

*Schmidt*

CAVITY PERTURBATION AND DIPOLE RESONANCE

BY A PLASMA

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## CAVITY PERTURBATION AND DIPOLE RESONANCE BY A PLASMA

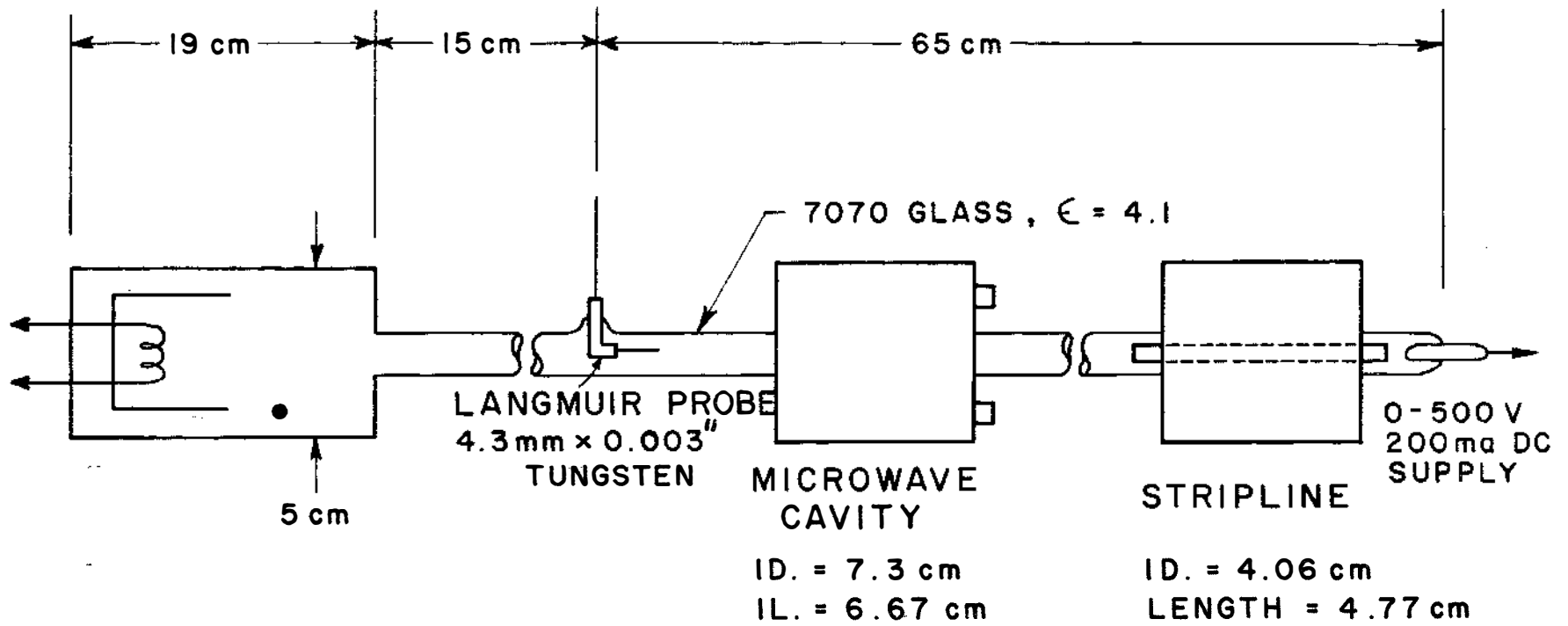
## I. Introduction

The Langmuir probe is a useful diagnostic tool because of its simplicity and its ability to make highly localized measurements in a plasma. It suffers from the disadvantage that the plasma is often disturbed by the physical presence of the probe. To overcome this difficulty, rf diagnostic techniques are useful whenever spatial resolution can be sacrificed. This experiment<sup>1</sup> introduces you to two such rf techniques. The cavity perturbation method depends upon the fact that the resonant modes of a microwave cavity are shifted in frequency when the cavity is partly filled with a plasma. The dipole resonance method involves the fact that a transmission line partially filled with plasma strongly absorbs waves of frequency near the plasma frequency. Both of these methods are used to determine the plasma density and the results are compared with that obtained from a Langmuir probe.

## II. Discharge tube

## A. Description

A mercury plasma is produced in a discharge tube 80 cm long with an inside diameter of  $.235 \pm .002$ " (5.97 mm) and outside diameter of  $.335 \pm .001$ " (8.50 mm). The glass is low loss type 7070 and has a dielectric constant of  $\epsilon = 4.1$ . A continuous stream of electrons is supplied by a National Electronics, Inc., NL-619 oxide coated cathode which requires 2.0 V, 12 A for operation and is capable of supplying a 6 amp beam of electrons. The anode is a hollow aluminum cup. The tube contains enough liquid mercury to insure that the mercury gas pressure is equal to the vapor pressure of mercury over the range of operating temperatures. A drawing of the tube and associated diagnostics is shown in Fig. 1.



DISCHARGE TUBE

Fig. 1

## B. Theory

In the absence of plasma, the electric field around the positive anode falls off as  $1/r^2$  and hence is too weak to pull electrons away from the cathode. Consequently to start the discharge it is necessary to produce a large electric field by means of a tesla coil which produces a plasma in the column. The conducting plasma then looks like a resistance with a constant voltage drop/unit length and the electric field at the cathode is just the anode voltage divided by the tube length. This electric field is sufficiently strong to accelerate electrons away from the cathode in the direction of the anode. When the electrons acquire an energy of  $> 10.6$  eV, they reach the threshold for ionization of mercury. Soon thereafter, the electrons create electron-ion pairs and loose 10.6 eV of energy. The new electrons undergo the same process, leading to an avalanche of ionization.

When the tube is operating, the voltage drop along the column is constant independent of discharge current. Consequently the electron temperature is nearly constant and the density varies approximately linearly with discharge current (see Section VI, D) .

The ions are almost entirely singly ionized mercury atoms. Because of their large mass ( $200.6 M_p$ ), they gain relatively little energy from the electric field and tend to be in thermal equilibrium with the walls of the tube.

## C. Operation

To start the discharge, turn on the filament and allow the tube to warm up for several minutes. The cathode reaches operating temperature in about 1 minute but it is necessary to heat the mercury to raise its vapor pressure. The vapor pressure of mercury is about  $10^{-3}$  torr (1 micron) at  $18^\circ\text{C}$  corresponding to a neutral density of  $4 \times 10^{13} \text{ cm}^{-3}$  . The vapor pressure increases by a factor of 2 for each  $8^\circ\text{C}$  temperature rise in this region. After five minutes turn on the anode power supply and set the voltage to 350 volts. Activate the tesla

coil to start the discharge. The anode current should rise to  $\sim 150$  ma. Reduce the voltage until the discharge current is  $\sim 100$  ma and allow it to run for several minutes in order to reach equilibrium. The discharge can be run with a current of  $\sim 20 - 200$  ma giving an order of magnitude variation in density. When changing the discharge current, allow a minute or two between each change for the tube to reach a new equilibrium temperature.

In all cases there may be slow thermal drifts which change the neutral pressure and, hence, the electron density. Mercury deposits on the tube walls may cause some trouble. These can be removed from the area of interest by preliminary use of a warm air flow. The mercury will then deposit at a colder point of the tube. It is desirable to perform the experiments close together in time. If they are performed on separate occasions then each rf method should be checked against the Langmuir probe.

### III. Langmuir probe<sup>2,3</sup>

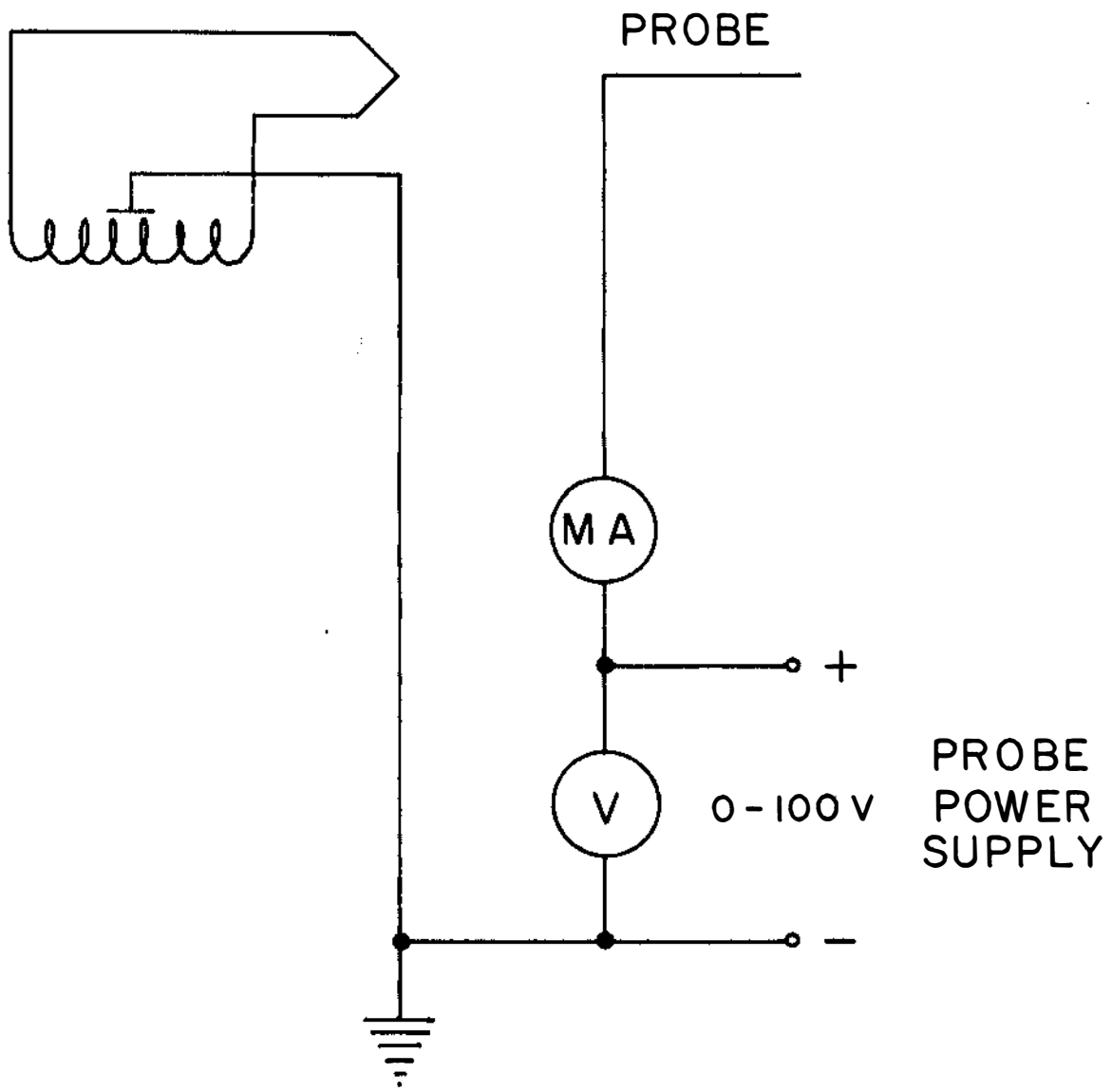
#### A. Description

A tungsten electrode is sealed into the tube to serve as a Langmuir probe. The electrode is 4.3 mm long and has a diameter of  $\sim 0.076$  mm. The probe is aligned along the axis of the tube so that it measures temperature and density on the tube axis. Since the rf techniques measure the average density it is necessary to assume something about the radial density profile. There is some theoretical justification<sup>4</sup> for the approximate description of the electron density profile by

$$n \approx n_0 [1 - 0.6(r/a)^2] \quad (1)$$

where  $n_0$  is the axial density and  $a$  is the plasma radius. The average of  $n$  is then about  $0.7 n_0$ .

A diagram of the Langmuir probe circuit is shown in Fig. 2.

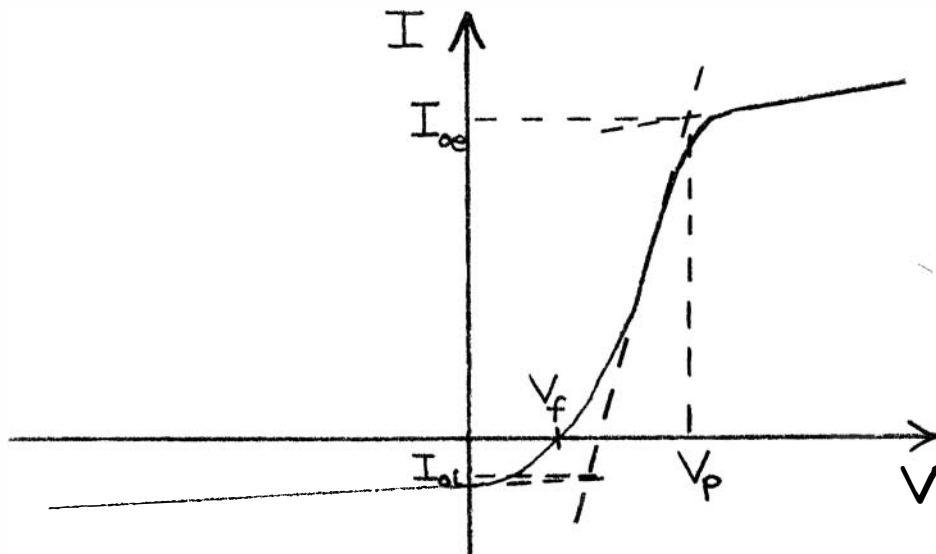


LANGMUIR PROBE CIRCUIT

Fig. 2

## B. Theory

A typical  $V - I$  characteristic for a probe in a plasma is shown below:



For  $V$  large negative, the probe collects all of the ions which strike the Debye sheath which surrounds the probe but none of the electrons since they are repelled by the large negative potential. In this region the probe draws saturated ion current,  $I_{oi}$ . As the probe potential is increased the high energy tail of the electron distribution is collected. At  $V = V_f$  the flux of electrons is equal to the ion flux and this potential is called the floating potential since it is the potential to which an isolated conductor in the plasma will come. As the probe potential is further increased, the flux of electrons increases sharply. When  $V = V_p$ , where  $V_p$  is the potential of the plasma, all particles which strike the sheath are collected. The current at this potential is highly positive since the electron flux is much greater than the ion flux because of the much larger velocity of the electrons. A further increase in potential does not appreciably increase the current above  $I_{oe}$ .



If we assume thermal equilibrium, the current in the transition region ( $V_f < V < V_p$ ) is given by

$$I = -I_{o_i} + I_{o_e} \exp\left[\frac{e(V-V_p)}{kT_e}\right] \quad (2)$$

where  $T_e$  is the electron temperature. Taking the log of both sides gives

$$\ln(I + I_{o_i}) = \ln I_{o_e} + \frac{e(V-V_p)}{kT_e} \quad (3)$$

Therefore by plotting  $\ln(I + I_{o_i})$  vs.  $V$ , one should obtain a straight line whose slope is  $\frac{e}{kT_e}$ . This method allows one to determine the electron temperature.

The plasma density can be determined from either the electron or ion saturation current. However, since  $I_{o_i} \ll I_{o_e}$ , the plasma is disturbed less if  $I_{o_i}$  is used. The ion current due to thermal motion is given by

$$I_{o_i} = \frac{1}{4} n e \bar{v}_i A \quad (4)$$

where  $A$  is the collecting area and  $\bar{v}_i$  is the average ion thermal velocity. The  $\frac{1}{4}$  is a geometrical factor.

For a Maxwellian ion velocity distribution with temperature  $T_i$ , we expect the average velocity to be

$$\bar{v}_i = \sqrt{\frac{8kT_i}{\pi M}} \quad (5)$$

where  $M$  is the ion mass. This is indeed the case provided  $T_i \geq T_e$ .

For  $T_i < T_e$  as is the case in this experiment, it can be shown that a stable sheath cannot exist unless the ions are collected with a velocity greater than that given by Equation (5). In this case, the ions flow into the probe as if they had a temperature equal to  $T_e$ . With this substitution, Equation (4)

becomes

$$I_{o_i} = n e A \sqrt{\frac{kT_e}{2\pi M}} \quad (6)$$

where  $M$  is the ion mass. Hence knowing the electron temperature and ion saturation current allows one to calculate the plasma density.

Alternately, the density can also be obtained from

$$I_{o_e} = n e A \sqrt{\frac{kT_e}{2\pi m}} \quad (7)$$

where  $m$  is the electron mass.

C. Experimental procedure.

1. Plot the  $V$  vs.  $I$  characteristic of the Langmuir probe at several values of discharge current. Use Equation (3) to calculate the electron temperature.

CAUTION: DO NOT LET THE PROBE CURRENT EXCEED 20 MA AS THIS WILL VAPORIZE THE PROBE.

2. Plot the plasma density vs. discharge current using Equation (6). Be sure to wait a few minutes between each change of discharge current so that the temperature of the glass can reach equilibrium.
3. Compare the density obtained from Equation (6) with that obtained from the electron saturation current using Equation (7). This should be done at a low discharge current since it may not be possible to reach electron saturation without overheating the probe. If there is a discrepancy, try to suggest some reasons for it.

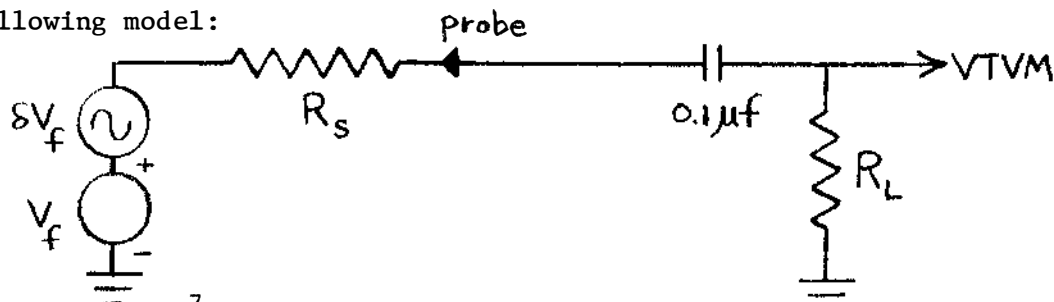
D. Optional experiments and questions.

1. Calculate the Debye length for the plasma. How does it compare with the probe radius? Since the probe actually collects particles which strike the sheath, and since the sheath is the order of a Debye length thick, the collecting area is larger than the probe area. You might wish to make a correction for this fact when calculating the density.
2. From Equation (2), show that

$$V_p - V_f = \frac{kT}{2e} \ln(M/m) \quad (8)$$

Compare this value with what is actually observed. Note that  $V_f$  can be measured in either of two ways: 1) by adjusting the probe voltage until the current drawn is zero, or 2) directly with a voltmeter. Do they give the same answer? Under what conditions would each method be preferred?

3. If an oscilloscope is available, use it to look at fluctuations in floating potential and ion saturation current with the Langmuir probe. At what frequencies do the fluctuations occur? Evaluate  $e\delta V_f/kT_e$  and  $\delta n/n$ . Would you expect the d.c. floating potential read by the probe to be different from the floating potential in the absence of fluctuations? Calculate the magnitude of this effect from Eq.(2). (See Ref. 6 if necessary.)
4. Using an oscilloscope or a VTVM with a sensitivity of a few mV and a frequency response of a few hundred kHz (such as a Hewlett Packard HP 400 VTVM), test the effect of various load resistors on the magnitude of the a.c. component of  $V_f$ . How low can the load resistance be and still read  $\delta V_f$  accurately? Assume the plasma has a source resistance  $R_s$ . Determine  $R_s$  using the following model:



By calculating<sup>7</sup> the inverse slope of the V-I characteristic at  $V = V_f$  from Eq.(2), show that

$$R_s = kT_e/eI_{o_i} \quad (9)$$

Compare the calculated and observed slope of the curve at this point.

5. If an oscilloscope is available, try to display the  $V - I$  characteristic directly on the oscilloscope screen by sweeping the probe voltage at some low frequency with the sawtooth output of the oscilloscope. Be careful not to exceed 20 ma probe current unless you do so only for a brief period by using the scope on single sweep. Watch the probe for any sign of a red glow. You will need either a differential amplifier or pulse transformer in order to measure current to the probe. You may see oscillations develop when electron saturation current is drawn. This is a common occurrence with probes. The reason, however, is not well understood.

#### IV. Microwave cavity<sup>8,9,10</sup>

##### A. Description

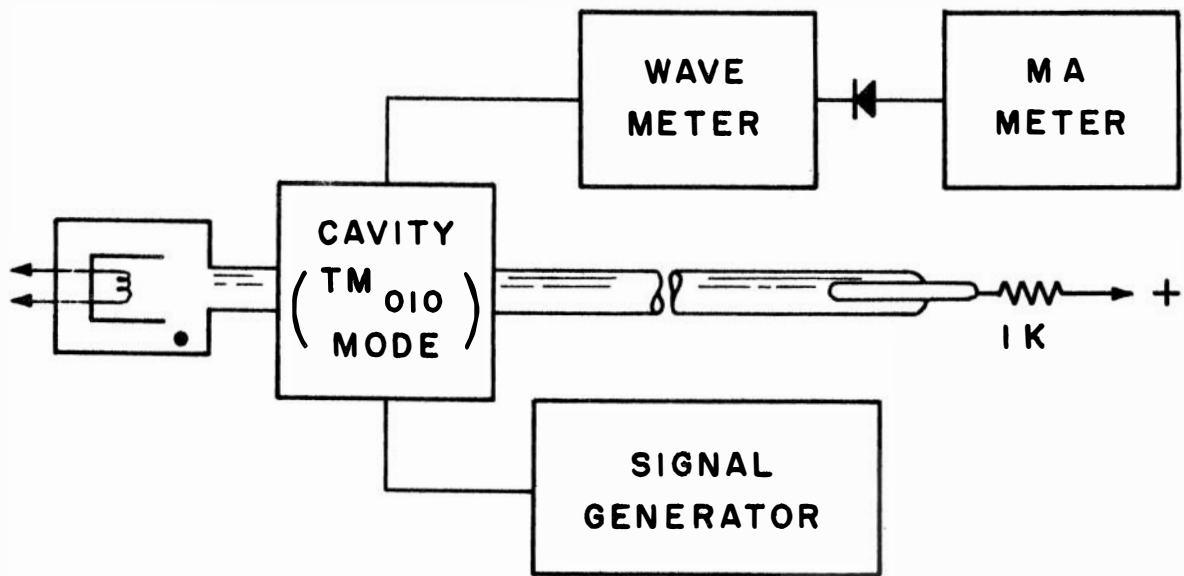
The microwave cavity is a hollow copper cylinder 3.65 cm in radius and 6.67 cm in length. The discharge tube passes along the cylinder axis through holes in each end. The cavity is excited by a loop of wire at one end oriented so as to excite the  $TM$  modes. A second loop oriented in the same direction is used to detect the amplitude of the oscillation in the cavity. The lowest excitable mode is the  $TM_{010}$  which has an axial electric field which is uniform and maximum on the axis. The magnetic field is purely circumferential. In the absence of plasma, the cavity with the glass tube inserted resonates at about 2830 MHz with a  $Q$  of several hundred.

The circuit used in the cavity perturbation experiment is shown in Fig. 3.

##### B. Theory

For plasma frequencies well below the working frequency, the plasma constitutes a small perturbation of the cavity mode. In this case, the wave equation can be solved with the appropriate boundary conditions to give the frequency shift  $\Delta\omega$  of the  $TM_{010}$  mode. More simply, Slater's perturbation theorem<sup>11</sup> can be used:

$$\Delta\omega = \int_V \frac{j}{P} \sigma E^2 dV / 4U \quad (10)$$



CAVITY PERTURBATION CIRCUIT

Fig. 3

where  $U$  is the average stored energy in the mode,  $E$  is the unperturbed electric field and  $\sigma$  is the conductivity of the plasma:

$$\sigma = \epsilon_0 \omega_p^2 / j\omega_0 \quad (11)$$

Substitution leads to

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{2} \frac{\omega_p^2}{\omega_0^2} \frac{\int_V E^2 dV}{\int_V E^2 dV} \quad (12)$$

where  $\omega_p$  is the electron plasma frequency:

$$\omega_p = \sqrt{\frac{ne^2}{\epsilon_0 m}} \quad (13)$$

and the integrations are taken over the plasma volume,  $V_p$ , and the cavity volume,  $V_c$ , respectively. For the  $TM_{010}$  mode, the electric field is entirely axial and is given by<sup>12</sup>

$$E = E_0 J_0 \left( \frac{2.405 r}{R} \right) e^{-i\omega t} \quad (14)$$

where  $J_0$  is the zero order Bessel function and  $R$  is the cavity radius.

Substituting Eq.(14) into Eq.(12) and performing the integrations gives:

$$\frac{\Delta\omega}{\omega_0} = 1.85 C \frac{\omega_p^2}{\omega_0^2} \quad (15)$$

$$\text{where } C = \alpha^2 [ J_0^2(2.405 \alpha) + J_1^2(2.405 \alpha) ] \cong \alpha^2 \quad (16)$$

and  $\alpha = \frac{d_p}{d_c}$  where  $d_p$  and  $d_c$  are the diameters of the plasma and cavity respectively. Eq.(15) allows one to calculate  $\omega_p$  from the observed frequency shift  $\Delta\omega$ , and the density can then be determined from Eq. (13).

### C. Experimental procedure

1. Connect the rf equipment to the cavity taking care with the

crystal detector to see that the opposite ends of all cables are grounded before connecting them to the detector. Four feet of coaxial cable charged to 14 volts by static electricity store enough energy to destroy the crystal diode. Use the lowest rf signal level which will give a satisfactory deflection on the most sensitive current scale. Locate the  $TM_{010}$  mode with the discharge off. Be careful not to use one of the many false resonances. Plot the amplitude vs. frequency using the wavemeter to measure the frequency. At resonance, power is absorbed by the wavemeter cavity, producing a dip in the coaxial line power. Determine the  $Q$  of the unloaded cavity.

2. Turn on the discharge and measure the frequency shift of the mode vs. discharge current. Remember to wait a few minutes between each current change for the tube walls to reach thermal equilibrium. It would be wise to measure ion saturation current at the same time to facilitate comparison with the probe results. Calculate the density from Eq.(15) and compare with the average density obtained from the Langmuir probe assuming a density profile as in Eq.(1).

#### D. Optional experiments and questions

1. Calculate the resonant frequency of the  $TM_{010}$  mode for the hollow cavity<sup>12</sup> using

$$\omega_{010} = \frac{2.405 C}{R} \quad (17)$$

Estimate the shift in frequency when the glass tube is inserted into the cavity using the perturbation method in Section B. Is the frequency shifted up or down? How does the observed frequency



compare with this calculated value? List some causes for any discrepancy.

2. Measure the  $Q$  of the resonance when the cavity is loaded with plasma. Try to estimate the electron neutral collision frequency  $\nu$  by adding to the conductivity in Eq.(11) a term to include collisions:

$$\frac{1}{\sigma} = \frac{j\omega_0}{\epsilon_0 \omega^2} + \frac{\nu}{\epsilon_0 \omega^2} \quad (18)$$

From this conductivity show that the change in  $\frac{1}{Q}$  is given by

$$\Delta \frac{1}{Q} = 3.70 C \frac{\nu/\omega_0}{1+(\nu/\omega_0)^2} \frac{\omega^2}{\omega_0^2} \quad (19)$$

where  $C$  is given in Eq.(16). Since it is difficult to obtain satisfactory results in this pressure and frequency range, do not take your result too seriously.

3. If an oscilloscope is available, display the cavity resonance by sweeping the oscillator frequency. The internal sweep only covers a 3 MHz interval, but by connecting the sawtooth output of the scope to the external fm input (sensitivity  $\sim 0.2$  MHz per volt) of the signal generator, the entire resonance can be displayed. Alternately, unground the discharge tube cathode and connect it to an ac voltage source such as the 6.3 VAC output of the power supply. By feeding the same ac signal into the horizontal input of the scope and using the signal generator on cw, the resonance can be observed by modulating the plasma density. Can you see any evidence of high frequency density fluctuations?

4. In many experiments, plasmas are contained in metal vacuum systems which are large compared to the wavelength of the microwaves. In this case, the cavity can be resonated in some very high mode and the frequency shift of the mode observed when the plasma is injected. Show that if the electric field in the cavity is random, the average plasma density in the cavity can be calculated from

$$\frac{\Delta\omega}{\omega_0} = \frac{\bar{n} e^2}{2\epsilon_0 m \omega_0^2} \quad (20)$$

## V. Stripline<sup>13, 14, 15</sup>

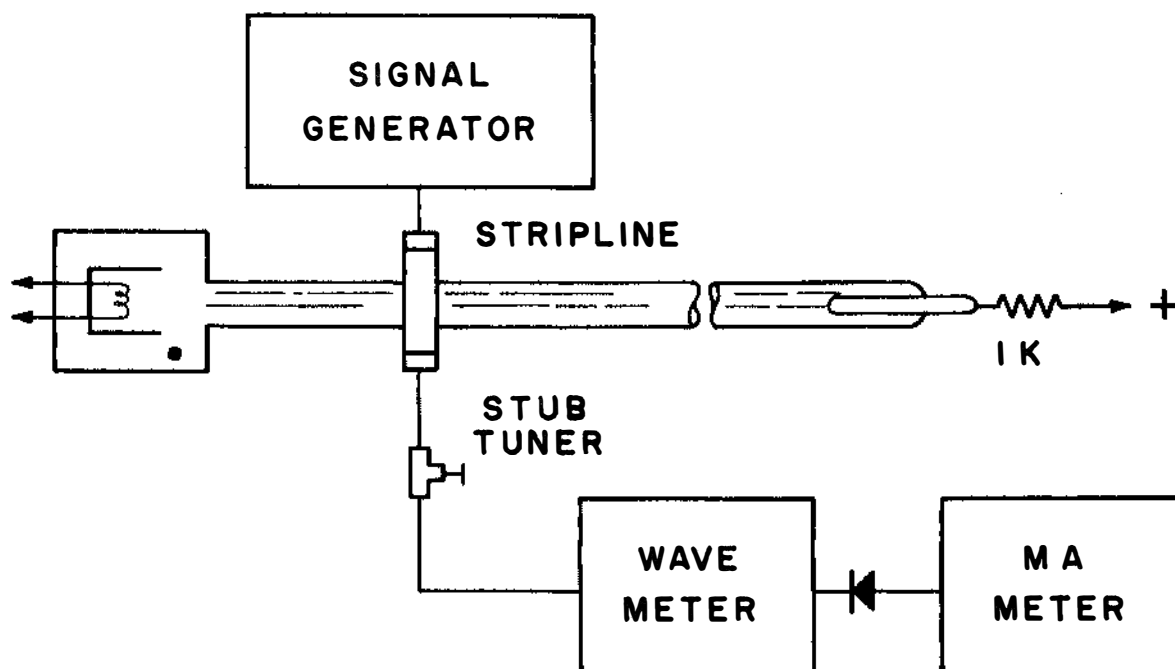
### A. Description

The stripline is a section of a two-conductor transmission line with the plasma column inserted between the conductors. The conductors are made out of copper sheet 4.77 cm wide and are bowed outward to form a cylinder with an inside diameter of 4.06 cm. The plasma column is placed on the axis so that the electric field across the plasma is transverse and essentially uniform. RF power is fed into one end of the stripline, propagated across the plasma, and detected on the other side by the same crystal detector used for the cavity perturbation measurements. A stub tuner is also used as a transformer to match impedances over the wide range of frequencies required and to decouple the signal source from the detector. The circuit used in the stripline method is shown in Fig. 4.

### B. Theory

The electric field  $\vec{E}_i$  inside a dielectric rod in a uniform transverse electric field  $\vec{E}_0$  can be found by solving Laplace's equation in cylindrical coordinates with appropriate boundary conditions<sup>16</sup>. The result is

$$\vec{E}_i = \frac{2 \vec{E}_0}{1+\epsilon} \quad (21)$$



STRIPLINE CIRCUIT

Fig. 4

where  $\epsilon$  is the relative dielectric constant of the rod. If the rod is a plasma,

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} \quad (22)$$

and Eq.(21) becomes

$$\vec{E}_1 = \frac{\vec{E}_0}{1 - \frac{\omega_p^2}{2\omega^2}} \quad (23)$$

Since the power absorbed by the plasma is proportional to  $\sigma E_1^2$ , we expect resonance absorption when  $\omega = \omega_0 = \frac{\omega_p}{\sqrt{2}}$ .

In the present experiment, the situation is somewhat complicated by the presence of the glass tube. The problem can be solved in the same way as before with the result that resonance is expected at

$$\omega_0^2 = \omega_p^2 / k \quad (24)$$

where 
$$k = 1 + \epsilon_g \left[ 1 - \frac{(1-K)}{(1+K)} \left(\frac{a}{b}\right)^2 / 1 + \frac{(1-K)}{(1+K)} \left(\frac{a}{b}\right)^2 \right] \quad (25)$$

and 
$$K = \frac{1}{\epsilon_g} \left[ 1 + \left(\frac{b}{c}\right)^2 / 1 - \left(\frac{b}{c}\right)^2 \right] \quad (26)$$

$\epsilon_g$  is the relative dielectric constant of the glass and  $a$ ,  $b$ , and  $c$  are the radii (or diameters) of the plasma, the outside of the glass tube, and the inside of the stripline, respectively. After  $k$  has been determined from the geometry, the density is easily found by measuring  $\omega_0$  and using Eq.(13) to get  $n$  from  $\omega_p$ .

The  $Q$  of the resonance can be used to calculate the electron neutral collision frequency. Considering the plasma to be an ensemble of harmonic oscillators, the power loss per cycle divided by the energy stored is just equal to the fraction of particles which undergo a collision during a period of oscillation. Hence from the definition of  $Q$  we obtain the result that

$$Q = \frac{\omega}{\nu} . \quad (27)$$

### C. Experimental procedure

1. Connect the signal generator to the stripline input and the detector to the stripline output with a stub tuner in series. Again, be careful to avoid connecting unterminated cables to the detector. The precision of this part of the experiment probably does not warrant the use of the wavemeter. Set the signal generator to 1.7 GHz and adjust the stub tuner for maximum output while reducing the rf output until a suitable reading is obtained on the 0.2 ma scale. Start the discharge and adjust the current until a resonance dip is observed. Increase the frequency and readjust the stub tuner in order to plot  $\omega_0$  vs. discharge current. Use Eq.(24) to calculate the density and compare with the Langmuir probe and microwave cavity data.
2. Set the signal generator at some constant frequency and plot the amplitude of the detected signal vs. discharge current. Calculate the  $Q$  of the resonance and use it to obtain the electron neutral collision frequency from Eq.(27). Note that the  $Q$  obtained in this way is one half the actual  $Q$  since

$$\frac{\Delta I}{I} \cong \frac{\Delta n}{n} = \frac{2\omega\Delta\omega}{\omega^2} = 2\frac{\Delta\omega}{\omega} . \quad (28)$$

### D. Optional experiments and questions

1. Derive Eq.(21) from Laplace's equation<sup>16</sup>. Show that the internal field is unidirectional. If you feel ambitious, solve the more complicated case in Eq.(24). Is  $\omega_0$  a strong function of  $a$ ,  $b$ , and  $c$ ? Show from Eq.(25) that for  $a = b \ll c$ , that  $k = 2$ . Show also that for  $a \neq b$ ,  $a, b \ll c$ ,  $\epsilon_g = 1$ , that  $k = 2$ .

2. If an oscilloscope is available, display the stripline resonance by applying a 60 cycle voltage between the cathode and ground and to the horizontal amplifier of the scope. You should see a horizontal broadening of the peak caused by density fluctuations. How does the presence of these fluctuations affect your previous measurement of the  $Q$  of the resonance?
3. It is possible to excite nondegenerate, higher order resonances<sup>17</sup> with suitable electrode systems. These can be observed by moving the tube off center in the stripline system. How are these related to the fundamental mode?
4. Suggest a method whereby you could determine the electron temperature from the  $Q$  measurement. What quantities would you need to know? This same method is used in VI.D. to obtain the neutral density from the electron temperature.

#### VI. Optional experiments and general questions

- A. Estimate theoretically the electron temperature by considering the equation of motion of an electron in a uniform electric field in the presence of ionizing collisions. Is your predicted distribution Maxwellian? A non-Maxwellian distribution should become Maxwellian in a time the order of the electron-electron collision time which is given by Spitzer<sup>18</sup> as

$$\tau_{ee} \text{ (sec)} = 0.266 \frac{T^{3/2} (\text{°K})}{n(\text{cm}^{-3}) \ln \Lambda} \quad (29)$$

where  $\ln \Lambda \approx 10$  for this plasma. How far would a 3 eV electron go in this time? Would you expect the distribution to be Maxwellian? The fact that a stream of electrons in a plasma becomes Maxwellian in a time short compared to the electron-electron collision time was first observed by Langmuir and is known as the "Langmuir paradox". Its explanation



## FIGURES

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## TABLE OF DIMENSIONS

## DISCHARGE TUBE

Length	80	cm
Inside diameter	0.597	cm
Outside diameter	0.850	cm
Dielectric const of glass	4.1	

## LANGMUIR PROBE

Length	0.43	cm
Diameter	0.0076	cm
Area	$1.03 \times 10^{-2}$	cm <sup>2</sup>

## MICROWAVE CAVITY

Length	6.67	cm
Inside diameter	7.3	cm

## STRIPLINE

Length	4.77	cm
Inside diameter	4.06	cm

## MERCURY

Mass	200.6	AMU
Ionization potential	10.6	eV
Vapor pressure	$10^{-3}$	torr @ 18° C