Phase Shift of Capacitance Probes

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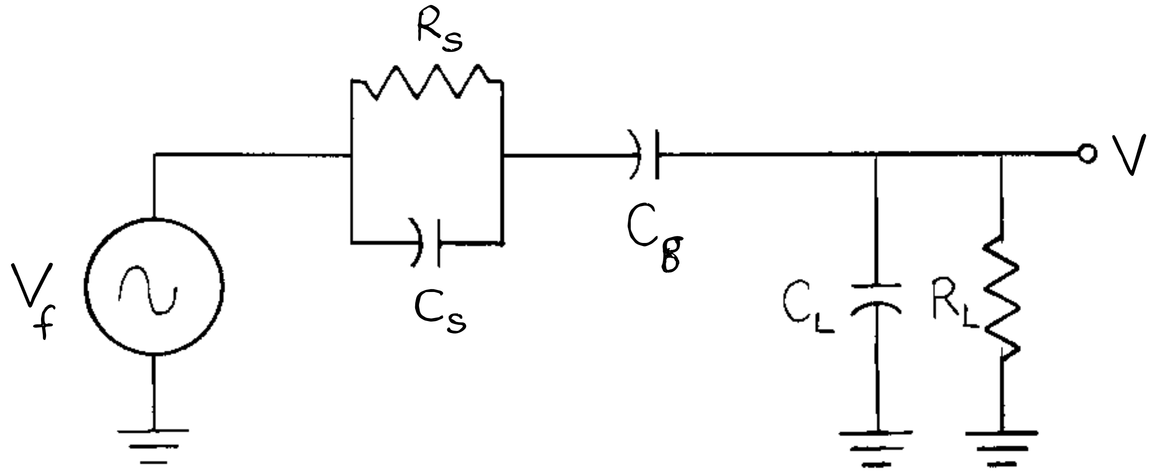
The capacitance probe was developed by J.A. Schmidt and described in PLP 159. In most respects it is superior to all other floating probes. This note contains a calculation of the phase shift which results when a capacitance probe is used to measure floating potential fluctuations. A knowledge of this phase shift is essential when making correlation measurements.

Since the glass enclosing the probe tip comes to the floating potential, it is surrounded by a plasma sheath identical to that around the usual conducting floating probe. This sheath has a resistance and capacitance associated with it which can be calculated in the usual manner[1,2].

The electrical equivalent circuit of the capacitance probe is as follows:



R_s and C_s are the sheath resistance and capacitance respectively. C_g is the capacitance between the outside of the glass and the probe tip. R_L and C_L are the load resistance and capacitance respectively including the capacitance of the probe cable. We will assume that the floating potential V_f is sinusoidal since an arbitrary floating potential can be fourier decomposed into a sum of sine waves.

The output voltage V is related to the floating potential by

$$V = \frac{\dfrac{R_L/j\omega C_L}{R_L + 1/j\omega C_L} V_f}{\dfrac{R_L/j\omega C_L}{R_L + 1/j\omega C_L} + \dfrac{1}{j\omega C_g} + \dfrac{R_s/j\omega C_s}{R_s + 1/j\omega C_s}}$$

and the phase shift is given by

$$\delta = \tan^{-1}\left(\frac{\mathrm{Im}\ V/V_f}{\mathrm{Re}\ V/V_f}\right) \tag{1}$$

or

$$\tan\delta = \frac{\omega^2 R_s[R_s C_s(C_s + C_g) - R_L C_g C_L] + 1}{\omega^3 R_L R_s^2 C_s[C_s(C_g + C_L) + C_g C_L] + \omega[R_s C_g + R_L(C_g + C_L)]} \tag{2}$$

This expression can be greatly simplified by making the reasonable approximations:

$$R_L >> R_s, \qquad C_L >> C_s, C_g$$

In this case,

$$\boxed{\tan\delta = \frac{1 - \omega^2 R_s R_L C_g C_L}{\omega R_L C_L + \omega^3 R_L R_s^2 C_L C_s (C_s + C_g)}} \tag{3}$$

Note that the phase shift is zero at only two values of ω:

$$\omega = \infty$$

and

$$\omega_o = \sqrt{\frac{1}{R_s R_L C_g C_L}}. \tag{4}$$

Also, note that $\delta = 90°$ at $\omega = 0$. From the form of equation (3), it is clear that the phase shift must have a maximum at some frequency ω_m between ω_o and ∞. Assuming that $\omega_m >> \omega_o$, equation (3) becomes

$$\tan\delta = -\frac{\omega R_s C_g}{1 + \omega^2 R_s^{\,2} C_s (C_s + C_g)}. \tag{5}$$

The value of ω_m can be determined from

$$\frac{d}{d\omega}\tan\delta\Big|_{\omega_m} = 0,$$

or

$$\omega_m = \frac{1}{\sqrt{R_s{}^2 C_s (C_s + C_g)}} \tag{6}$$

The assumption that $\omega_m >> \omega_o$ is indeed justified so long as $R_L >> R_s$ and $C_L >> C_s, C_g$. The magnitude of the max phase shift δ_m can be calculated by substituting ω_m into equation (3):

$$\tan \delta_m = - \frac{1}{3} \sqrt{\frac{2C_g{}^2}{C_s (C_s + C_g)}} \tag{7}$$

The phase shift is negative and is minimized by making $C_g << C_s$. Since

$$C_s \sim \frac{\varepsilon_o A}{\lambda_D}$$

where A is the probe area and λ_D is the Debye length, this condition is hard to satisfy in low density plasmas where λ_D is large. For $C_g = C_s$, δ_m is -21°.

To solve equation (3) in detail, it is convenient to assume that $R_s C_s$ is many orders of magnitude smaller than $R_L C_L$ and that C_S and C_g are of the same order. These conditions are nearly always well satisfied. These restrictions allow equation (3) to be solved independently in the range

$$\omega \sim \frac{1}{R_L C_L} \ll \omega_o$$

and

$$\omega \sim \frac{1}{R_s C_s} \sim \frac{1}{R_s C_g} \gg \omega_o$$

For the low frequency case, equation (3) takes the form

$$\tan\delta = \frac{1}{\omega R_L C_L}$$

which is plotted in the graph at the end of this paper.

In the high frequency case, it is convenient to define

$$\alpha \equiv \frac{C_g}{C_s}.$$

Furthermore, it can be shown[3] that $R_s C_s \approx \frac{1}{\omega_{pi}}$ where ω_{pi} is the ion plasma frequency. Then equation (3) becomes

$$\tan\delta = -\frac{\alpha\xi}{1 + \xi^2(1 + \alpha)}$$

where $\xi \tilde{=} \frac{\omega}{\omega_{pi}}$. This equation is also plotted in the graph at the end for several values of α.

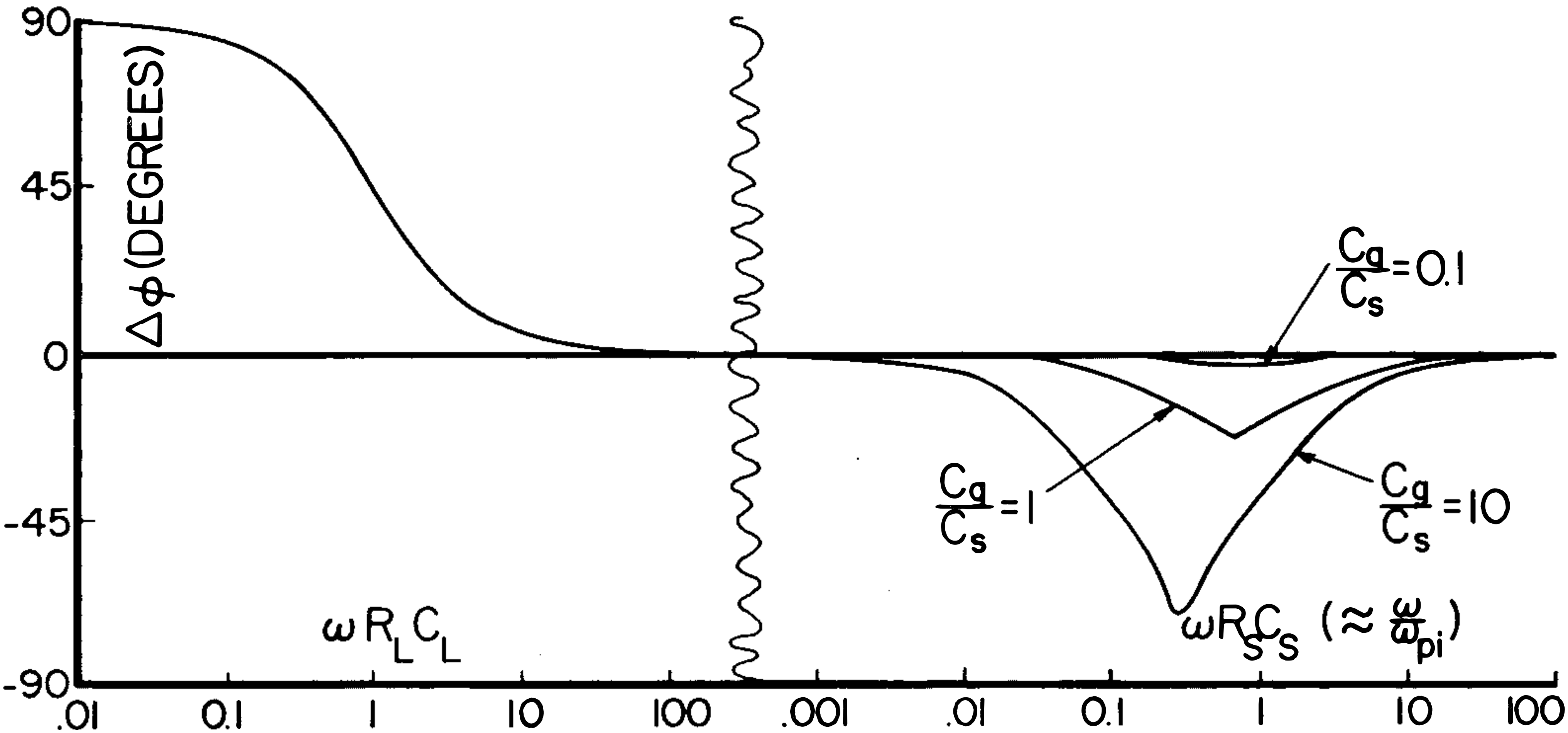
In conclusion, we have shown that for $C_g \leq \frac{1}{10} C_s$, the phase shift is negligible for $\frac{10}{R_L C_L} \leq \omega < \infty$. For a typical $R_L C_L$ of 10^{-1} sec, the phase shift is important only below ~ 20 Hz. For $C_g > \frac{1}{10} C_s$, an appreciable phase shift exists near the ion plasma frequency.

REFERENCES

1. J.C. Sprott, Rev. of Sci. Instr. 37, 897 (1966)

2. F.W. Crawford and R. Grard, Jour. Appl. Phys. 37, 180 (1965)

3. J.C. Sprott, PLP 88, Univ. of Wis.



90
45
0
-45
-90
Δφ (DEGREES)
.01
0.1
1
10
100
.001
.01
0.1
1
10
100
ωR_LC_L
ωR_SC_S (≈ ω/ω_pi)
Cg/Cs = 0.1
Cg/Cs = 1
Cg/Cs = 10