COMPARISON OF ELECTRON CYCLOTRON HEATING IN VARIOUS DEVICES

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I. Introduction

The electron cyclotron heating rate of a plasma in an arbitrary magnetic field can be written as

$$\frac{d\langle W\rangle}{dt} = \pi\int neE_{\perp}^2\delta(B-B_o)dV/2\int ndV, \quad (1)$$

where n is the electron density, $E_{\perp}$ is the rms electric field strength at frequency ω perpendicular to $\vec{B}$, B_o is the magnetic field strength at resonance ($B_o = m\omega/e$), $\langle W\rangle$ is the average electron energy, and the integrals are over the volume of plasma. Recent experimental observations (PLP 520) have confirmed the validity of this result for a wide range of parameters provided the density is low ($\omega_{pe}^2/\omega^2 \lesssim 0.01$). This paper will compare the heating rates in a toroidal octupole, a bumpy torus, and a Tokamak. For purposes of comparison, it is convenient to write the heating rate as

$$\frac{d\langle W\rangle}{dt} = \frac{e\langle E^2\rangle}{B_o} G, \quad (2)$$

where

$$G = \pi B_o\int n\delta(B-B_o)dV/3\int ndV \quad (3)$$

is a dimensionless quantity which depends only on the magnetic field shape, the position of the resonance zone, and the spatial distribution of plasma density.

II. Toroidal Octupole

The integrals is equation (3) have been evaluated numerically in PLP 282 for the Wisconsin Supported Toroidal Octupole assuming the density to be constant in space. At small values of B_o, the resonance occurs on a toroidal surface of circular cross section centered on the minor axis, and it is shown in PLP 282 that $G \propto B_o^{2/3}$. At high values of B_o (resonance surface in the bridge), G falls approximately as B_o^{-4}. A reasonable fit to the numerical calculation gives

$$G = [1.31\alpha^{-2/3} + 5.07\alpha^4]^{-1}, \qquad (4)$$

where $\alpha = B_o/B_{MAX}$

and B_{MAX} is the spatially maximum value of magnetic field in the machine.

III. Bumpy Torus

Equation (3) has been evaluated numerically for an idealized bumpy torus field with a mirror ratio of 2:1 and an infinite aspect ratio:

$$\left.\begin{aligned} B_\theta &= \frac{3}{2} - \left(\frac{1}{2} + \frac{r^2}{8}\right)\cos N\theta \\ B_r &= r\left(\frac{1}{4} + \frac{r^2}{32}\right)\sin N\theta \end{aligned}\right\} \qquad (5)$$

The density is assumed to be constant out to a flux surface on which B at the midplane of a mirror section is half its value on axis. The conditions approximate those expected in the ELMO Bumpy Torus (EBT) at Oak Ridge. The heating rate for a simple mirror with a 2:1 mirror ratio would be similar. The result is to show that $G \simeq 1$ over most of the range of magnetic fields in the device.

IV. Tokamak

If the poloidal field is neglected, the field in a Tokamak is

$$B = B_A R_A / R.$$

Assuming a parabolic density profile,

$$n = n_A[1 - (r/a)^2],$$

and a large aspect ratio ($A = R_A/a >> 1$), the heating rate is given by

$$G = \frac{8(A - 1)^2}{9A\alpha^5} [\alpha^2 - (\alpha A - A + 1)^2]^{3/2} \qquad (6)$$

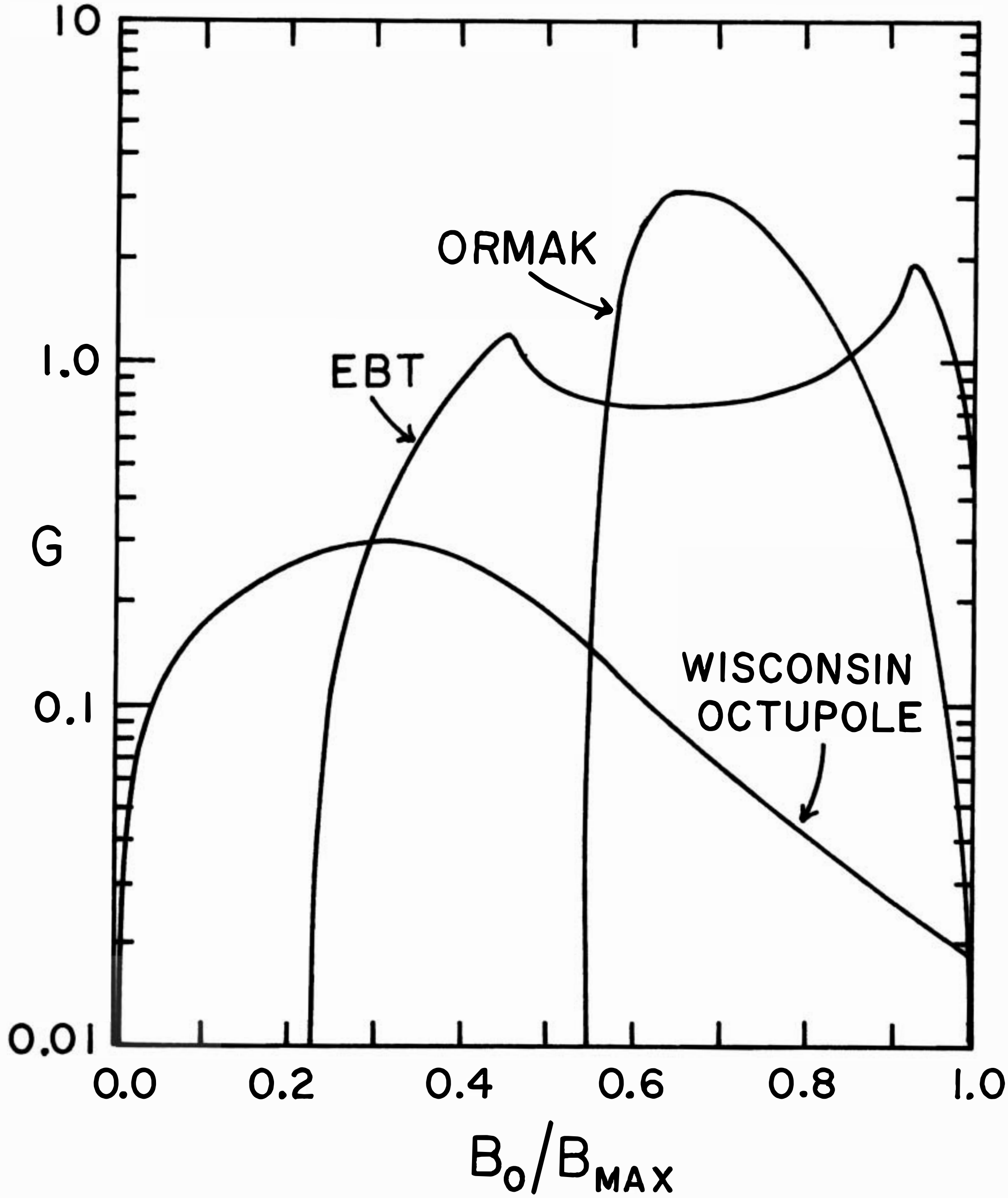
where α is the ratio of the field at the resonance surface to the maximum field (inside wall midplane). For the ORMAK device ($A = 3.4$) G has a maximum of about 3.0 at $\alpha \simeq 0.67$.

V. Conclusion

The normalized heating rates for these three devices are compared in Fig. 1. The Tokamak is relatively good and the octupole relatively bad in agreement with our intuition that a nearly uniform field enhances the electron cyclotron heating. For most cases of experimental interest, however, the absorption in ~ 100%, so that the actual average heating rate is

$$\frac{d\langle W\rangle}{dt} = P_o/\langle n\rangle V, \tag{7}$$

and the value of G merely gives the rms electric field within the cavity.



10
1.0
G
0.1
0.01
0.0
0.2
0.4
0.6
0.8
1.0
B_0/B_{MAX}
ORMAK
EBT
WISCONSIN OCTUPOLE