A MICROWAVE CAVITY PERTURBATION TECHNIQUE FOR PLASMA DENSITY MEASUREMENT

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April 25, 1968

PLP 196

Plasma Studies
University of Wisconsin

General Theory

Numerous techniques have been devised whereby microwave radiation can be used to determine plasma densities.[1-4] All of these methods rely upon the fact that the dielectric constant of a plasma at microwave frequencies is a function of the electron density. In the absence of a magnetic field, the relative dielectric constant of a collisionless plasma is given by

$$K = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{ne^2}{\pi m f^2} \qquad (1)$$

where ω_p is the electron plasma frequency, $\omega(=2\pi f)$ is the microwave frequency, n is the electron   The method to be described is useful over a limited range of densities for plasmas confined in a closed conducting cavity. It consists of resonating the cavity in a very high order mode and counting the number of modes through which a constant frequency source is shifted when the cavity is filled with plasma.

The number of modes N in a cavity of volume V filled uniformly with an isotropic dielectric can be calculated in a manner similar to that used to derive the Rayleigh-Jeans law for blackbody radiation.[5] The result is

$$N = \frac{8\pi}{3}\left(\frac{\sqrt{K}\, f}{c}\right)^3 V. \qquad (2)$$

For $\omega >> \omega_p$, $K \approx 1$, and the number of modes/unit frequency can be found by differentiating equation (2):

$$\frac{dN}{df} = 8\pi \frac{f^2}{c^3} V. \qquad (3)$$

The average frequency between adjacent modes is then

$$\Delta f = \frac{c^3}{8\pi f^2 V} \quad . \tag{4}$$

The number of modes through which an initially empty cavity shifts when plasma is injected is found from equation (2):

$$\Delta N = \frac{8\pi}{3}\left(\frac{f}{c}\right)^3 V[1-K^{3/2}] \cong 4\pi \left(\frac{f}{c}\right)^3 V \frac{\omega_p^2}{\omega^2} = \frac{f}{\Delta f}\,\frac{\omega_p^2}{2\omega^2} \; . \tag{5}$$

The choice of operating frequencies is limited by the requirement that ω be much greater than the electron plasma frequency, the electron cyclotron frequency, and the electron collision frequency. In a large cavity, this condition, unfortunately, leads to a situation in which the average frequency between adjacent modes is less than the average width of a mode. In this case, equation (5) can still be used but Δf much be determined emperically rather than from equation (4).

Application to the Toroidal Octupole

The most serious limitation in the case of the toroidal octupole is that the microwave frequency should be much greater than the electron cyclotron frequency in order for equation (1) to apply. Since the magnetic field has a maximum value of about 7 kG, the frequency should be above 20 GHz. Using available K band klystrons (f=24 GHz) we therefore avoid electron cyclotron resonance and satisfy the requirement that $\omega \gg \omega_c$ over most of the volume of the cavity.

At 24 GHz, the mode number is about 10^6 and the measured Q of the cavity is about 10,000. The halfwidth of a mode is then typically

$$\delta f = f/Q = 2.4 \text{ MHz}.$$

For a linearly polarized source, only half of the modes are excitable and equation (4) predicts an average mode spacing of

$$\Delta f = \frac{c^3}{4\pi f^2 V} = .012 \text{ MHz/mode. (Theoretical)}$$

Clearly, most of the modes will be unobservable, and it is necessary to determine emperically the average spacing between adjacent modes. To do this, the klystron frequency was slowly tuned between two frequencies and the number of peaks counted. This method gave an average mode spacing of

$$\Delta f = 3.0 \text{ MHz/mode. (Emperical)}$$

From equation (5) this corresponds to a density of

$$n = \frac{2\pi m}{e^2} f\Delta f = 1.8 \times 10^9 \text{ cm}^{-3}\text{/mode.} \qquad (6)$$

Measurements of the gun injected plasma give a shift of about one mode in agreement with Langmuir probe data which show an average density of $\sim 1 \times 10^9$ cm^{-3}.

The mode counting technique is not useful in the octupole for densities at which the shift is less than one mode nor at densities where f_p is greater than about f/3. For 24 GHz, the useful density range is

$$2 \times 10^9 \text{ cm}^{-3} < n < 10^{12} \text{ cm}^{-3} .$$

Fortunately the plasma produced by cyclotron resonance breakdown at 9 GHz is well within this range.

Experimental Results

Figure 1 shows an oscilloscope trace of the klystron and toroid modes obtained by sweeping the klystron repeller with a sawtooth voltage. The klystron mode at 24.2 GHz is about 55 MHz wide. Also in fig. 1 is an oscilloscope trace of the modes obtained with a fixed microwave frequency in the presence of a microwave produced plasma along with a trace of ion saturation current obtained from a probe on the minor axis.

The density obtained by the mode counting technique is compared with the density obtained from the Langmuir probe in fig. 2. The density was varied by changing the power level of the 9 GHz microwaves which produced the plasma. The average density was obtained from probe measurements by determining the density profile in ψ space and using

$$\bar{n} = \frac{\int n(\psi)\, V'(\psi)\, d\psi}{\int V'(\psi)\, d\psi} \tag{7}$$

where $n(\psi)$ was determined from the ion saturation current I_{oi} by

$$I_{oi} = neA\sqrt{\frac{kT_e}{2\pi M}} \tag{8}$$

where A was taken as the geometric area of the probe (.236 cm^2). The electron temperature was determined from admittance probe measurements to be 2 eV. It was assumed that the density profile and electron temperature are independent of the microwave power level. The agreement between the two methods is probably better than could be expected especially since several recent papers[6-9] indicate that a considerable correction is necessary to conventional Langmuir probe theory. Microwaves apparently indicate a somewhat higher density than probes but the error never exceeds a factor of two.

The lifetime of the microwave produced plasma as indicated by the mode counting technique was determined by plotting the mode position from fig. 1 in fig. 3. A lifetime of 1 msec was obtained by this method.

After 4 msec the magnetic field decays quite fast and the density decay is correspondingly increased.

The microwave technique was checked by measuring the dielectric constant of air. The klystron frequency was swept and the mode pattern displayed on an oscilloscope. When the toroid was let up to air, all modes shifted without appreciable distortion by ~ 8.6 MHz. The dielectric constant was determined from

$$\frac{\Delta f}{f} = \frac{1}{2}(K - 1) \tag{9}$$

to be $K = 1.00072 \pm .00014$. This value is in reasonable agreement with the accepted dielectric constant of 1.00054 for dry, CO_2 free, air at microwave frequencies.

Summary

The mode counting technique can suffer from several difficulties:

1. The method is statistical, i.e., it assumes a constant frequency separation between modes. The error involved becomes proportionately less as the mode shift becomes greater.
2. The method is useful only when the average density increases monatonically with time and the decreases monatonically to zero. The maximum density can be determined by dividing the total number of modes observed by two.
3. Since < 1% of the modes are observable, one might suspect that these large modes represent special modes in which only a small fraction of the cavity volume is filled with electric fields. This problem can be reduced by separating the transmitter and detector as much as possible.
4. If the waves protrude significantly into the drift tank or diffusion pumps, the average density measured will be too

low. Moving a large metallic baffle around in the drift tank did not change the appearance of the modes but bubbling pump oil does cause a considerable fluctuation in the mode pattern.

5. Rapid plasma motion could produce additional modes especially when the mode shift is large.

6. The cavity may have degenerate modes which are removed by the plasma. In cases where the mode spacing is limited by the Q of the cavity this problem is probably not serious.

7. A plasma with a collision frequency $\nu \gtrsim f/Q$ would change the cavity Q and hence the apparent mode spacing.

Although the above difficulties warrant careful consideration, most are probably not serious for the plasmas studied in the octupole. The mode counting technique has two advantages over other microwave techniques which assure its usefulness in many different circumstances.

1. There is no necessity to stabilize the klystron frequency or amplitude.

2. Apart from an initial determination of the frequency spacing between modes, no calibration is necessary.

(NOTE ADDED IN PROOF: The mode counting technique has previously been used by T. J. Fessenden and L. D. Smullin at MIT and is described in the *Proceedings of the Seventh International Conference on Phenomena in Ionized Gases*, Vol. III (Beograd, 1966), with results in essential agreement with those presented here.)

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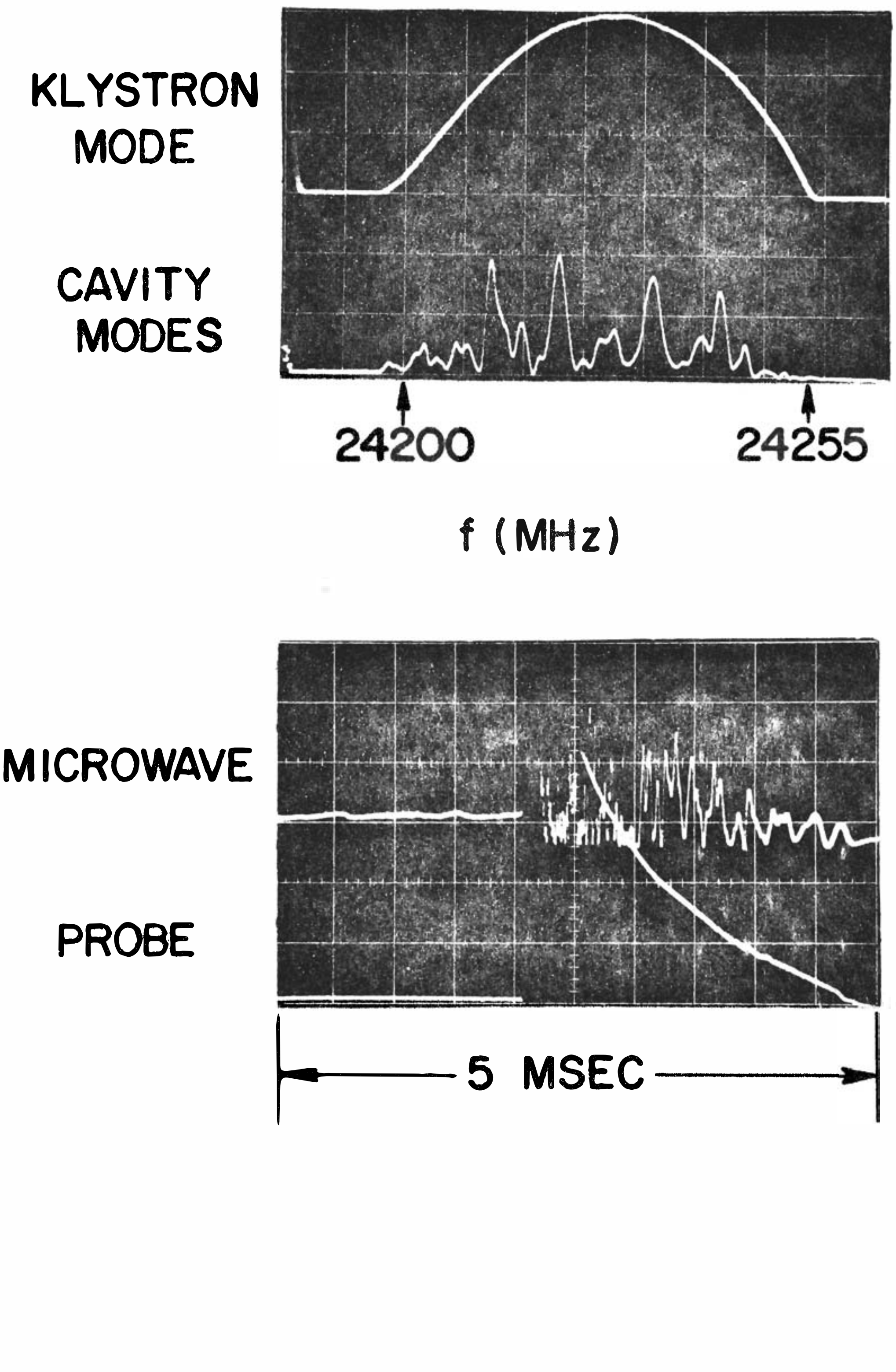
KLYSTRON
MODE
CAVITY
MODES
24200
24255
f (MHz)
MICROWAVE
PROBE
5 MSEC

FIGURE 1

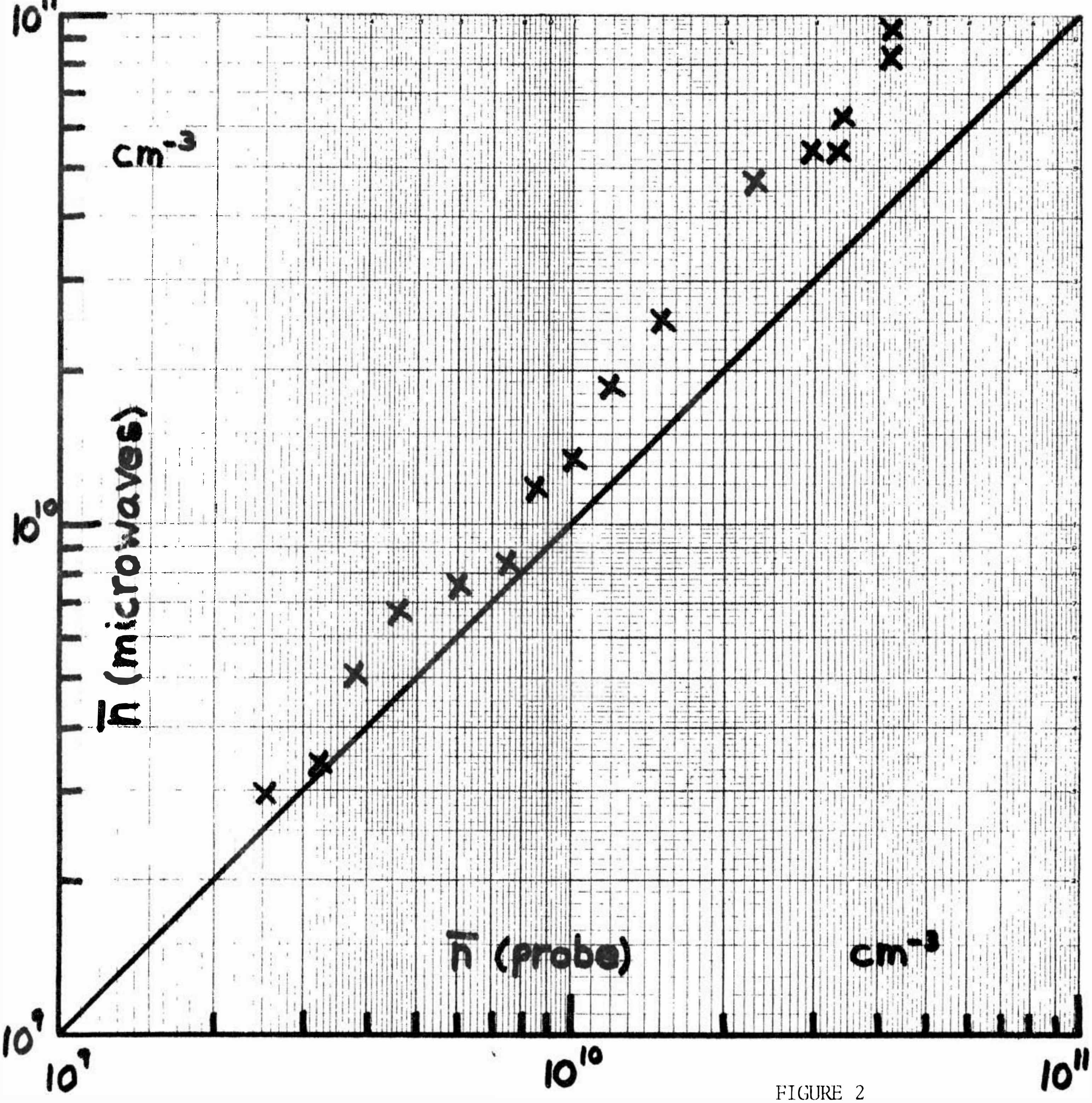

FIGURE 2

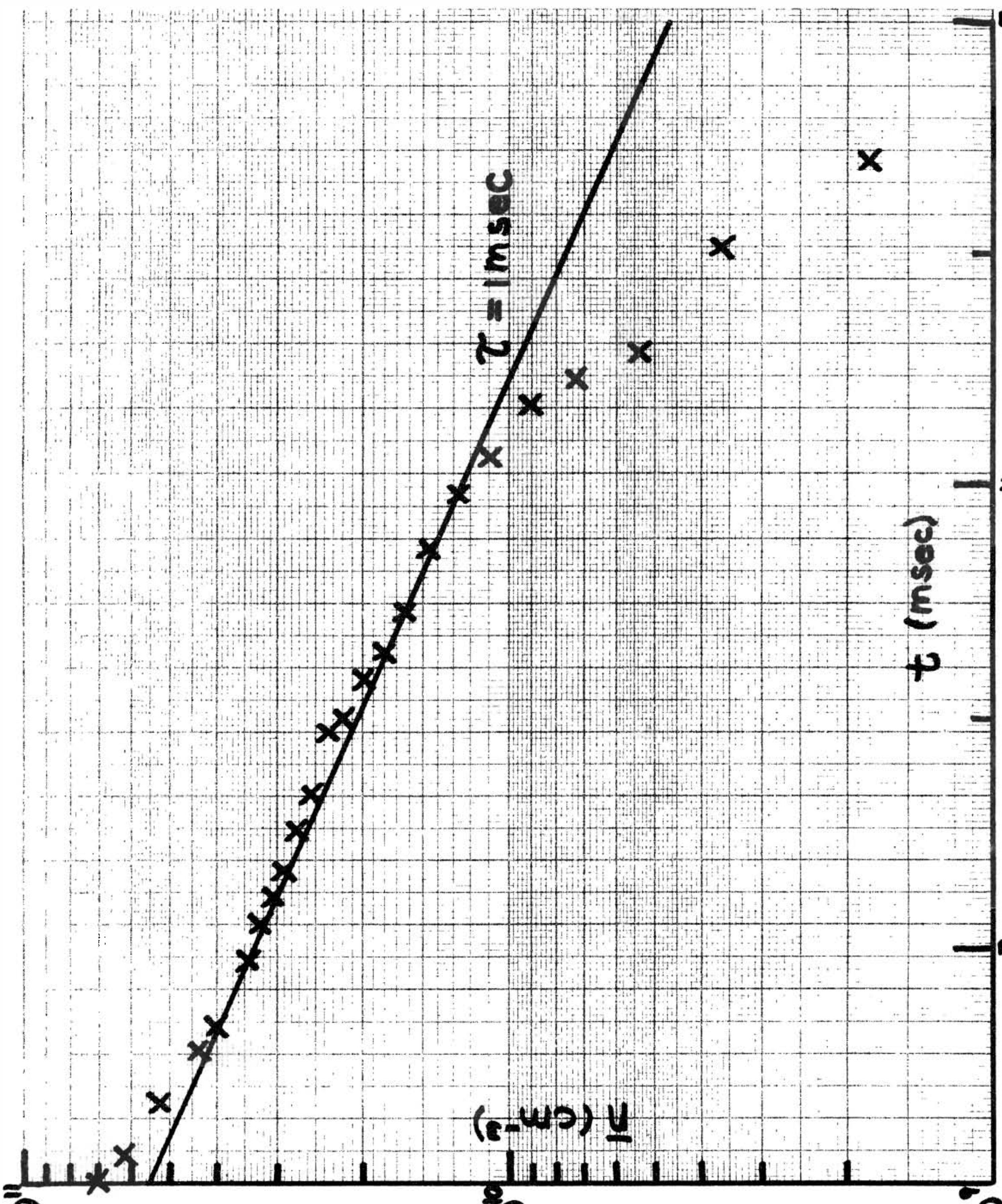

FIGURE 3