Design of Delay Lines and Pulse Forming Networks

by

J.C. Sprott

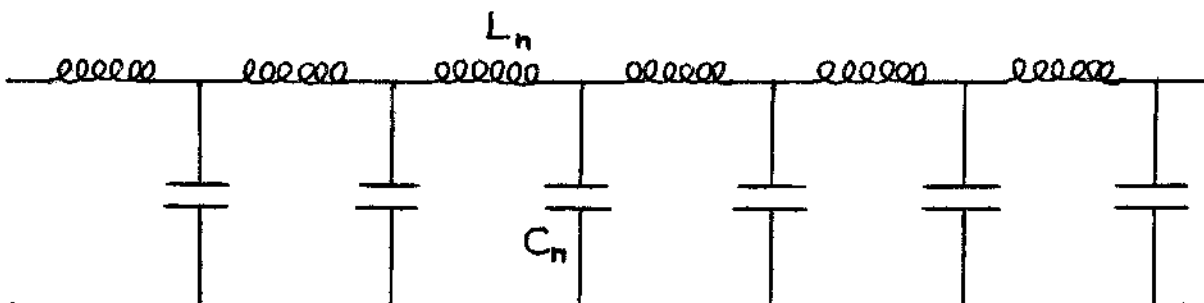July 1970

Plasma Studies

University of Wisconsin

Lumped-constant transmission lines are useful for delaying pulses, phase shifting sine waves, and storing energy for devices such as magnetrons that require a rectangular voltage pulse. Handy formulas and considerations for designing such lines are collected here for convenience.

Transmission lines are characterized by an electrical length T (time required for an electromagnetic wave to propagate from one end to the other) and by a characteristic impedance Z (the resistance required to terminate the line without causing reflection of the wave). A transmission line is a four terminal device that can be approximated by the electrical equivalent circuit:

An "ideal" transmission line has an infinite number of such LC networks, so that L_n and $C_n \to 0$. The fundamental equations for a transmission line are

$$T = \sqrt{LC}$$

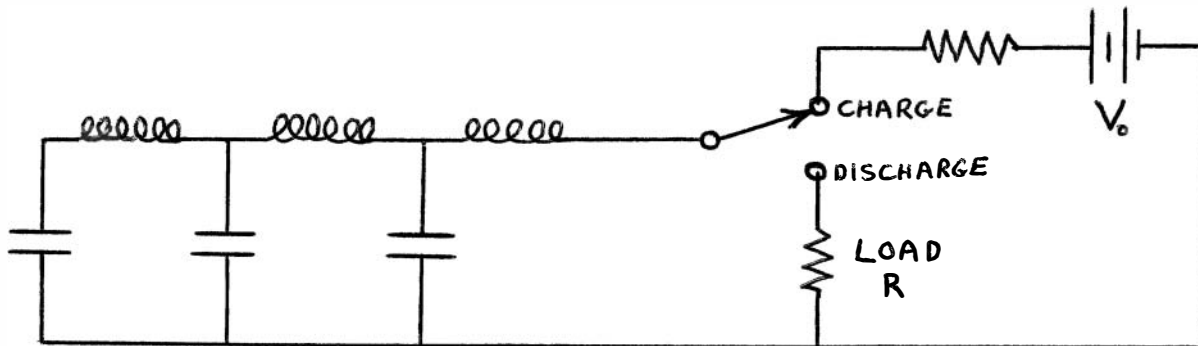$$Z = \sqrt{L/C},$$

from which we calculate

$$\begin{cases} L \ (= nL_n) \ = \ TZ & \text{(1)} \\[2ex] C \ (= nC_n) \ = \ T/Z, & \text{(2)} \end{cases}$$

where n is the number of identical LC networks in the line.
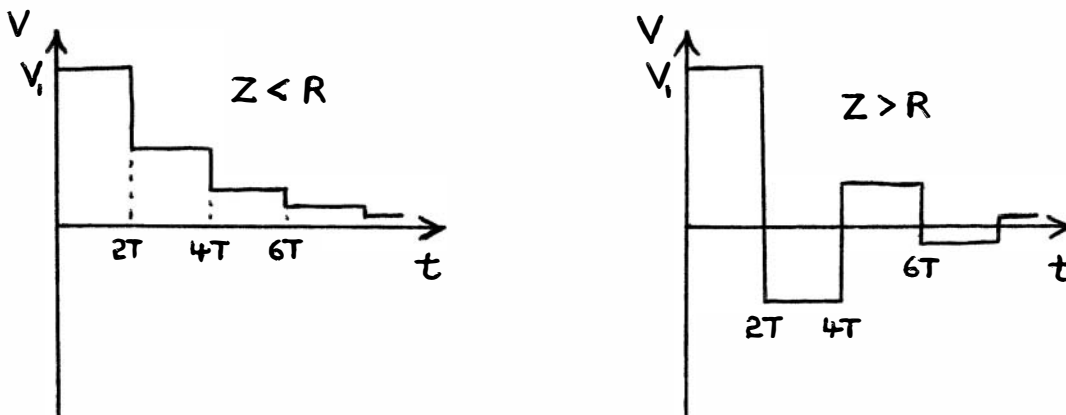
For use as a pulse forming network, the capacitance of the line is charged to a voltage V_o and then switched on to the load, R:

If the line is matched to the load (Z = R), the voltage developed across R is $V_o/2$ and has a duration of 2T. For Z $\neq$ R, a reflected wave is produced, the initial voltage at the load is

$$V_1 = \frac{RV_o}{R + Z},$$

and the waveform is as follows:

For a "nonideal" line consisting of a finite number n of LC networks, each network can be considered as a low pass filter with a cutoff frequency of

$$\omega_c = 1/\sqrt{L_n C_n} = n/T. \tag{3}$$

This cutoff causes dispersion in a delay line, and causes a pulse forming network to produce a pulse with a rise and fall time of

$$\tau = 1/\omega_c = T/n, \tag{4}$$

and oscillations at a frequency ω_c. The dispersion is minimized by designing the line so that the mutual inductance between adjacent coils is ~ 0.36 L. For a single layer, continuously wound, air core solenoid, this condition is satisfied if the length of each section is 0.8 times the coil radius r, so that a line with n sections would have a length

$$\ell = 0.8 \, nr. \tag{5}$$

Having chosen a coil radius and determined the length for the desired number of sections the total number of turns required can be calculated from

$$N = \frac{\sqrt{L(9r+10\ell)}}{r}, \tag{6}$$

where L is in μhy, and r and ℓ are in inches. To allow for end effects, the end sections should have 20% more turns than a section in the middle.

For optimum Q in the inductor, a wire should be chosen with a diameter at least 1/2 as great as the winding pitch. A convenient way to space the turns evenly is to cut shallow grooves (depth ~ 1/4 wire diameter) in a

plexiglass (or other dielectric) cylinder using a lathe. The winding pitch must then be one of the discrete values available on the lathe. Close-winding the turns, if adequately insulated with enamel or other low loss dielectric, is generally adequate. We note in this connection that the peak voltage between adjacent turns is

$$\frac{dV}{dN} = \frac{\omega T}{N} V_o,$$

where ω is frequency and V_o is the peak voltage of the wave propagating down the line. Since ω can never exceed the cutoff frequency of the line ($\omega_c = n/T$), the turn-to-turn voltage is never greater than

$$\frac{dV}{dN} = \frac{n}{N} V_o, \tag{7}$$

i.e., the peak voltage divided by the number of turns per section. If the coil is to be shielded, the shield should consist of a high conductivity non-permeable material and should have a diameter at least twice that of the inductor and a length longer than the inductor by at least one coil diameter. The coil form should be non-conducting to reduce eddy current losses and should consist of a dielectric that has a low loss at the highest frequencies for which the line is designed.

The capacitors chosen should have a voltage rating greater than the peak voltage across the line (V_o), and a capacitance of $C_n = C/n$. Electrolytic capacitors are useful at very low frequencies (< 10 kHz) if the voltage across the line never reverses and if stability and distortion of the waveform and high efficiencies are unimportant. Oil and paper capacitors

are better, and are generally satisfactory up to about 100 kHz. Ceramic capacitors are useful up to about 100 MHz, but if very high Q's are required, mica capacitors will in most cases produce losses much less than those in the associated inductor. For high precision work, the capacitances should be closely matched and installed in order of increasing or decreasing capacitance.

Assuming that the line has been designed so that resistive losses in the inductor dominate the Q, the loss can be determined by measuring the line resistance, R_L. For a line terminated in its characteristic impedance (Z), the voltage delivered to the load is

$$V = \frac{Z}{Z + R_L} V_o.$$

Mismatch at the load will increase the loss, but as a general rule, we want $R_L \ll Z$. At low frequencies, where the skin depth is greater than the wire diameter, the resistance can be determined with a dc ohmeter. At high frequencies, the currents flow on the surface of the wire, and a coil wound as prescribed here has an effective resistance of

$$R_L \simeq 10^{-5} \sqrt{f} \; r \, N^2/\ell, \tag{8}$$

where r and ℓ are the coil radius and length in inches. In practice, one determines R_L by both methods and uses the larger of the two values.

In conclusion, we summarize the steps required to design a delay line or pulse forming network.

1) Decide on a time delay T and impedance Z. Remember that a pulse forming network produces a pules of length 2T.

2) Calculate the required total inductance and capacitance using Eqs. (1) and (2).

3) Determine the number of sections from the desired cutoff frequency (Eq. (3)) or risetime (Eq. (4)).

4) Choose a radius and determine the optimum length for the inductor using Eq. (5).

5) Calculate the number of turns using Eq. (6).

6) Choose a wire diameter at least 1/2 as great as the winding pitch, and make sure the turn-to-turn voltage (Eq. (7)) is not excessive, and the total winding resistance (calculated from Eq. (8) at high frequencies) is much less than the line impedance. If these conditions are not satisfied, return to step 4) and choose a larger coil radius.

7) Choose n capacitors with capacitance C/n and adequate voltage rating of a type suitable for the frequency at which the line will operate.

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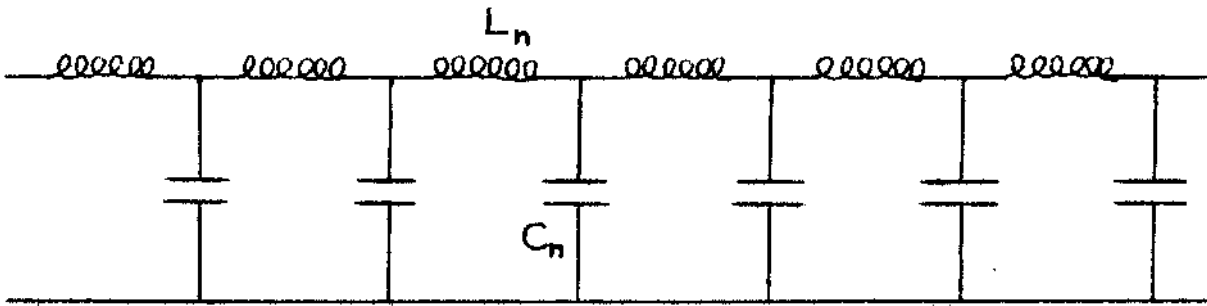
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(Revised January 1977)

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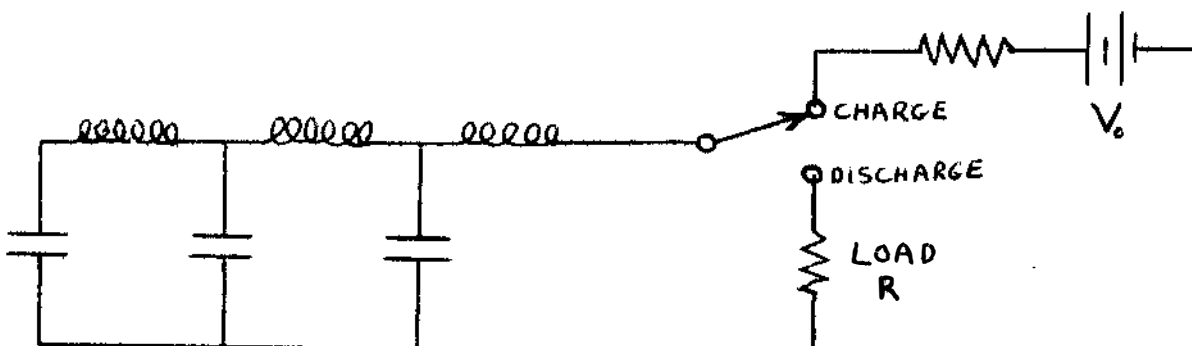$$Z = \sqrt{L/C},$$

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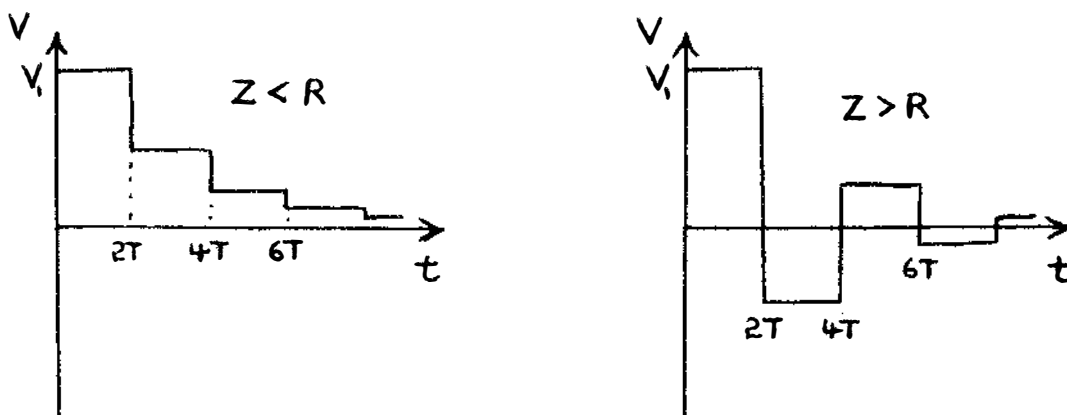
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For a "nonideal" line consisting of a finite number of LC networks, each network can be considered as a low pass filter with a cutoff frequency of

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This cutoff causes dispersion in a delay line, and causes a pulse forming network to produce a pulse with a rise time of

$$\tau \sim T/3n \tag{4}$$

and oscillations at a frequency $2\omega_c$. The peak-to-peak amplitude ΔV of these oscillations is given by $\Delta V \sim V/2n$. The dispersion is minimized by designing the line so that the mutual inductance between adjacent coils is $\sim 0.36\ L$ although the exact value is not very critical. For a single layer, continuously wound, air core solenoid, this condition is satisfied if the length of each section is 0.8 times the coil radius r, so that a line with n sections would have a length

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Having chosen a coil radius and determined the length for the desired number of sections the total number of turns required can be calculated from

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For optimum Q in the inductor, a wire should be chosen with a diameter at least 1/2 as great as the winding pitch. A convenient way to space the turns evenly is to cut shallow grooves (depth $\sim$ 1/4 wire diameter) in a

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Assuming that the line has been designed so that resistive losses in the inductor dominate the Q, the loss can be determined by measuring the line resistance, R_L. For a line of impedance Z terminated with a resistance R, the voltage delivered to the load is

$$V = \frac{V_o R}{R+Z+R_L t/2T} \quad .$$

For a 10% droop in the pulse with a matched (R=Z) load, we require $R_L \sim 0.2Z$. At low frequencies, where the skin depth is greater than the wire diameter, the resistance can be determined with a dc ohmeter. At high frequencies, the currents flow on the surface of the wire, and a coil wound as prescribed here has an effective resistance of

$$R_L \simeq 10^{-5} \sqrt{f} \ r \ N^2/\ell \text{ ohms}, \tag{8}$$

where r and ℓ are the coil radius and length in inches. In practice, one determines R_L by both methods and uses in the larger of the two values.

If a line designed in the above manner turns out to be impractically large, the individual sections can be wound with several layers. The formulas above should be sufficiently accurate as long as the winding thickness is much less than the coil radius. Note, however, that the insulation problems

are more severe because of the large voltage between layers and between adjacent sections of the line. The voltage standoff of the line can be improved considerably if the line is immersed in (bubble-free) transformer oil.

PLP 649 describes a computer code which has proved useful for calculating the behavior of pulse forming networks with up to 50 sections of variable R, L, and C. It has been used to check most of the results described above as well as to predict the behavior of lines which were subsequently constructed.

In conclusion, we summarize the steps required to design a delay line or pulse forming network.

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2) Calculate the required total inductance and capacitance using Eqs. (1) and (2).

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4) Choose a radius and determine the optimum length for the inductor using Eq. (5).

5) Calculate the number of turns using Eq. (6).

6) Choose a wire diameter at least 1/2 as great as the winding pitch, and make sure the turn-to-turn voltage (Eq. (7)) is not excessive, and the total winding resistance (calculated from Eq. (8) at high frequencies) is much less than the line impedance. If these conditions are not satisfied, return to step 4) and choose a larger coil radius, or wind the coil in multiple layers using a wire size that will give the desired resistance.

7) Choose n capacitors with capacitance C/n and adequate voltage rating of a type suitable for the frequency at which the line will operate.