,DNEUT,TE)*EXP(6.98/(TE+0.1))
C     PE4 IS DUE/DT DUE TO BREMSSTRAHLUNG
      PE4(DENS,TE)=1. E-4*DENS*DENS*SQRT(TE)
C     PE5 IS DUE/DT DUE TO SYNCHROTRON RADIATION
      PE5(DENS,TE)=3. 7E-3*DENS*B*B*TE*(1.0+TE/2.04E5)
C     PE6 IS DUE/DT DUE TO THERMAL CONDUCTION
      PE6(DENS,TE)=(2.5*D2(DENS,TE)+2.0*D3(DENS,TE)+D4(DEA))*(TE-TWALL)
C     PI3 IS DUI/DT DUE TO CHARGE EXCHANGE
      PI3(DENS,DNEUT,TI)=0.01 6*DENS*DNEUT*TI*TI*(TI-TWALL)/(TI+100.0)
C     PI6 IS DUI/DT DUE TO THERMAL CONDUCTION
      PI6(DENS,TE,TI)=(2.5*D2(DENS,TE)+2.0*D3(DENS,TE)+D4(DEA))*(TI-TWAL
     2L)

      DEA=X(1)
      DENS=DEA
      TEA=X(2)
      IF(TEA.LT.20.*TWALL)TEA=.25*TWALL*(ALOG(EXP(4.*TEA/TWALL)+1.)+.01)
      TE=TEA
      TIA=X(3)
      IF(TIA.LT.20.0*TWALL) TIA=0.25*TWALL*ALOG(EXP(4.0*TIA/TWALL)+1.0)
      TI=TIA
      DNEUT=X(4)
      B=SQRT(ABS(B0*B0-4.05E- *DEA*(TEA+TIA)))
      BOLD=B
      DT=1.0
      GO TO (1,2,3,4),K
    1 CONTINUE
      AUXFCN=(D1(DEA,DNEUT,TEA)-D2(DEA,TEA)-D3(DEA,TEA)-D4(DEA))/DENS
      RETURN
    2 CONTINUE
      DEO=F*F*(0.0045*(F/ )** +123.0*(B/F)**0.66667)/
      AUXFCN=(PE1(P)-PE2(DEA,TEA,TIA)-PE3(DEA,DNEUT,TEA)-PE4(DEA,TEA)
     2-PE5(DEA,TEA)-PE6(DEA,TEA))/DENS
      RETURN
    3 CONTINUE
```

```
      AUXFCN=(P22(DEA,TEA,TIA)-P13(DEA,DNEUT,TIA)-P16(DEA,TEA,TIA))/DENS
      RETURN
4     CONTINUE
      DNT=322.0*PRES*EXP(-1.2 E-6*A*DI(DEA,DNEUT,TEA)/DNEUT)
      DFC=12. 8*PRES*EXP(-6.9 E-8*A*DI(DEA,DNEUT,TEA)/DNEUT)
      AUXFCN=DNEUT-DNT-DFC
      RETURN
      END

COMPILATION:          NO  DIAGNOSTICS.
```

```
 IPWR   IERR       POWER       DENSITY        TE            TI           DNEUT        FIELD        JSAT
   1      4       1.0000         .1957      0.0160        .0014        334.8780      1.0000        .0456
   2      6       1.5849         .4287      0.0171        .0286        334.8757      1.0000        .0999
   3      5       2.5119         .7974      0.0189        .0497        334.8720      1.0000        .1858
   4      4       3.9811        1.3800      0.0215        .0667        334.8661      1.0000        .3217
   5      4       6.3096        2.3011      0.0256        .0927        334.8568      1.0000        .5365
   6      4      10.0000        3.7514      0.0317        .1319        334.8420      1.0000        .8752
   7      4      15.8489        6.0304      0.0407        .1906        334.8185      1.0000       1.4082
   8      4      25.1189        9.6002      0.0538        .2786        334.7811      1.0000       2.2445
   9      4      39.8107       15.1633      0.0722        .4002        334.7217      1.0000       3.5517
  10      4      63.0957       23.7914      0.0971        .5715        334.6272      1.0000       5.5862
  11      4     100.0000       37.0982      0.1296        .8009        334.4767      1.0000       8.7384
  12      4     158.4893       57.5274      0.1698       1.0[illegible]3   334.2370      1.0000      13.6035
  13      4     251.1887       88.785       0.21[illegible]     1.45[illegible]1   333.8551      1.0000      21.0911
  14      4     398.1072      138.547       0.27[illegible]     1.8[illegible]4    333.2467      1.0000      32.6039
  15      4     630.9574      209.594       0.32[illegible]     2.35[illegible]0   332.2774      1.0000      50.3230
  16      3    1000.0000      321.662       0.39[illegible]8    2.8[illegible]4    330.7314       .9994      77.6785
  17      4    1584.8933      494.4685      0.45[illegible]     3.37[illegible]2   328.2583       .9999     120.1277
  18      8    2511.8865      762.6521      0.525[illegible]    3.86[illegible]7   324.2830       .9999     186.4579
  19      4    3981.0719     1131.8592      0.60        4.3[illegible]6    317.8264       .9998     291.0134
  20      4    6309.5736     1842.0377      0.70        4.7[illegible]1    307.1487       .9996     457.6786
  21      4   10000.0000     2889.8392      0.862       5.1[illegible]7    288.8464       .9994     727.7125
  22      5   15848.9329     4568.4417      1.168       5.59[illegible]2   254.6765       .9989    1179.9796
MODIFIED JACOBIAN IS SINGULAR
TRY A DIFFERENT INITIAL APPROXIMATION
```

```
                                                  5
                              7+                  7
                              12                  1
                              98                  6
                              1                   1
                              89                  3
```

```
    RUNID:  CWJ_58   PROJECT: 0_900          .     USER: 41288l021

  ITEM                                    AMOUNT     COST(DOLLARS)

CPU TIME                          00:00:05.074         $0.31
I/O REQUESTS                               144         $0.24
I/O-WORDS TRANSFERRED                   220,28         $0.11
CORE USAGE                               0. 51         $0.09
CARDS IN                                   143         $0.04
PAGES PRINTED                                5         $0.09
JOB CHARGE                                   1         $0.10

TOTAL COST                                             $0.98

    THE ABOVE DOLLAR AMOUNTS ARE APPROXIMATE AND ARE BASED ON RATES FOR WHILE-YOU-WAIT

    INITIATION  TIME:   09:06:4 -OCT 12,1973

    TERMINATION TIME:   09.07:00-OCT 12,1973

   PREVIOUS RUN TIME:   11:07:41-OCT 11,1972
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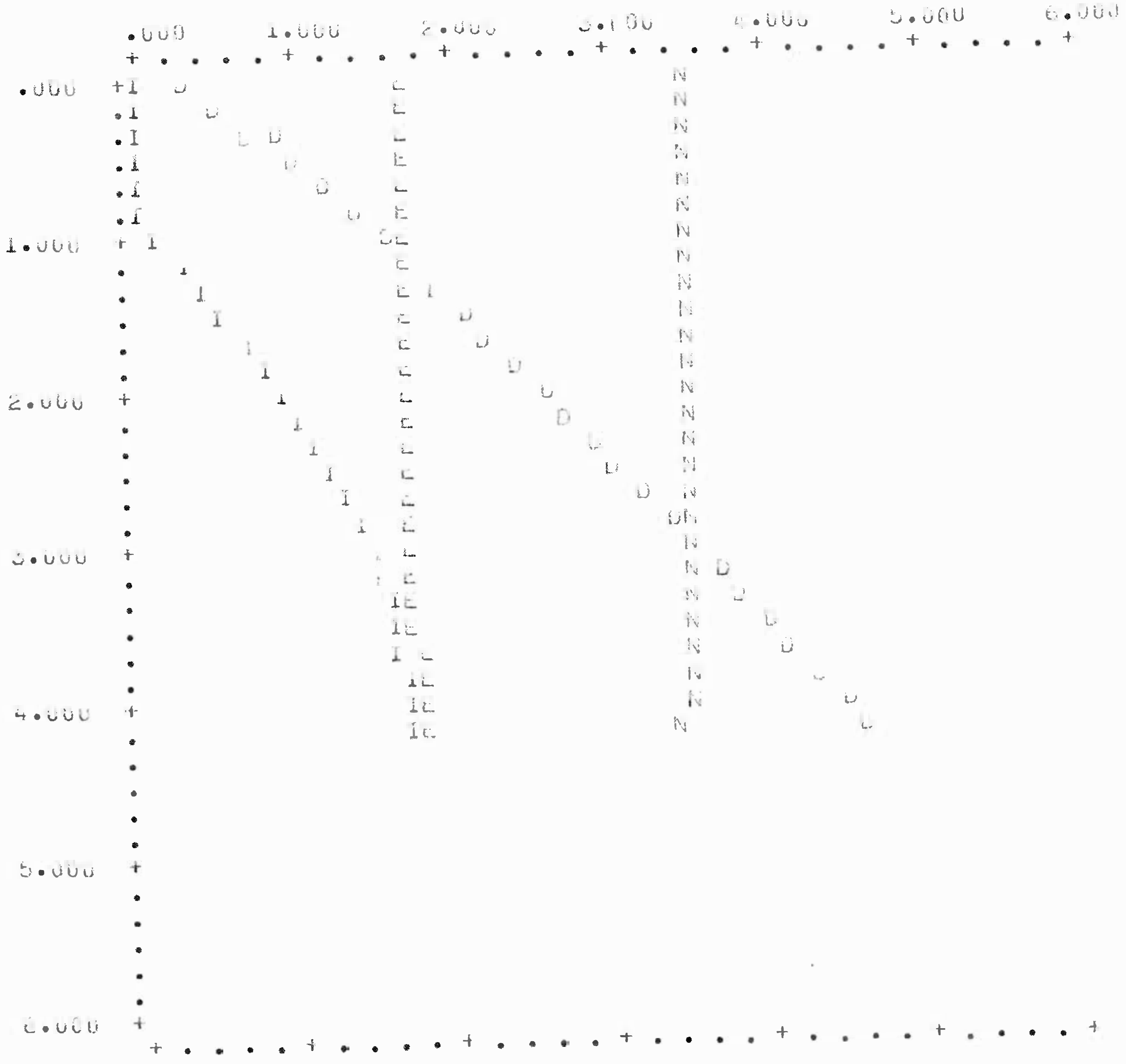
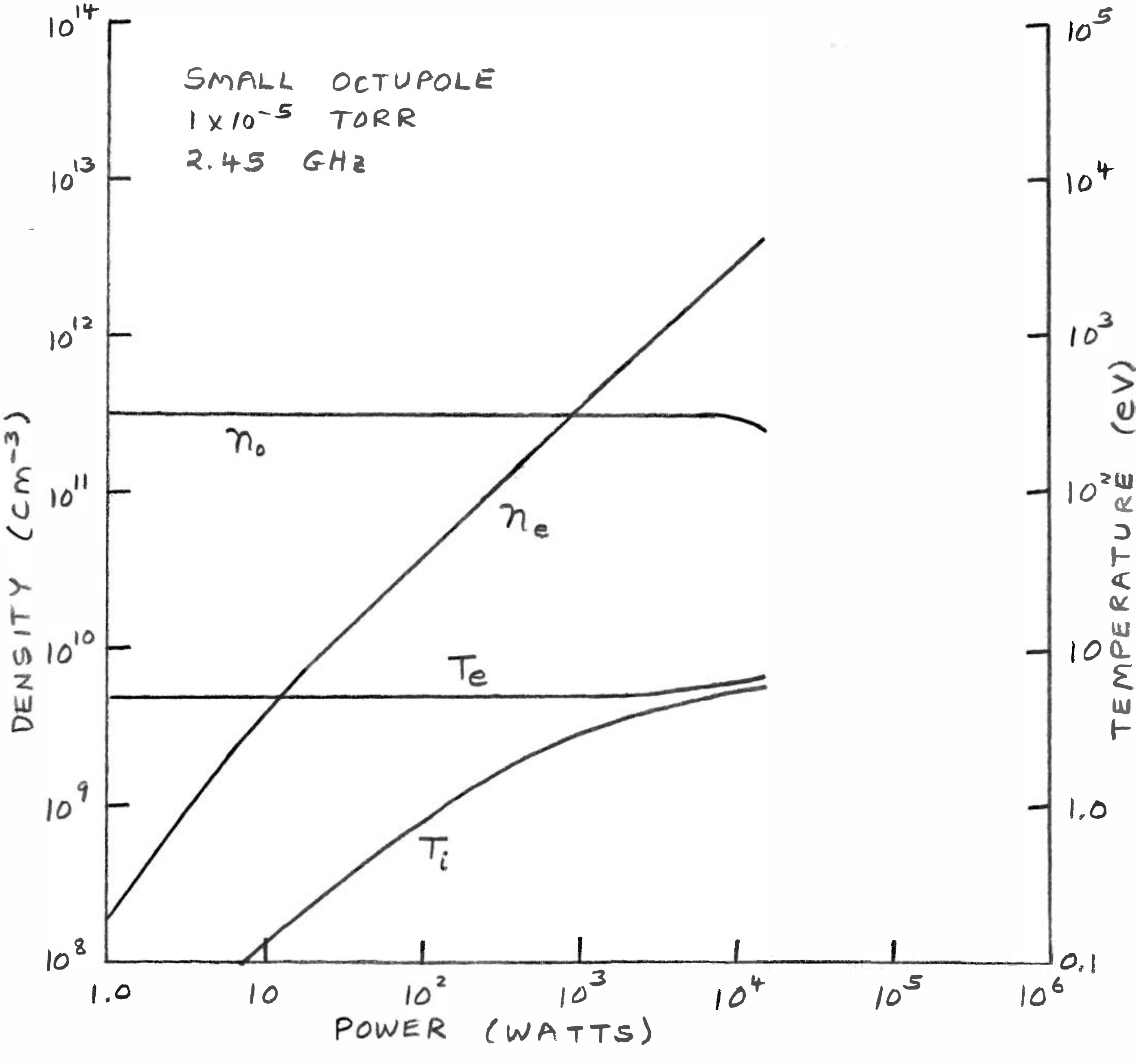
ZERO-D TIME INDEPENDENT SIMULATION

DENSITY AND TEMP





SMALL OCTUPOLE
1 x 10^-5 TORR
2.45 GHz
DENSITY (cm^-3)
TEMPERATURE (eV)
POWER (WATTS)
10^14
10^13
10^12
10^11
10^10
10^9
10^8
10^5
10^4
10^3
10^2
10
1.0
0.1
1.0
10
10^2
10^3
10^4
10^5
10^6
n_o
n_e
T_e
T_i

FIG 1

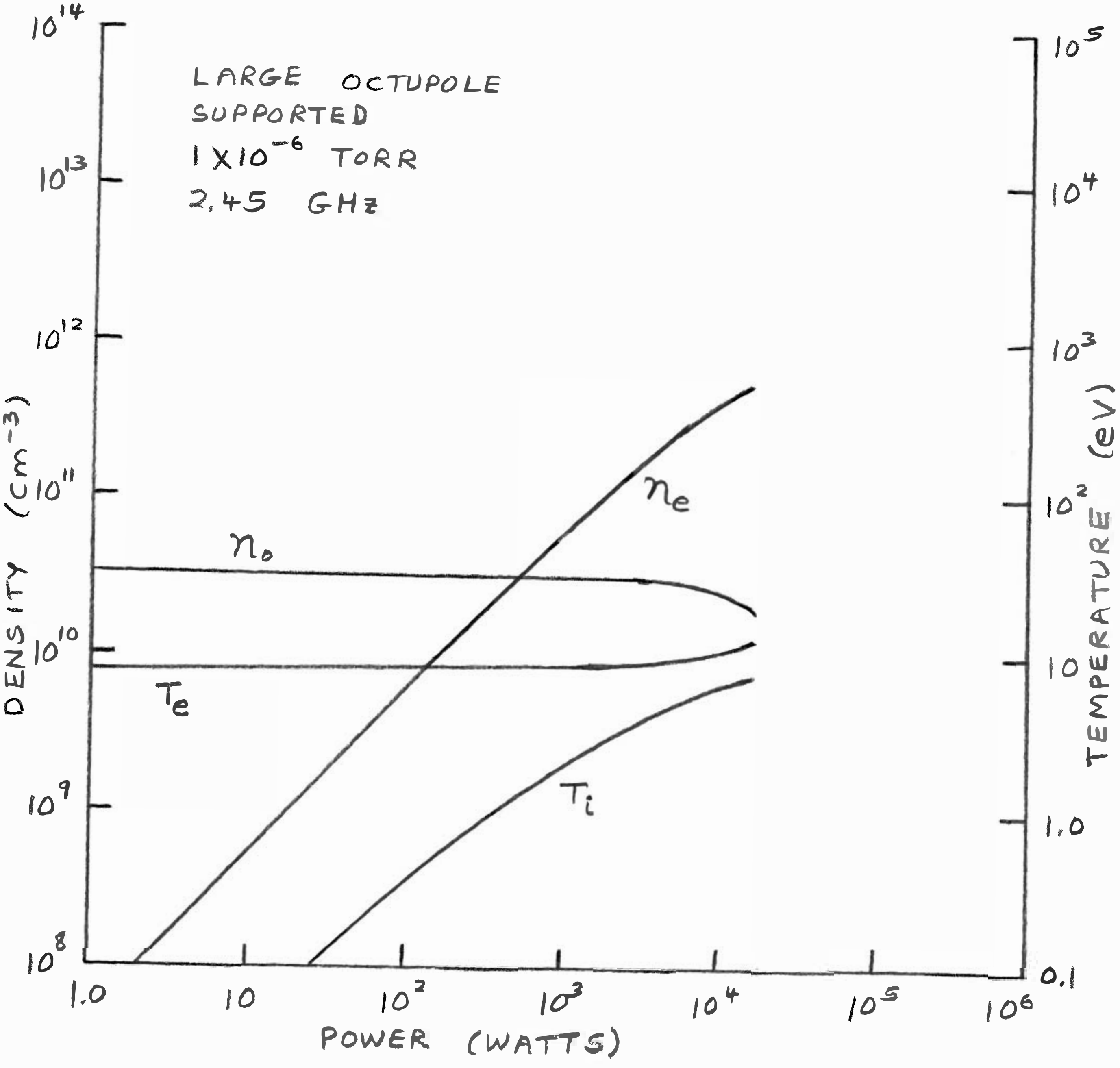
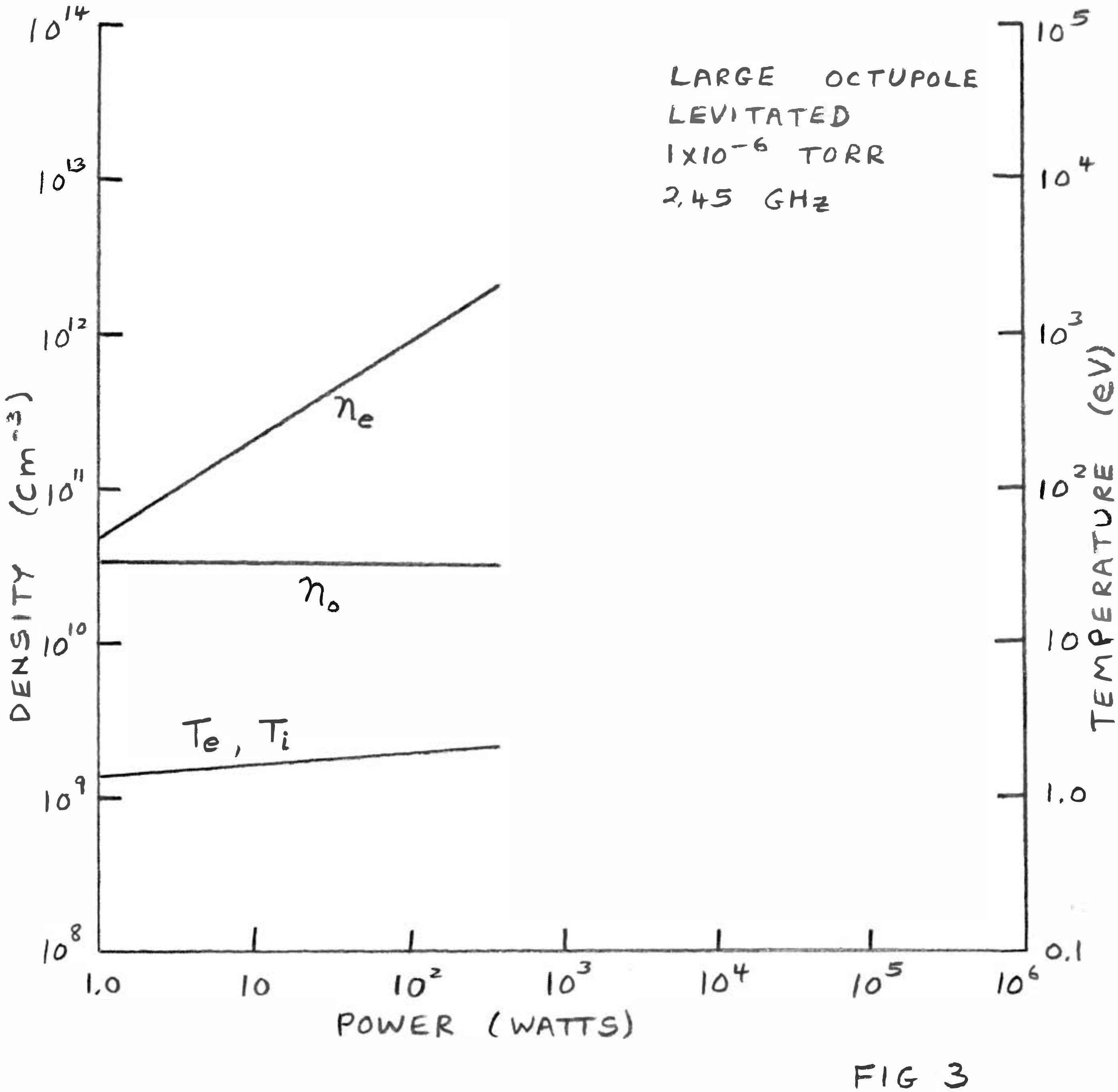
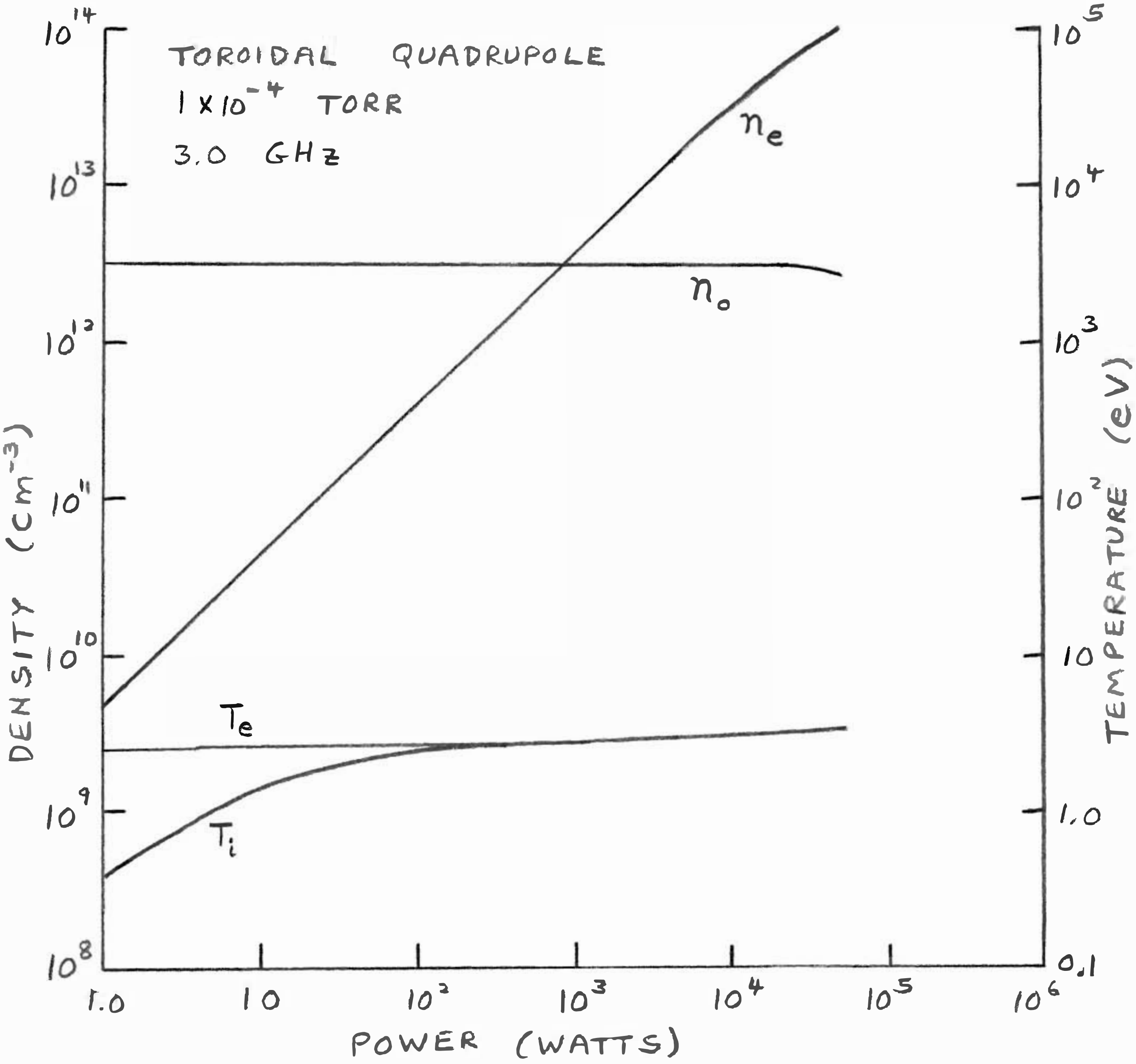

LARGE OCTUPOLE
SUPPORTED
1 X 10^{-6} TORR
2.45 GHz
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
DENSITY (cm^{-3})
10^{5}
10^{4}
10^{3}
10^{2}
10
1.0
0.1
TEMPERATURE (eV)
n_e
n_o
T_e
T_i
1.0
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
POWER (WATTS)

FIG 2



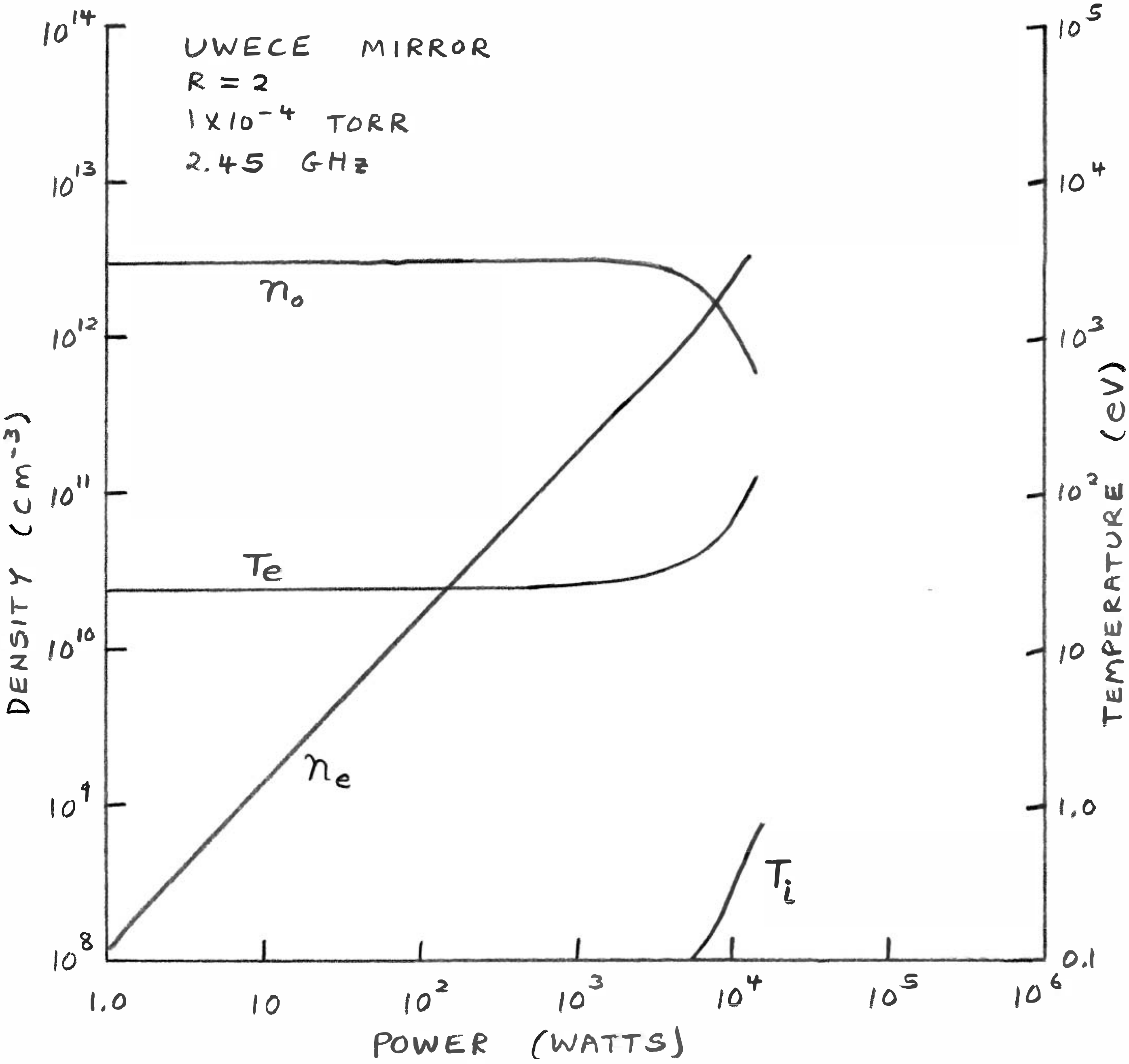
LARGE OCTUPOLE
LEVITATED
1x10⁻⁶ TORR
2.45 GHz
n_e
n_o
T_e, T_i
DENSITY (cm⁻³)
TEMPERATURE (eV)
POWER (WATTS)
10¹⁴
10¹³
10¹²
10¹¹
10¹⁰
10⁹
10⁸
10⁵
10⁴
10³
10²
10
1.0
0.1
1.0
10
10²
10³
10⁴
10⁵
10⁶

FIG 3



TOROIDAL QUADRUPOLE
1 x 10^-4 TORR
3.0 GHz
n_e
n_o
T_e
T_i
10^14
10^13
10^12
10^11
10^10
10^9
10^8
DENSITY (cm^-3)
10^5
10^4
10^3
10^2
10
1.0
0.1
TEMPERATURE (eV)
1.0
10
10^2
10^3
10^4
10^5
10^6
POWER (WATTS)

FIG 4



UWECE MIRROR
R = 2
1x10⁻⁴ TORR
2.45 GHz
n₀
nₑ
Tₑ
Tᵢ
DENSITY (cm⁻³)
10¹⁴
10¹³
10¹²
10¹¹
10¹⁰
10⁹
10⁸
TEMPERATURE (eV)
10⁵
10⁴
10³
10²
10
1.0
0.1
POWER (WATTS)
1.0
10
10²
10³
10⁴
10⁵
10⁶

FIG 5

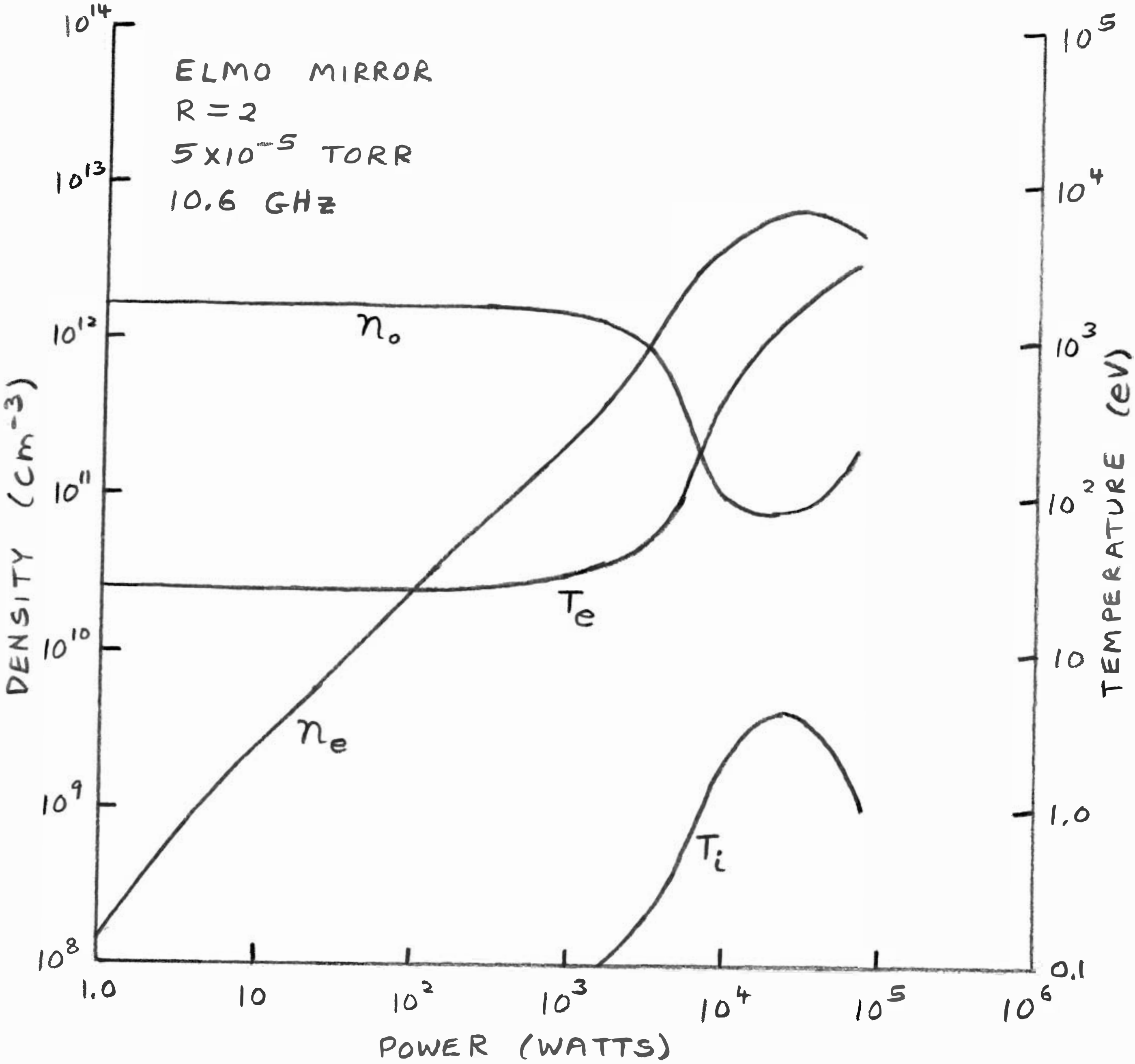

ELMO MIRROR
R = 2
5 X 10^{-5} TORR
10.6 GHz
DENSITY (cm^{-3})
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
TEMPERATURE (eV)
10^{5}
10^{4}
10^{3}
10^{2}
10
1.0
0.1
n_o
n_e
T_e
T_i
1.0
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
POWER (WATTS)

FIG 6

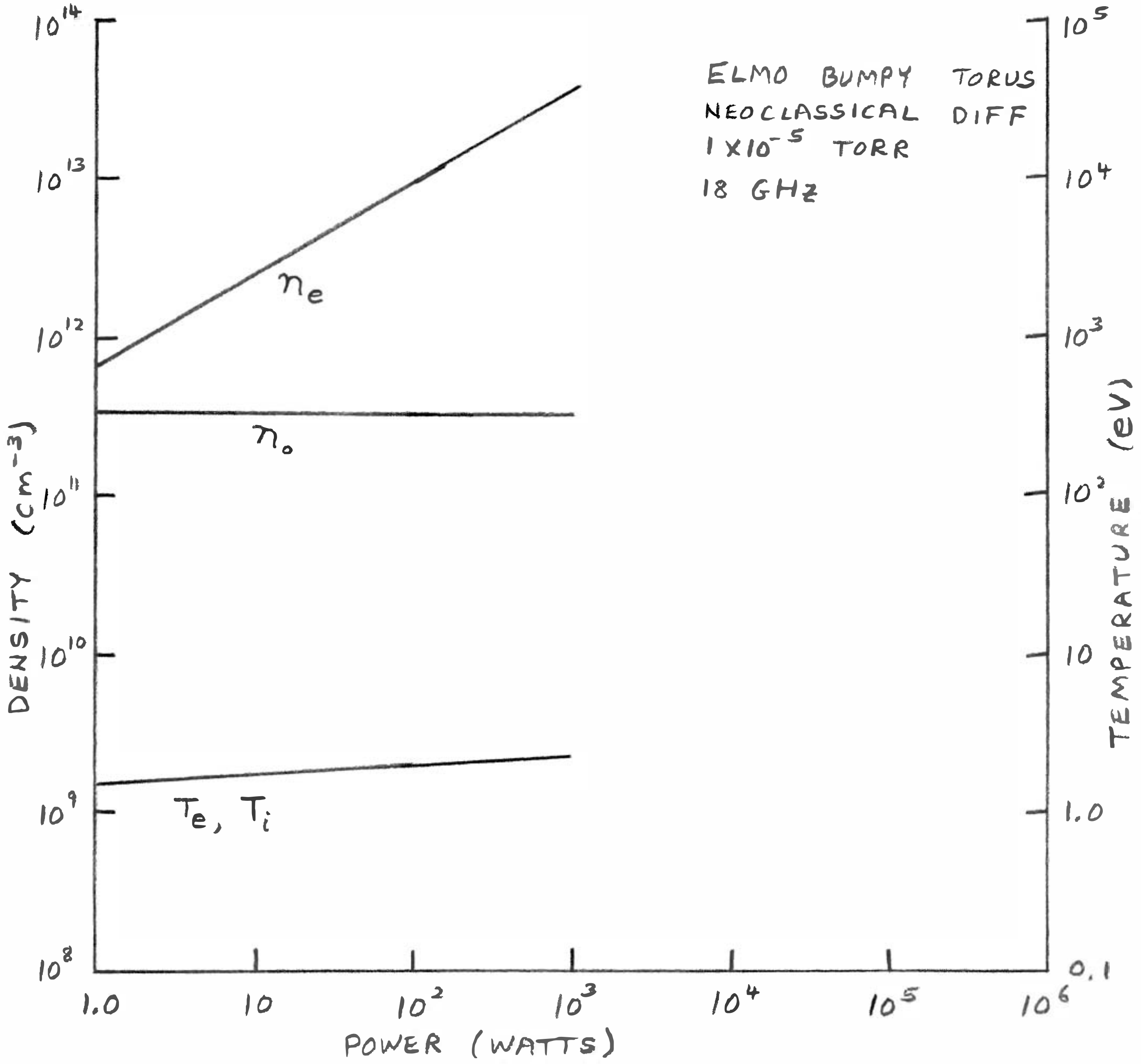

ELMO BUMPY TORUS
NEOCLASSICAL DIFF
1 X10^-5 TORR
18 GHz
DENSITY (cm^-3)
TEMPERATURE (eV)
POWER (WATTS)
10^14
10^13
10^12
10^11
10^10
10^9
10^8
10^5
10^4
10^3
10^2
10
1.0
0.1
1.0
10
10^2
10^3
10^4
10^5
10^6
n_e
n_o
T_e, T_i

FIG 7

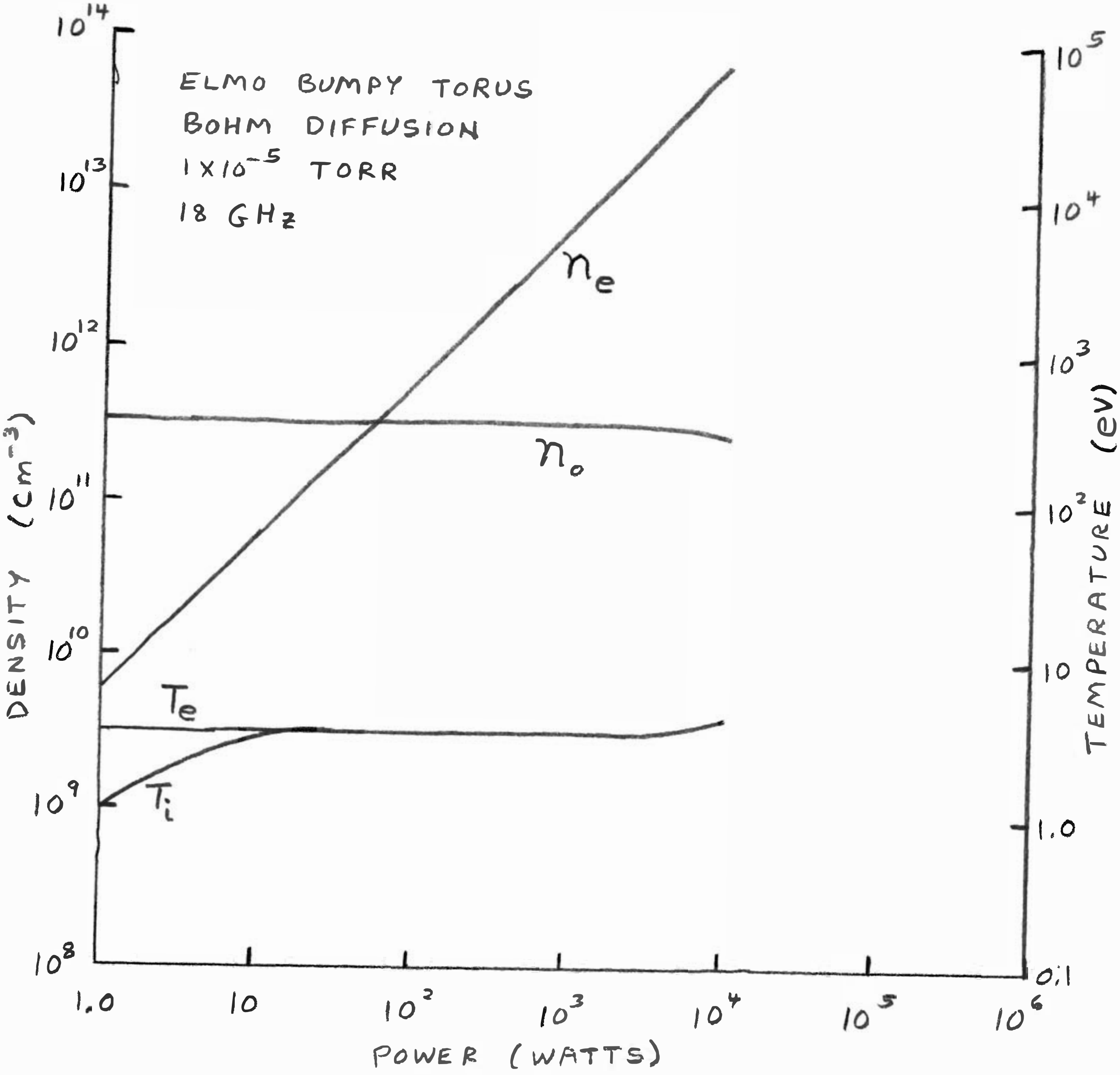

ELMO BUMPY TORUS
BOHM DIFFUSION
1×10⁻⁵ TORR
18 GHz
n_e
n_o
T_e
T_i
DENSITY (cm⁻³)
10¹⁴
10¹³
10¹²
10¹¹
10¹⁰
10⁹
10⁸
TEMPERATURE (eV)
10⁵
10⁴
10³
10²
10
1.0
0.1
1.0
10
10²
10³
10⁴
10⁵
10⁶
POWER (WATTS)

FIG 8