Plasma Potential in a Magnetic Mirror

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If the particle loss from a magnetic mirror is dominated by classical scattering into the loss cone, the plasma potential in steady state assumes a value such that electrons and ions are lost at an equal rate. It is the purpose of this note to calculate that potential, and to derive an expression for the particle confinement time in the presence of this ambipolar potential.

If electron-ion pairs are produced isotropically at the mirror midplane, and if the mean free path is greater than the length of the mirror, the electron current into the loss cone is given by

$$I_{oe} = Ne \left(\nu_{ioniz} \frac{\Omega_{LC}}{4\pi} + \nu_{scatt} \right),$$

where N is the number of electrons in the mirror, ν_{ioniz} is the ionization rate, ν_{scatt} is the scattering rate into the loss cone, and Ω_{LC} is the loss cone solid angle. It is simple to show that

$$\nu_{scatt} = \nu_e / \log R$$

and

$$\frac{\Omega_{LC}}{4\pi} = 1 - \sqrt{1 - 1/R} ,$$

where ν_e is the 90^o scattering time for electrons and R is the mirror ratio. Substitution gives

$$I_{oe} = Ne \left[\nu_{ioniz} (1 - \sqrt{1 - 1/R}) + \nu_e / \log R\right] .$$

Similarly, for ions,

$$I_{oi} = Ne \left[\nu_{ioniz} (1 - \sqrt{1-1/R}) + \nu_i / \log R\right] .$$

If the particles flow along the field to conducting ends, a sheath will develop at the wall, and the plasma will charge up to a potential such that the electron and ion currents are equal. For maxwellian electrons and ions, these currents are given by

$$I_e = I_{oe} \max [1, \exp(- e\Phi/kT_e)] \equiv I_{oe} A_e$$

and $$I_i = I_{oi} \max [1, \exp(e\Phi/KT_i)] \equiv I_{oi} A_i ,$$

where Φ is the potential of the plasma relative to the end walls. We also note that in steady state $\nu_{ioniz} = \nu_{loss}$, and

$$I_e = I_i = Ne\, \nu_{loss} = Ne\, \nu_{ioniz} .$$

Setting $I_e = I_i$ and eliminating ν_{ioniz} gives

$$\nu_e A_e [1 - A_i (1 - \sqrt{1 - 1/R})] = \nu_i A_i [1 - A_e (1 - \sqrt{1 - 1/R})] .$$

For coulomb collisions,

$$\frac{\nu_e}{\nu_i} = \frac{m_i^{1/2}\, T_i^{3/2}}{m_e^{1/2}\, T_e^{3/2}} .$$

For hydrogen ($m_i = 1836\, m_e$) ,

$$\frac{\nu_e}{\nu_i} = 43\, \alpha^{3/2} ,$$

where $\alpha \equiv T_i/T_e$, and T_i and T_e are the ion and electron temperatures respectively. For $\Phi<o$,

$$\frac{e\Phi}{kT_i} = - \ln \left[1 - \sqrt{1 - 1/R} + \frac{\sqrt{1 - 1/R}}{43\, \alpha^{3/2}}\right] ,$$

and for $\Phi>o$,

$$\frac{e\Phi}{kT_i} = \frac{1}{\alpha} \ln [43\alpha^{3/2} \sqrt{1 - 1/R} + 1 - \sqrt{1 - 1/R}] .$$

These equations are plotted in Fig. 1 for various mirror ratios. Note that the plasma potential is positive for

$$\alpha = T_i/T_e > \alpha_o \equiv (m_e/m_i)^{1/3} = 0.08167$$

independent of the mirror ratio. For $\alpha = \alpha_o$, electrons and ions scatter into the loss cone at the same rate, and no space charge develops. In the large mirror ratio limit, the potential can be written as

$$e\Phi = \frac{3}{2}\left[kT_e \max\,(0,\ \ln\,(\alpha/\alpha_o)) + kT_i \min\,(0,\ \ln\,(\alpha/\alpha_o))\right] .$$

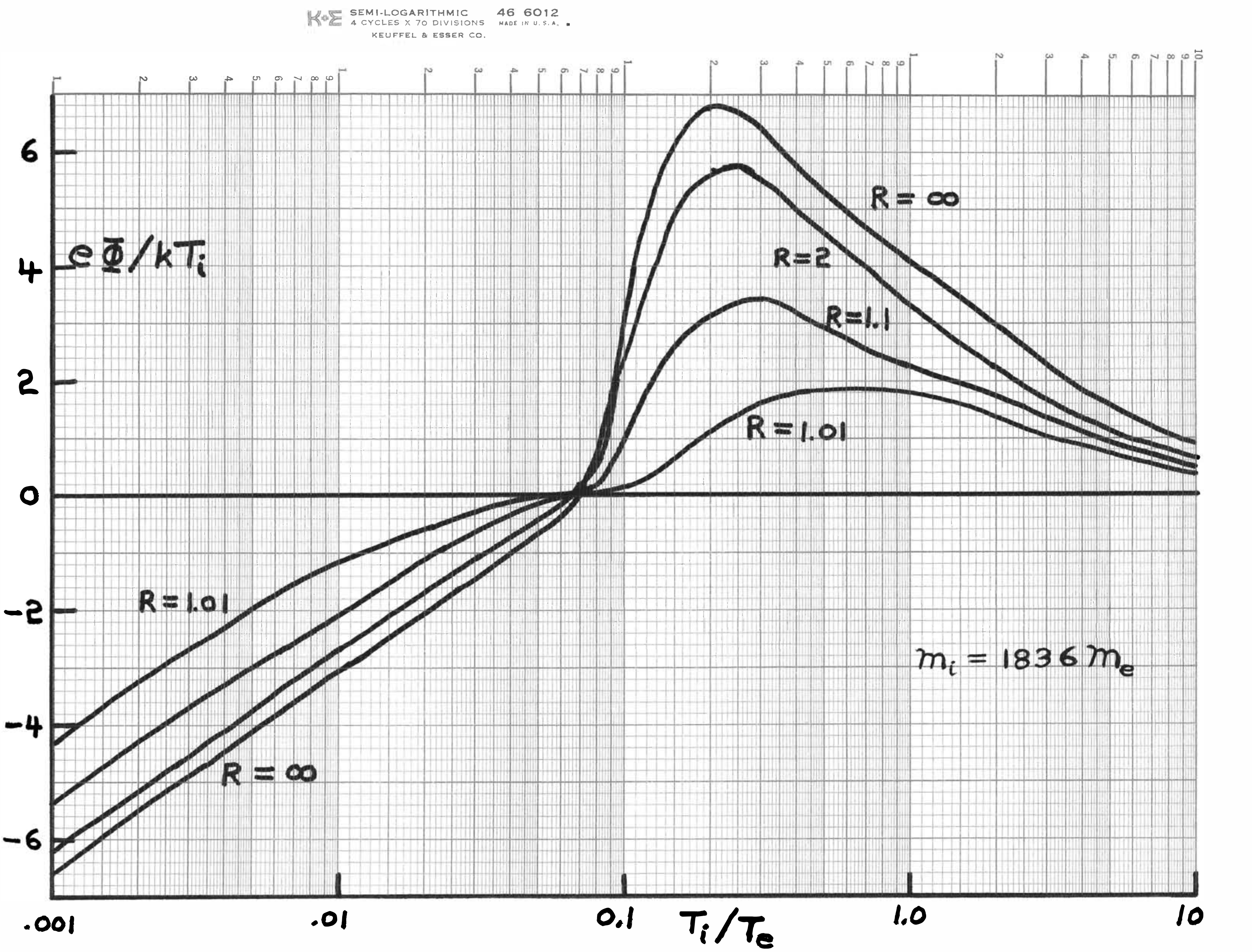
Finally, we can calculate the particle loss rate from

$$\nu_{loss} = \frac{I_e}{Ne} = \frac{I_i}{Ne} ,$$

with the result that

$$\nu_{loss} = \frac{\min\,\{\nu_i,\ \nu_e\}}{\sqrt{1 - 1/R}\,\log R} .$$

This result displays the fact that the more slowly diffusing specie dominates the loss, and that the loss is enhanced when some particles are born in the loss cone as is the case with a microwave produced plasma.



K+E SEMI-LOGARITHMIC 46 6012
4 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.
$e\Phi/kT_i$
6
4
2
0
-2
-4
-6
R = ∞
R=2
R=1.1
R=1.01
R=1.01
R = ∞
$m_i = 1836\, m_e$
.001
.01
0.1
1.0
10
T_i/T_e