A LOW-FIELD LOW-ASPECT RATIO TOKAMAK

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I. INTRODUCTION

The objective of this report is to consider the type of experimental program utilizing a tokamak configuration that would be commensurate with a university level research program. The goals of such a program are: low cost, meaningful experimental parameters, flexibility, reasonable technological level, and the training of students.

Indications in current tokamak research are that low aspect ratio devices[1] will be most important. The lowest aspect ratio tokamak at present is (probably) ORMAK or PLT, with an aspect ratio slightly greater than 3. The problem has been that as higher and higher toroidal fields were required, it became impossible to lower the aspect ratio, due to the increased amount of copper and/or iron for the core for the corresponding circulating current, were needed. Thus, the possibility of a low aspect ratio device could only be realized if the toroidal field was lowered. This would have several advantages. (1) The amount of copper would be much smaller, providing for a lower aspect ratio, and at substantially lower cost, both in power supply and coil design. (2) A lower toroidal field means that microwave and r.f. heating, e.g., electron cyclotron resonant heating, might cost much less than that required for the high field tokamaks.

The low aspect ratio would also then permit a large minor radius and/or convenient modification of the minor cross section by time and spatially shaped magnetic fields. It would also lower the impurity levels, since a low aspect ratio implies a low surface to volume ratio.

In addition, the low aspect ratio, low field configuration is also most useful for another configuration, the E-beam spherator. It is easy to increase the circulating current so that the tokamak field becomes a spherator field when the toroidal field is low, provided that mainly electrons are being accelerated. This can be done by lowering the base pressure in the system.

An important question is whether such a low-field-low-aspect-ratio tokamak could actually operate in the tokamak mode. The following analysis attempts to answer this question.

II. PARAMETRIC ANALYSIS

Let us consider the feasibility of a tokamak designed to have the following parameters:

Aspect ratio	3
Toroidal field	5 kG
q at limiter	3
Minor radius	40 cm

With these values the poloidal magnetic field is 555 gauss and the plasma current is 111 kA. If q could be lowered to 2, then the plasma current is 166 kA.

At a toroidal field of 5 kilogauss it appears to be necessary to keep the plasma frequency down to the order of the electron cyclotron frequency if supplementary electron cyclotron heating is to be utilized. At 5 kG, this suggests $n_e \approx 2.5 \times 10^{12}/cm^3$ for the limit on the electron density. This is somewhat low for typical tokamaks, but, as will be shown subsequently, the device may still be operable as a tokamak. On the other hand, if this density were exceeded at the center of the plasma, microwave radiation could still be absorbed on the edge; this would produce a leveling of the temperature profile in the center and a steepening near the limiter. The reduced gradient in the core of the plasma should reduce heat conduction out of the ohmically heated zone. It should also be noted that other forms of r.f. heating, e.g., ion cyclotron, lower hybrid parametric heating, are not limited by $\omega_p=\omega_{ce}$.

The time scales for this configuration are developed as follows. Assume q=3 and β_p (poloidal beta)≃.25, $n_e=2.5\times10^{12}/cm^3$, and $T_i=\frac{1}{2}T_e$. Then

$$n_e k(T_e + \frac{T_e}{2}) = \beta_p \frac{B_p^2}{2\mu_o} \tag{1}$$

gives T_e≈500 eV. If we assume a pure hydrogen plasma (Z=1) the corresponding electron-ion equipartition time is 83 msec and the electron-ion scattering time τ_{ei} is

$$\tau_{ei} = \frac{1}{\nu_{ei}} = \frac{T_e^{3/2}}{11.4\ n\ \ell n\Lambda} \text{ in c.g.s. units} \tag{2}$$

$$\tau_{ei} = 2.9 \times 10^{-5} \text{ sec.}$$

The bounce time for trapped electrons is

$$\tau_b \simeq \frac{4\pi Rq}{V_{\parallel}}$$

$$V_{\parallel} \leq V_{\perp}(\frac{2}{A})^{1/2} = (\frac{2kT}{mA})^{1/2} \quad . \tag{3}$$

Thus

$$\tau_b \simeq \frac{2\pi Rq}{(\frac{kT}{m})^{1/2}} \sqrt{A} = 4.2 \times 10^{-6} \text{ sec.} \tag{4}$$

The circulation time for a transiting electron in the poloidal direction is

$$\tau_c = \frac{2\pi Rq}{\sqrt{\frac{2kT}{m}}} = \left(\frac{A}{2}\right)^{1/2} \qquad \tau_b \simeq \tau_b \tag{5}$$

The various regimes of neoclassical diffusion[2,3] are

$$\tau_{ei} > A\tau_b \quad \text{banana regime}$$

$$\tau_c < \tau_{ei} < A\tau_b \quad \text{plateau regime}$$

$$\tau_{ei} < \tau_c \quad \text{Pfirsch-Shlüter regime}$$

For the device considered here, $\tau_{ei} \approx A\tau_b$ and thus the electrons are on the verge of the banana regime. The neoclassical particle diffusion coefficient is

$$D_{NC} \cong \frac{2\rho_e^{\,2}}{\tau_{ei}} A^{-1/2} \left(\frac{B_T}{B_P}\right)^2 \tag{6}$$

or

$$D_{NC} = \frac{2\rho_e^{\,2}}{\tau_{ei}} q^2 A^{3/2} = .073 \text{ m}^2/\text{sec.}$$

where ρ_e, the electron gyroradius, is

$$\rho_e = \sqrt{\frac{2m_e kT_e}{e^2 B_T^{\,2}}} = 1.51 \times 10^{-4} \text{ m.} \tag{7}$$

This gives a neoclassical particle confinement time of

$$\tau_p = \frac{a^2}{D_{NC}} = 2.2 \text{ sec.} \tag{8}$$

The neoclassical <u>ion</u> energy confinement time is

$$\tau_E^{\,i} \approx \tau_p \sqrt{\frac{m_e}{M_i}} = 51 \text{ msec.} \tag{9}$$

If this is the dominant loss mechanism, then total plasma energy confinement time is comparable to $\tau_E^{\,i}$.

Pseudoclassical diffusion[4,5] gives

$$D_{pc} = c^2 \ (D_{c\ell})_p$$

where

$$(D_{c\ell})_p = \frac{2\pi_e^{\ 2}}{\tau_{ei}} \ \frac{B_T}{B_P} \qquad (10)$$

and c^2 is an unknown coefficient. It we take $c^2 \approx 1$, then the pseudoclassical confinement time is comparable to the neoclassical value for A=3, in the banana regime. Tt should be recalled that pseudoclassical theory extends the banana regime scaling into the plateau regime with a higher diffusion coefficient. From the above expression, it can be seen that, as the aspect ratio is lowered, the pseudoclassical diffusion coefficient approaches the neoclassical diffusion coefficient.

III. SCALING FROM CURRENT EXPERIMENTS

We use pseudoclassical scaling in which

$$\tau_E \propto \frac{a^2 T_e^{\ 1/2} B_T^{\ 2}}{n \ q^2 \ A^2} \qquad (11)$$

A. From T-6.[6] The parameters of this experiment are: τ_E=1.5 m sec., B_T=7.5 kG, q=4.9, A=3.5, n≈10^{13}/cm^3, a=20 cm and T_e≅200 eV.

This gives an energy confinement time of ~62 m sec for the low field, low density device considered here.

B. From ORMAK. For ORMAK, the parameters are: τ_E=15 m sec., B_T=20 kG, q≅5, A=3.4, n≅3x10^{13}/cm^3 a=23.5 cm, T_e≅1000 eV.

This scales to τ_E≅64 m sec for the device considered here.

C. From ST. For ST, the parameters are: τ_E=12 m sec., B_T=37 kG, q≃3, A=8.4, n=2x10^{13}/cm^3, T_e≃2000 V, a=13 cm.

This scales to τ_E≃65 m sec.

Based on these results, a conservative guess for the energy confinement time is ~60 m sec for the low field, low density device considered here.

IV. WILL THE MACHINE WORK AT THE LOW DENSITY?

In order for this to be the case, it must be required that the following inequality be satisfied:

$$P_{RF} + P_{OH} \geq P_{\substack{\text{thermal} \\ \text{conductivity}}} + P_{\substack{\text{charge} \\ \text{exchange}}} + P_{\substack{\text{impurity} \\ \text{radiation}}} \tag{12}$$

We assume an initial ion temperature of 5 eV. Then, the ohmic heating is greater than the electron-ion rethermalization losses whenever

$$\sigma E^2 > \frac{3}{2} \frac{nK(T_e - T_i)}{\tau_{eq}} \quad . \tag{13}$$

Now

$$\sigma = \frac{T_e^{3/2}\ 100}{6.53 \times 10^3\ \ell n\ \Lambda} \text{ ohm}^{-1}\ \text{m}^{-1} \tag{14}$$

and

$$\tau_{eq} = \frac{2.5 \times 10^{-2}\ T_e^{3/2}}{10^{-6}\ n\ \ell n\ \Lambda} \tag{15}$$

Using values of n_e=2.5x10^{18}m^{-3} and $\ell n\ \Lambda$ of 12 then Equation (13) implies

$$E^2 > 4.87 \times 10^{12} \frac{(T_e - T_i)}{T_e^3} \quad \text{volts}^2/\text{m}^2 \tag{16}$$

where T is in ^{o}K. We can solve for the maximum value of the quotient $\frac{(T_e-T_i)}{T_e^3}$ and find that this occurs whenever T_e=3/2 of T_i, which is 7.5 eV, when T_i=5 eV. The corresponding minimum value for E is thus

$$E_{min} > 14.7 \text{ volts/meter} .$$

The next question to be asked is whether this field causes electron runaway. The critical field for electron runaway is

$$E_{max} = E_c = \frac{q_e \ln \Lambda}{8\pi\varepsilon_o \lambda_D^2} \tag{17}$$

where λ_D is the Debye length, $\left(\frac{\varepsilon_o kT_e}{ne^2}\right)^{1/2}$.

For the parameters assumed above, the critical field is:

$$E_c = 51.8 \text{ volts/meter} .$$

To avoid runaway electrons in the thermal regime, it is necessary that $E<E_c$. The range between 14.7 V/m and 51.8 V/m is not very big. However, both E_{min} and E_{max} scale like n/T so that no improvement in E_{max}/E_{min} is obtained by going to higher density. Note that one needs to turn down the electric field as T_e rises in order to avoid electron runaway. However, it should be pointed out here that if $E>E_c$ then runaway electrons may be accelerated to produce an E-beam spherator configuration.

V. WILL THE RESISTIVITY BE ANOMALOUS?

For ion acoustic anomalous resistivity, one needs

$$\bar{V} > V_s = \frac{2kT_e}{m_i} \tag{18}$$

where

$$\bar{V} = \frac{I}{\pi a^2 n q_e} = 4.72 \times 10^5 \text{ m/sec.} \tag{19}$$

Now

$$V_s = \left[\frac{2(500)\times 1.6\times 10^{-19}}{1.67\times 10^{-27}}\right]^{1/2} \tag{20}$$

$$= 3.1\times 10^{5}\ \text{m/sec.}$$

Hence, anomalous resistivity may be present, especially during the "semi-early stage" when $T_i \ll T_e$ and T_e is very low.

VI. CALCULATION OF POLOIDAL BETA

So far we have assumed T_e=500 eV, n=2.5x10^{12}/cm^3, q=3, I=111 kA. This corresponds to β_p=.25, which is somewhat low. In this section we attempt to determine if this value of β_p is consistent with ohmic heating as the only energy source. Of course, application of auxiliary heating, such as microwaves or r.f., will increase T_e and thus β_p.

Considering only ohmic heating, we equate the heating rate to the energy loss rate;

$$\frac{J^2}{\sigma} = \frac{3}{2}\,\frac{nK(T_e + T_i)}{\tau_E} \tag{21}$$

where

$$\tau_E = \frac{1}{n}\,c_0\,T_e^{\,1/2} \tag{22}$$

and

$$\sigma = \frac{1}{\alpha}\,c_1\,T_e^{\,3/2} \tag{23}$$

with c_0 and c_1 constants. α is the anomalous resistivity factor. If we set $T_i=\eta\,T_e$ with $\eta<1$ then

$$\frac{\alpha I_p^{\,2}}{\pi^2 a^4 c_1 T_e^{\,3/2}} = \frac{3}{2}\,\frac{n^2 K T_e(1+\eta)}{c_0 T_e^{\,1/2}}\ . \tag{24}$$

Now

$$I_p = \frac{2\pi a B_p}{\mu_o}$$

so

$$n^2 T_e^{\ 2} = \frac{2}{3K} \frac{\alpha}{1+\eta} \frac{4B_p^{\ 2}}{\mu_o^{\ 2} a^2} \frac{c_0}{c_1} . \tag{25}$$

Now

$$c_1 = \frac{1}{65.3 \ \ell n \Lambda} , \text{ and } c_0 = 1.83 \times 10^{16} \ B_p^{\ 2} \ a^2 \ \sqrt{A}$$

in MKS units using neoclassical scaling from ORMAK conditions.

Therefore

$$n^2 T_e^{\ 2} = \frac{3}{2K} \frac{\alpha}{1+\eta} \frac{1.83 \times 10^{16}}{c_1} \frac{B_p^{\ 4} \sqrt{A}}{\mu_o^{\ 2}}$$

Now

$$\beta_p = \frac{nKT_e(1+\eta)}{B_p^{\ 2}/2\mu_o} \tag{26}$$

which gives

$$\beta_p = 6K \frac{\alpha}{1+\eta} \left[\frac{1.83 \times 10^{16}}{c_1} A^{1/2} \right]^{1/2} \qquad \text{if } \ell n \Lambda \approx 16. \tag{27}$$

Therefore

$$\beta_p = .04 \frac{\alpha^{1/2}}{(1+\eta)^{1/2}} A^{1/2} . \tag{28}$$

If

$$\alpha = 10, \qquad \eta = 1/2, \qquad A = 3$$

then

$$\beta_p \simeq 0.13 . \tag{29}$$

From T-3, $\alpha \simeq 3$ if $\bar{V}/V_s < 1$ and $\alpha \approx 25-30$ if $\bar{V}/V_s >> 1$. Thus, if we can get into the ion sound anomalous resistivity range, one might expect

$$\beta_p \approx .25$$

implying $T_e \simeq 500$ eV. Thus our initial assumption of $T_e = 500$ eV is reasonable.

VI. ALLOWANCES FOR IMPURITY RADIATION

If the aspect ratio is allowed to go below 3, the impurity radiation will drop off, since the surface to volume ratio will be much improved for a fixed volume. Nevertheless, it is most important to allow for a very clean system.

VIII. NON-CIRCULAR CROSS SECTION

There are strong theoretical indications that circular minor cross section tokamaks may not be as desirable as non-circular ones.[7,8,9] First of all, the Kruskal-Shafranov limit is really determined not by the aspect ratio, but rather by the ratio of the major to minor circumferences. An elliptical minor cross section presents a higher q for given values of circulating current and toroidal magnetic field. Thus, higher circulating currents can be tolerated, and the heating for such a tokamak will be improved. Non-circular cross sections of the plasma can be obtained by appropriate shaping of the conducting shell and/or the transverse magnetic field.

IX. E-BEAM SPHERATOR

The basic scheme for this device is to substitute a circulating beam of relativistic electrons for the solid ring in current spherator experiments.[10,11] Typical parameters for the FM-1 device are: B_T=2-3 kilogauss, and I_{ring}=150 kA. The spherator configuration has a low toroidal field compared to most tokamaks, and thus should be readily obtainable in this low field device. Operation might be as follows:

1. Initialize with low neutral gas pressure (2×10^{-8} torr).
2. Apply toroidal electric field to establish ring of relativistic electrons. (A small amount of OH may be used in the opposite direction to the ring current.)
3. Increase pressure and produce lower temperature high density plasma with microwaves.

Since the spherator geometry has B_P greater than B_T, the Kruskal-Shafranov limit is exceeded. However, it is claimed that there is a good chance that the relativistic ring is stable against the kink instability.[12] This conjecture is supported by experimental data from the ORMAK experiment.[13]

An additional limitation is the Alfvén limit for the maximum current in the beam. The limit is:

$$I_A = 1.7 \times 10^4 \ \beta\gamma \text{ amperes}$$

where

$$\beta = v/c, \quad \text{and} \quad \gamma = \frac{1}{(1 - \beta^2)^{1/2}} \ .$$

The Alfvén limit may be exceeded when a toroidal magnetic field is present and work at ORNL has recently shown this to be the case experimentally.

For a circulating current of, say 100 kA, B_T must be less than a given value to exceed the Kruskal-Shafranov limit and operate as a spherator. We calculate this field to be 1.75 kilogauss for a q of 1. Thus, lowering the field to this value would enable the same circulating current to operate as a spherator.

X. R.F. HEATING OF THE TOKAMAK

In this section we consider the problem of supplying supplemental r.f. energy to an ohmically heated plasma. The prime candidates for consideration in this design are 1) electron cyclotron resonant and non-resonant microwave heating, and 2) other modes such as whistler mode, ion-cyclotron heating, or lower hybrid resonance with parametric decay instabilities, launched in the plasma by means of appropriately designed coupling structures. These latter modes offer the advantages of relatively high power levels at low cost, since they are of lower frequency than the electron cyclotron resonant sources. However, it is important that these modes penetrate into the plasma (accessibility) and thus special care must be devoted to the nature of the coupling device. An estimate of the r.f. power required to equal the power supplied by the ohmic heating may be made as follows. The toroidal current density can be calculated in terms of the poloidal magnetic field as

$$J_\theta = \frac{B_p}{2\pi r} = \frac{B_T}{2\pi Rq} \quad \text{in CGS units.}$$

The ohmic heating power to the plasma[14] is supplied using the Spitzer conductivity as

$$P_\eta = 1.32 \times 10^{-2} \frac{Z_{eff}}{R^2 q^2} \frac{B_T^{\,2}\, \ell n\, \Lambda}{(T_e)^{3/2}}\, \alpha\, \frac{\text{watts}}{\text{cm}^3} \tag{31}$$

where Z_{eff} is the effective ionic charge of the plasma. B_T is in gauss, R is the major radius in cm, and T_e is in eV. The factor α is the "anomalous resistivity" factor due to trapped particle effects. To double the power supplied to the plasma above that from the ohmic heating requires a total r.f. power of

$$P_{RF} = P_{\eta} 2\pi R \pi a^2 \quad . \tag{32}$$

Based on the Spitzer conductivity, B_T of 5 kg, $\ell n \Lambda$ of 15, R=120 cm, a=40 cm, A=3.0, q=3, α=1 and T_e=500 eV, we get an r.f. power requirement of 13,000 watts. This is a minimum power, for if the resistivity is anomalous, and α=5, the r.f. power jumps to 65 kilowatts. It should be pointed out that this is r.f. power at any frequency that can be coupled into the plasma, and it can be pulsed. At frequencies below 1 GHz, such powers are readily available at reasonable cost. Again, with a lower aspect ratio, the r.f. power requirements drop considerably.

XI. POWER SUPPLY REQUIREMENTS

Some rough calculations on power requirements are as follows: for 5 kilogauss on the magnetic axis we need 2.5×10^6 R ampere-turns, where R is the major radius. Using a coil radius of 50 cm and winding it out of 1/2" square copper wire with a 1/4" diameter cooling hole, we have a resistance of approximately 1.7 m ohm/turn. If we take 10 coils of 10x10 turns each or 10^3 turns, we have a DC resistance of 1.7 ohms and require 2500 R amperes. For a major radius of 1 meter we require a power of 11 MW at 4.25 kV. It is conceivable that direct rectification of the existing power lines might be feasible.

If the system is pulsed for 50 milliseconds, the energy required is 530,000 joules or perhaps a bit more, which is not an exhorbitant size for a capacitor bank.

XII. DIAGNOSTICS TO BE USED

The most important diagnostic especially in view of the recent results on ATC, is Thomson scattering of laser light to obtain the electron temperature. In addition, microwave interferometers, diamagnetic loops, probes, fast neutral particle, x-raidation and optical spectral analyses should be made.

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