TRIPLE PROBE MEASUREMENTS IN THE OCTUPOLE

J. C. Sprott

February 1967

PLP 109
Copy No._____

University of Wisconsin
Thermonuclear Plasma Physics

TRIPLE PROBE MEASUREMENTS IN THE OCTUPOLE

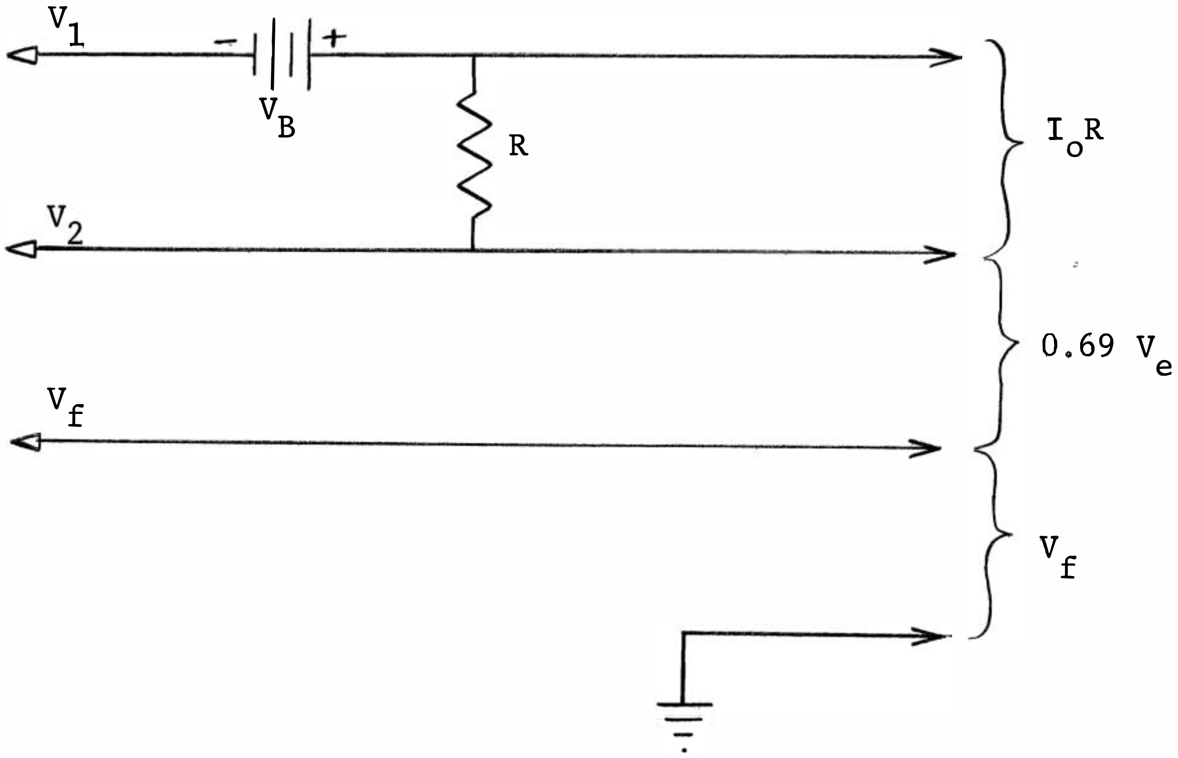
J. C. Sprott

INTRODUCTION

The triple probe is useful because it enables one to measure simultaneously density, electron temperature, and floating potential as a function of time without the complicated circuits that accompany other types of probes. It draws no net current and hence produces a minimum disturbance to the plasma. The first triple probe was used by Okuda and Yamamoto[1] but required continual adjustment, thereby limiting its usefulness to DC plasmas. An improvement was made by Aisenberg[2] but it was not until 1965 that a satisfactory triple probe for pulsed plasmas was developed by Chen and Sekiguchi[3]. The present paper presents a simplified theoretical description, and shows the results of some measurements made in the toroidal octupole with a triple probe.

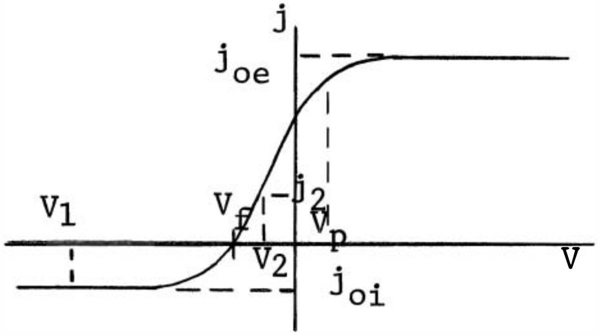
TRIPLE PROBE THEORY

The triple probe consists of three electrodes immersed in the plasma. One electrode floats electrically while the other two are biased in the usual Johnson and Malter[4] fashion.



The bias voltage V_B is made large compared to the electron temperature V_e. The voltage I_oR is proportional to the density for $I_oR \ll V_B$. The floating potential is read between the floating electrode and some reference ground. The difference voltage $V_2 - V_f$ is equal to 0.69 times the electron temperature in eV as will presently be shown.

Consider the well known[5] characteristic curve of a probe immersed in a plasma:



The voltage difference $V_2 - V_1 = V_B - I_oR \cong V_B$ is fixed by making R small. For $V_B \gg V_e$, $j_1 = j_{oi}$, the ion saturation current density. The particle density can then be found as follows:

$$I_o = j_{oi}A = \frac{1}{4}\, n e \overline{v}_i \quad , \tag{1}$$

where A is the probe area and $\overline{v}_i$ is the mean velocity with which ions enter the sheath. The floating potential V_f is defined by the condition that $j(V_f) = 0$.

If we assume a Maxwellian electron velocity distribution and a perfectly absorbing probe with a sheath sufficiently thin to preclude orbital motion, the portion of the characteristic curve for $V < V_p$ can be represented analytically as follows:

$$j = -j_{oi} + j_{oe}e^{(V-V_p)/V_e} \ , \tag{2}$$

where j_{oe} is the saturation electron current density. The floating potential is found by setting $j = 0$:

$$j_{oi} = j_{oe}e^{(V_f-V_p)/V_e} \ . \tag{3}$$

Since the total current to the probe must be zero, $j_2 = -j_1 = j_{oi}$. This condition defines V_2:

$$2j_{oi} = j_{oe}e^{(V_2-V_p)/V_e} \ . \tag{4}$$

Dividing equation (4) by (3) leads to the very nice result:

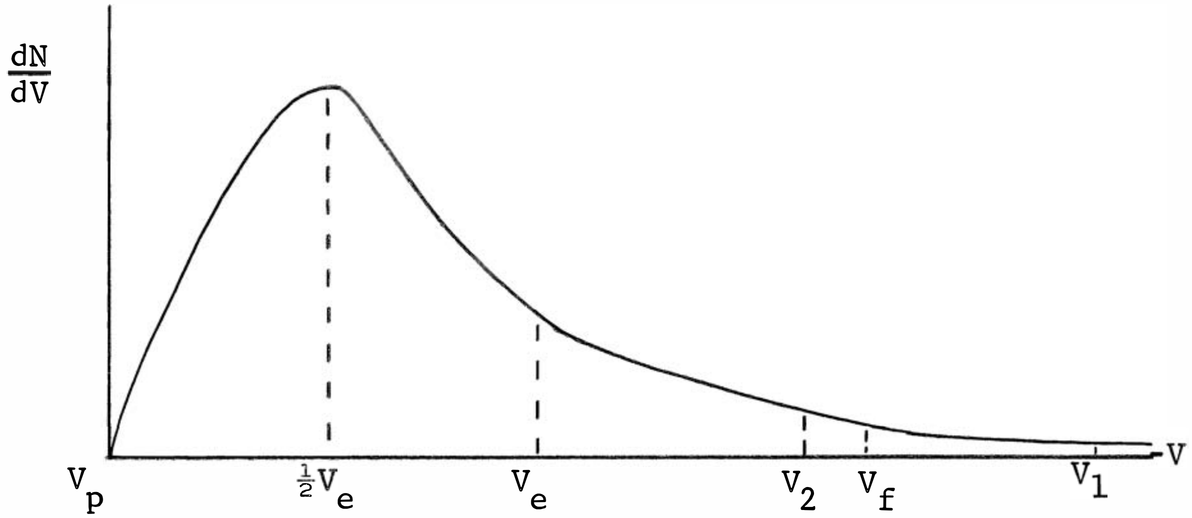
$$V_2 - V_f = V_e \ \ell n2 = 0.693V_e . \tag{5}$$

Equation (5) is the basis for electron temperature measurement with the triple probe. Note that the result is independent of any assumption about the ions, such as mass, energy, or velocity distribution. The only assumptions are that the electron velocity distribution is Maxwellian (necessary in order to define a temperature) and that the flux of ions is the same to all three tips (generally a reasonable assumption).

It is of interest to digress for a moment to consider how the triple probe works from a more physical standpoint. The number of electrons per unit energy interval is given by

$$\frac{dN}{dV} = \frac{2}{\sqrt{\pi}} \quad \frac{N_o}{V_e} \left[\frac{V-V_p}{V_e} \right]^{\frac{1}{2}} e^{(V-V_p)/V_e}. \qquad (6)$$

A graph of equation (6) is given below:



V_1, V_f, and V_2 are the potentials of the three probe electrodes. In most plasmas they are well out in the tail of the Maxwellian distribution. Each tip collects only those few electrons with energies greater than the potential at the probe tip. The previous derivation assumed that V_1 is sufficiently large that it collects a negligible electron flux, and that the number of electrons collected by the tip at V_2 is exactly twice that collected by the tip at V_f. Hence by measuring $V_2 - V_f$ one can determine V_e if the curve is assumed to be of the form given by equation (6). There is, however, an inherent danger with this kind of probe, because a measurement far out in the tail of the distribution is used to specify the entire distribution. The fraction of particles sampled by the tip at V_f is

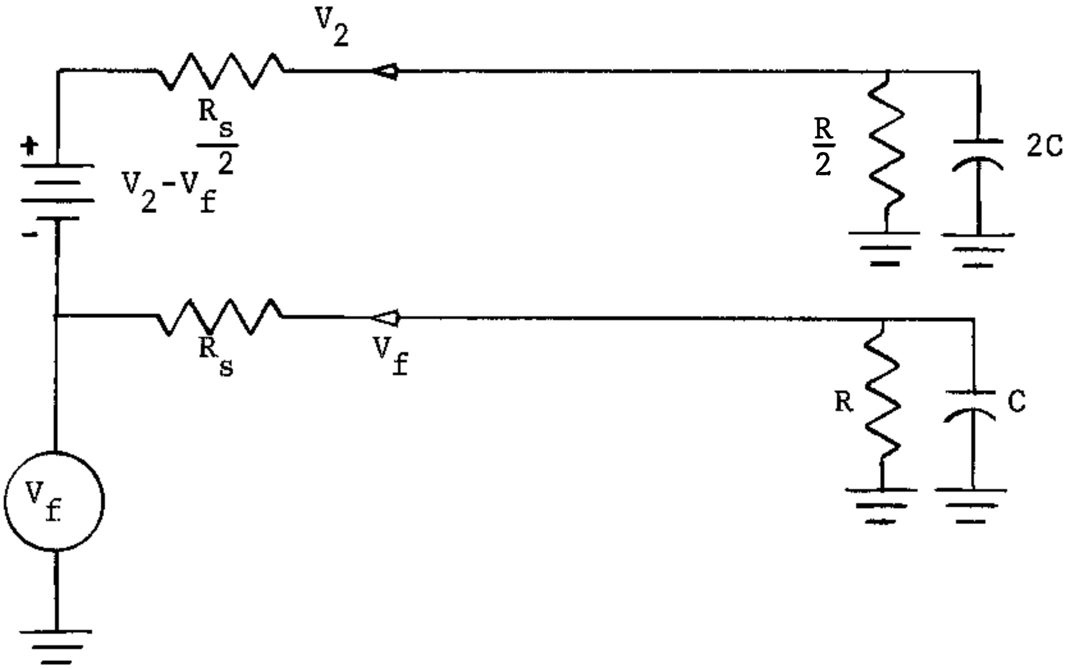
$$\frac{j_{oi}}{j_{oe}} = \frac{\bar{v}_i}{\bar{v}_e} = \sqrt{\frac{m_e V_i}{m_i V_e}} \, .$$

For plasma in the octupole ($V_i = 40$ eV, $V_e = 10$ eV, $m_i/m_e = 1836$) the above ratio is about 4.7%. The tip at V_2 samples less than 10% of the electrons. A slight departure from a Maxwellian can cause a large error in the temperature. In particular, if the electron distribution is depleted of high energy electrons as may be the case in the octupole[6], the triple probe will indicate an electron temperature which is lower than that of the rest of the distribution.

The limitation of the usefulness of the triple probe in a pulsed plasma depends on its frequency response characteristics and the independence of the three measured quantities. The single floating probe and double biased probe have been examined in these respects[7]. The response time of the temperature probe is the same as that of the floating probe, namely, $\tau \cong R_s C$, where C is the probe input capacitance and R_s is the sheath resistance:

$$R_s = \frac{V_e}{j_{oi}A} = V_e/I_o \ . \tag{7}$$

For floating potential fluctuations small compared to V_e, the following equivalent circuit is valid for the temperature probe:



The tip at V_2 is driven more strongly by the plasma than is the floating tip because the sheath resistance

$(\frac{dV}{di})_{V_2}$ is a as arge as $_s = \frac{\quad}{di}_{V_f}$. owever, since the tip at V_2 is connected by a low impedance to the tip at V_1, the impedance to ground is just half that of the floating tip provided the load on each probe tip is identical. Under these conditions, $V_2 - V_f$ is independent of V_f. Even if the load resistors are vastly different, the error voltage is of the order of $(R_s/R)V_f$ which for $R \gg R_s$ is negligible compared to V_e.

In the case of a thick sheath (Debye length $\gg$ probe radius), orbital motions of ions are important and the saturation ion current is voltage dependent. In particular, the ion current is given by

$$I_i = A\, j_{oi}(1 + \frac{V_p - V}{V_i})^k \qquad (8)$$

where

$$k = \begin{cases} 0 \text{ for a plane} \\ \frac{1}{2} \text{ for a cylinder} \\ 1 \text{ for a sphere.} \end{cases}$$

Revising equation (3) and (4) with the appropriate correction leads to the approximate result:

$$V_2 - V_f \cong V_e \,\ell n\, 2[1 + \frac{kV_e}{V_i + V_p - V_f}] \qquad (9)$$

For a cylindrical probe in the octupole, the error involved in equation (5) is

$$\frac{kV_e}{V_i+V_p-V_f} \cong 7\% \ .$$

This is an overestimate of the error since the sheath is usually not large compared to probe dimensions.

A more serious limitation is the fact that the sheaths of the three electrodes must not overlap (i.e., the electrodes must be many Debye lengths apart). Since good spatial resolution requires the tips to be close together, there exists a lower limit on the density for which a given probe will operate properly.

Aside from the limitations mentioned above, the temperature measurement should have the same accuracy as density and potential measurements.

MEASUREMENTS IN THE TOROIDAL OCTUPOLE

A triple probe was constructed with three identical electrodes placed on the corners of an equilateral trangle. Each electrode was 1 cm long by 1 mm in diameter. The distance between tips was 4 mm. Each tip was connected to a 1 meter length of 52Ω cable terminated with a pair of Tektronix P6008 probes (X10, 10MΩ, 7pF). Ion saturation current and electron temperature measurements used type 1A1 differential amplifiers with a Tektronix type 127 power supply. The probes and amplifiers were balanced by driving all three tips with a -10 volt square wave and observing the differential amplifier outputs. The temperature channel was calibrated by applying a 10 volt square wave between tip 2 and the floating tip and adjusting the scope gain to read 1.4 cm.

The probe density and frequency limitations are determined as follows: an input resistance of 5 megΩ requires the sheath resistance to be less than 5 megΩ or the density to be greater than $2 \times 10^7 \mathrm{cm}^{-3}$ by equation (7). The Debye length becomes comparable to the electrode spacing if the density falls below about $4 \times 10^7 \mathrm{cm}^{-3}$ The triple probe should not be trusted for densities below this value. At a density of $4 \times 10^7 \mathrm{cm}^{-3}$ the sheath resistance is 2.5 megΩ . With 100 pF of cable capacitance, the response time of the probe is about 250 μsec. At higher densities the response time goes down in inverse propotion to the density.

The triple probe was installed in the octupole at Port 2 ($\theta = +50$). In Figure 1 the density, electron temperature, and floating potential are shown as a function of the time at $\rho = 0$ (the zero field point) for $B_\theta = 0$ and full B_θ. The density traces are in agreement with those obtained earlier[8] with single probes, showing the initial fast decay, followed by a leveling off, and then a rapid decay as the magnetic field disappears. The floating potential traces follow very closely those obtained from our best attenuated probes. The crucial test of the triple probe is the behavior of the temperature curve. The temperature with $B_\theta = 0$ is seen to be exactly 10 volts over most of the containment period, in perfect agreement with log i_e vs V plots obtained from single probes[9]. Note that this result is at least twice the value of V_e determined by Erickson[6] with an electrostatic analyzer. The temperature with full B_θ appears to be 10-20% lower (8-9 volts) than with $B_\theta = 0$. There is evidence of a slight decay in temperature with time as would be expected for losses to hoop supports.

One curious feature of the electron temperature is the fact that it increases for the first 500 μsec of the containment time. During filling the large electric fields would disturb the probe, but since the density is $> 10^9 \text{cm}^{-3}$, the probe should recover in less than 10 μsec. With some reservations about the first 100 μsec, therefore, the electron temperature is increasing. It was first thought that since plasma was injected while the field was still increasing, that some heating by magnetic compression was taking place. To test this hypothesis the injection time was varied as shown in Figure 2. The quantity $\dot{B} = \frac{dB}{dt}$ is also shown on the lowest trace. Although the heating did seem to be more pronounced for $\dot{B} > 0$, it was still present for $\dot{B} < 0$ where a cooling would be expected. Either the electrons are gaining energy from the ions (which is unlikely because the equipartition time at 10^9cm^{-3}

is about 1 second[10]) or the electrons are initially non-Maxwellian. (The electron-electron collision time at $10^9 cm^{-3}$ is about 500 μsec[10]). Single probe characteristics show that by 600 μsec, the electrons are Maxwellian up to at least 4 V_e[9].

Figures 3 and 4 show the spatial variation of n, V_e, and V_f across the horizontal midplane at 600 μsec with $B_\theta = 0$ and with full B_θ. S = 0 is the inside wall and S = 14" is the outside wall. The density is lower and flatter with B_θ than without, in agreement with previous measurements[8]. The electron temperature is fairly constant across the midplane except within a few inches of the wall. Recall that triple probe measurements with densities below $0.4 \times 10^8 cm^{-3}$ are expected to be in error. The electron temperature with B_θ is also smaller and flatter than with $B_\theta = 0$. The floating potential with $B_\theta = 0$ is reasonably constant across the midplane except for some unexplained phenomenon near the outside wall. With full B_θ, the floating potential is very irregular near both walls due to the presence of magnetic islands[11].

The reproducibility of the data is shown by the spread of points in Figures 3 and 4. Three shots were taken at each position. When the three shots were nearly identical, only the average was plotted. The temperature and floating potential are quite reproducible while the density varies 20-30% between shots.

It has previously been verified that the potential[12] and density[8] are constant along magnetic field lines. The constancy of electron temperature along magnetic field lines is yet to be completely tested. Preliminary tests show the temperature to be somewhat higher in the high field region behind the hoops than in the center. This temperature gradient seems to be larger with B_θ than without. Such a gradient implies a non-isotropic electron velocity distribution. The larger temperature gradient with B_θ may be a manifestation of the lower mobility of electrons in the strong field which confines them near $\rho = 0$. A more thorough investigation of this behavior is needed.

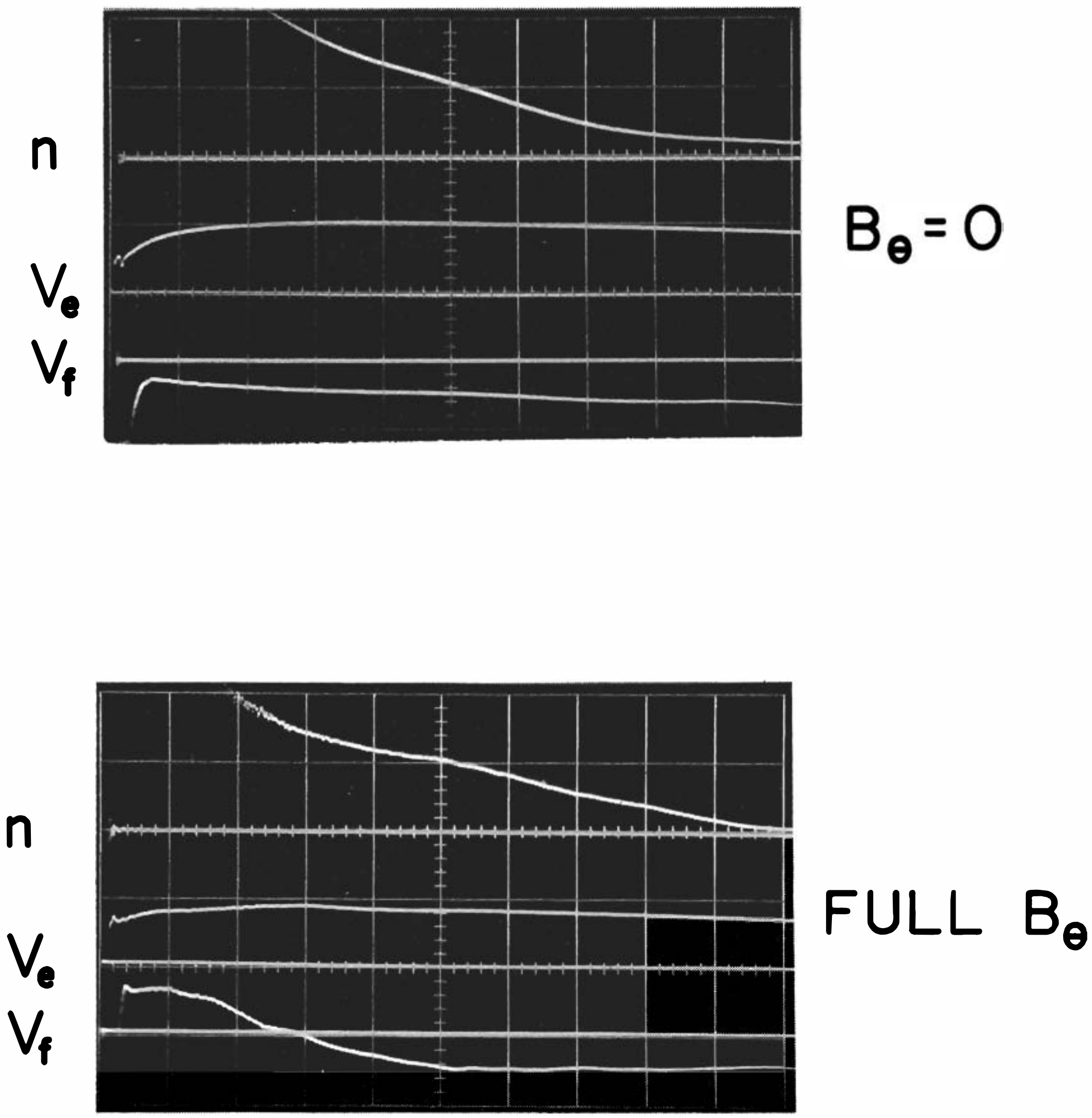
Triple probe measurements were made as a function of ψ at the point on the ψ-line where the field is weakest ($\phi = 0°$ for $+5 > \psi > -1.6$, and $\phi \cong 45°$ for $-1.6 > \psi > -5$). The results are plotted in Figures 5, 6, and 7. X's indicate $B_\theta = 0$ and O's indicate full B_θ. The density curves strongly resemble those obtained earlier[8] with single probes. The electron temperature, like the density, is maximum about 1 ψ unit inside the separatrix toward the hoops, and is generally lower with B_θ. The floating potential is relatively smooth for $B_\theta = 0$, but shows a marked deviation around $\psi = +1$. A potential gradient appears to exist across the separatrix in both cases.

An interesting application of the triple probe is the measurement of the effectiveness of electron cyclotron heating. For this purpose two magnetrons were mounted on the toroid. One was rated for 10 kW at 3500 MHz and the other for 500 kW at 1300 MHz. Figure 8 shows the result of pulsing on the smaller magnetron 900 μsec after injecting plasma. The triple probe was placed at $\rho = 5''$, $\phi = 0$ - the point where electron cyclotron resonance should occur. The spikes in the traces are due to the presence of strong r.f. fields in the vicinity of the probe. At these densities, the probe should require about 100 μsec to recover from such a perturbation, in agreement with what is observed. The temperature leveled off at a value about 50% higher than that of the plasma before heating. Note also that the density increased markedly, implying that plasma had been created. To test this assumption, the background hydrogen pressure in the octupole was raised considerably and the microwaves were pulsed on without any background plasma. Plasma was indeed generated as is shown by the single probe ion saturation current trace in Figure 8. This cold ion plasma possessed some rather remarkable characteristics in the octupole field, and will form the subject of a later paper.

REFERENCES

1. T. Okuda and K. Yamamoto, J. Appl. Phys. 31, 158 (1960).
2. S. Aisenberg, J. Appl. Phys 35, 130 (1964).
3. Sin-Li Chen and T. Sekiguchi, J. Appl. Phys. 36, 2363 (1965).
4. E. O. Johnson and S. Malter, Phys. Rev. 80, 58 (1950).
5. F. F. Chen in Plasma Diagnostic Techniques, R. H. Huddlestone and S. L. Leonard, Eds. (Academic Press: New York, 1965). Chapt. 4.
6. C. W. Erickson, Ph. D. Thesis, University of Wisconsin (1967).
7. J. C. Sprott, PLP 88.
8. D. E. Lencioni and J. C. Sprott, PLP 89.
9. J. C. Sprott, Data Book No. 5, page 47-49.
10. J. C. Sprott, PLP 92
11. J. A. Schmidt and D. E. Lencioni, PLP 85.
12. D. M. Meade, Bull. Am. Phys. Soc 11, 462 (1966).

P-2 $\rho = 0$



200 µsec/Div

n 2×10^8 cm^{-3}/Div

V_e 10 eV/Div

V_f 10 volts/Div

Figure 1

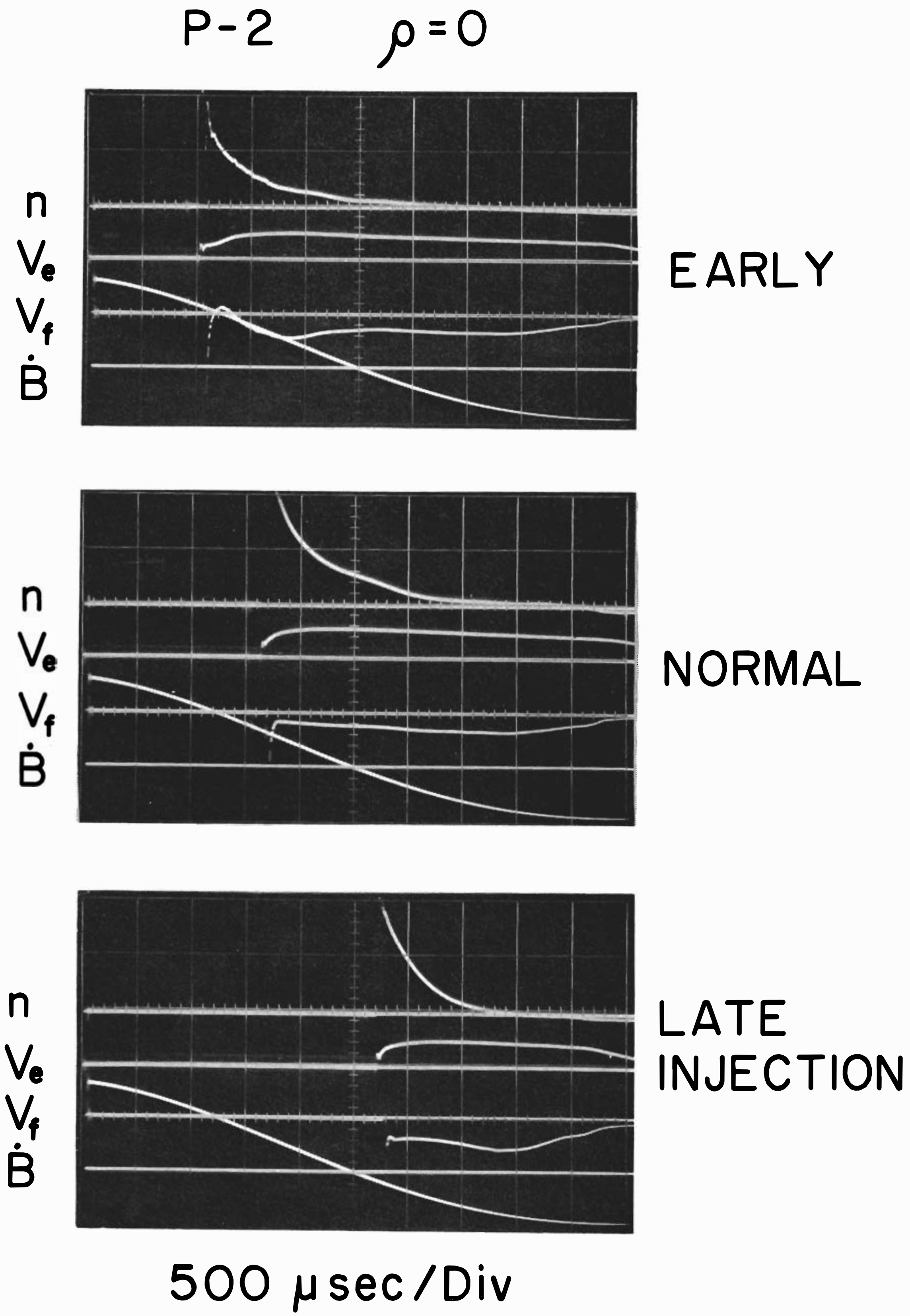

P-2 $\rho = 0$
n
V_e
V_f
$\dot{B}$
EARLY
n
V_e
V_f
$\dot{B}$
NORMAL
n
V_e
V_f
$\dot{B}$
LATE INJECTION
500 μsec/Div

Figure 2

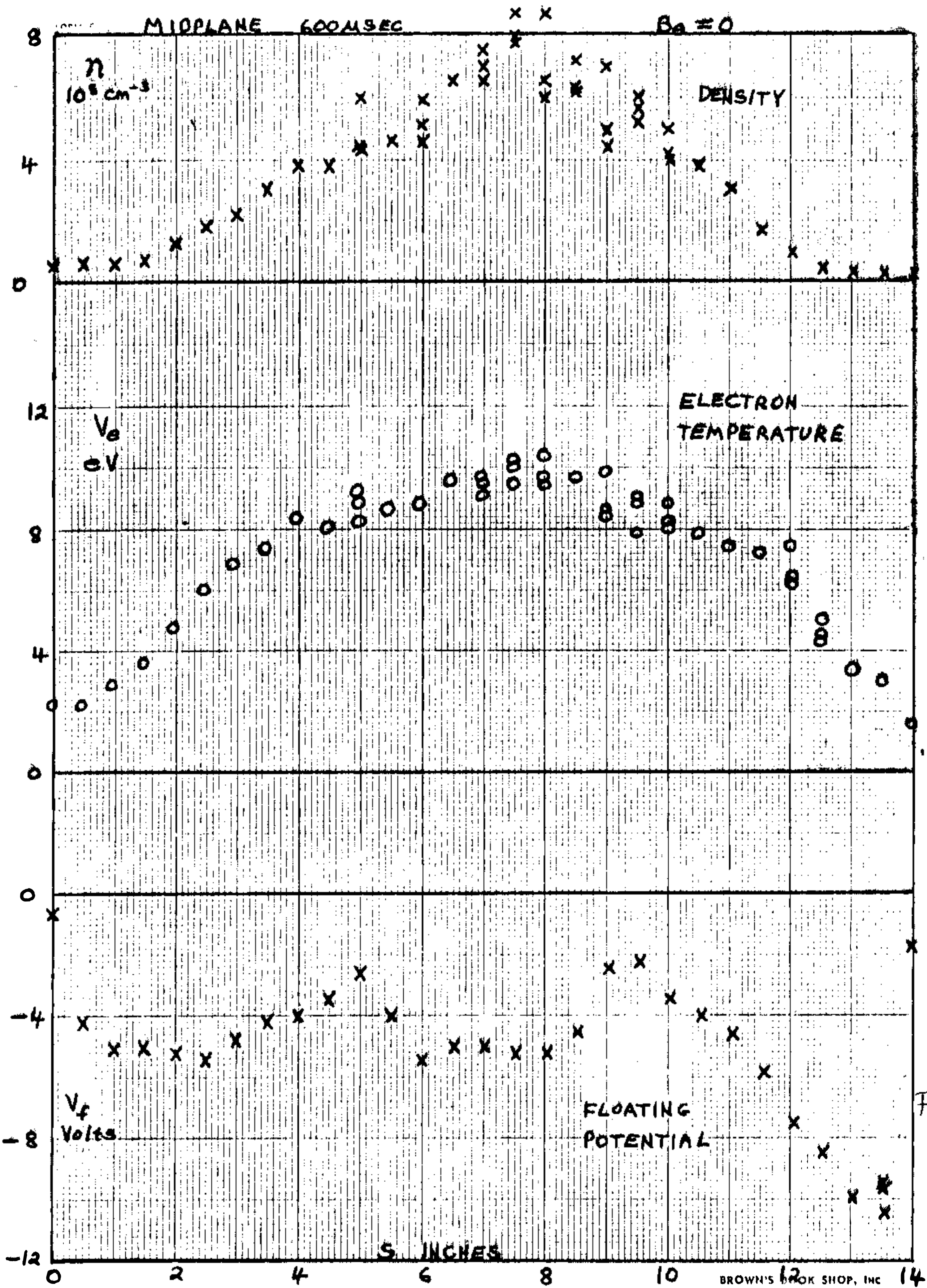

MIDPLANE 600μSEC
Ba ≠ 0
n
10⁸ cm⁻³
DENSITY
8
4
0
ELECTRON
TEMPERATURE
Ve
eV
12
8
4
0
0
−4
−8
−12
Vf
Volts
FLOATING
POTENTIAL
S, INCHES
0
2
4
6
8
10
12
14

Fig. 3

MIDPLANE 600 μSEC FULL B_0

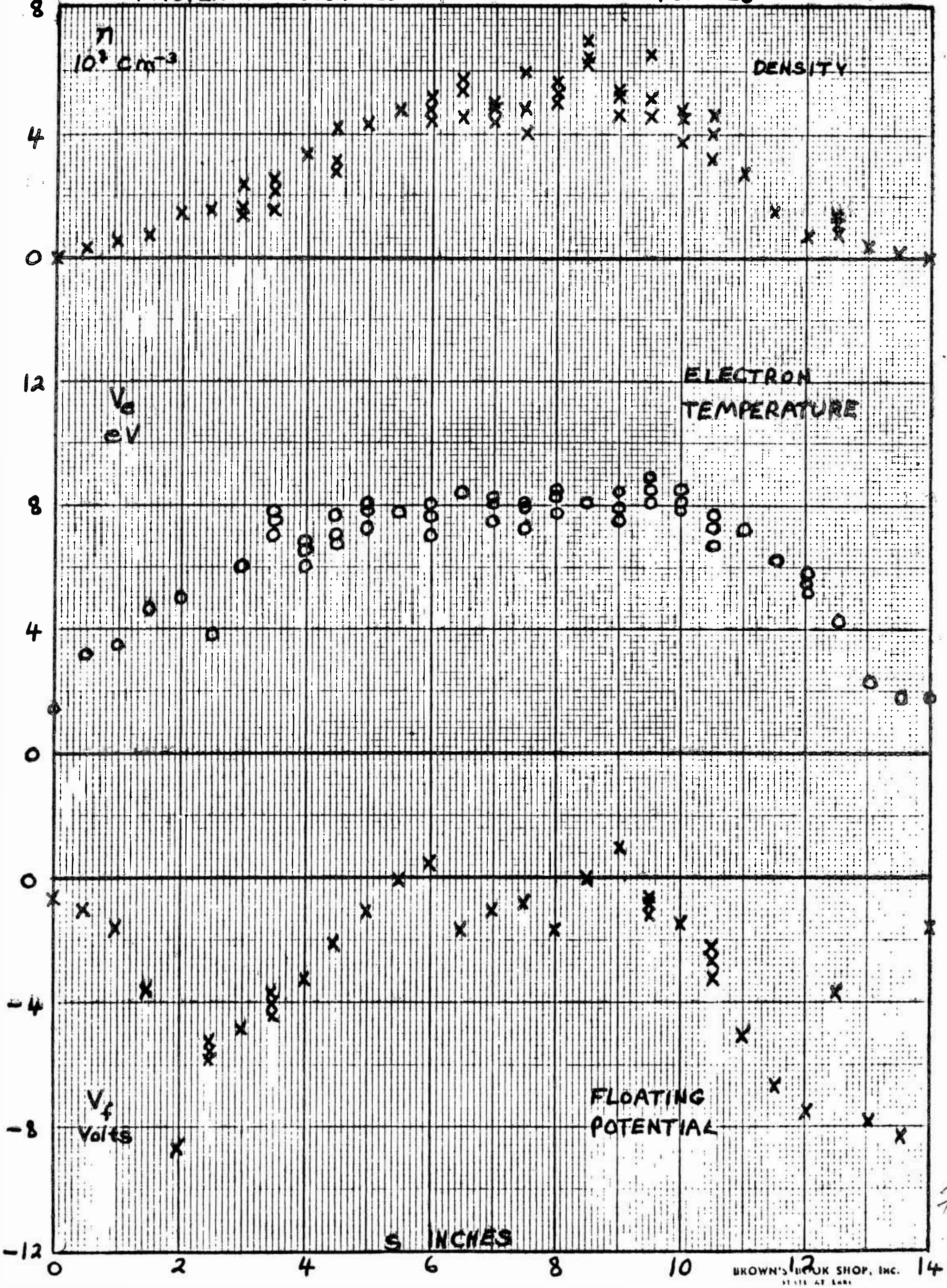


Fig 4

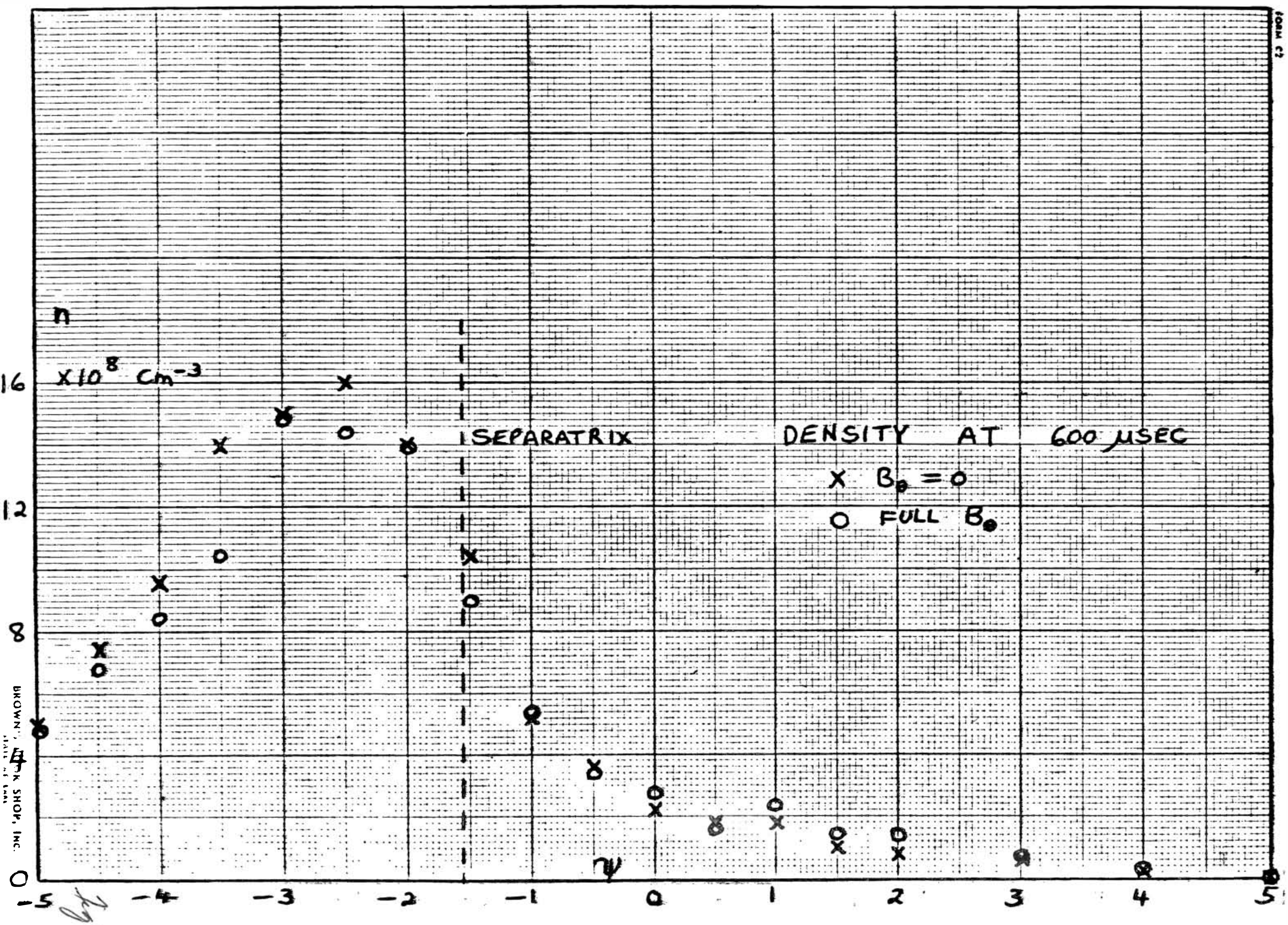

DENSITY AT 600 μSEC
X $B_\theta = 0$
O FULL B_θ
SEPARATRIX
n
X10^8 cm^{-3}
16
12
8
4
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
ψ
FORM C2
BROWN'S [illegible] SHOP, INC.
MADISON, WISCONSIN

Fig 5

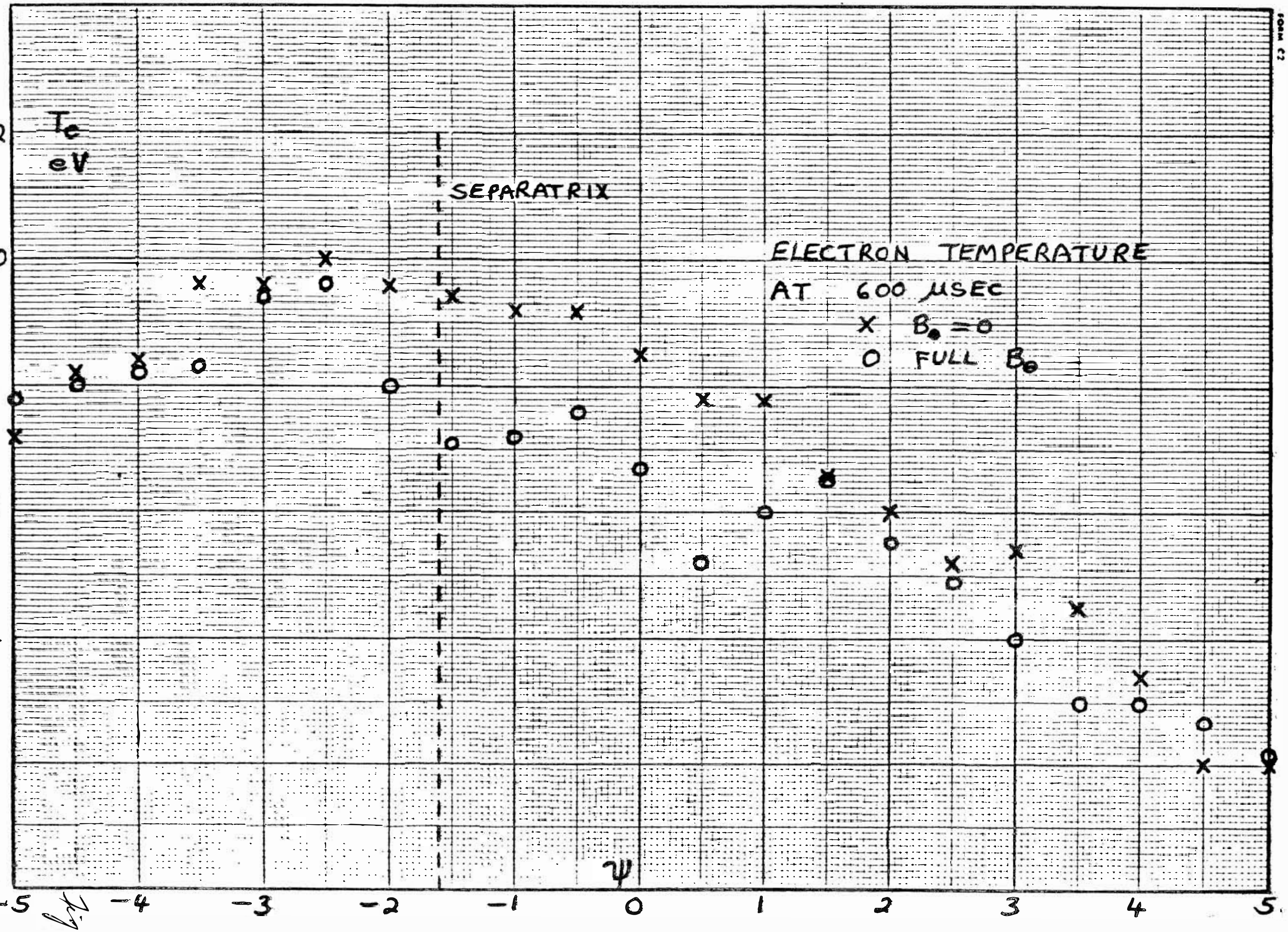

ELECTRON TEMPERATURE
AT 600 μSEC
X $B_0 = 0$
O FULL B_0
SEPARATRIX
T_e eV
12
10
8
6
4
0
ψ
-5
-4
-3
-2
-1
0
1
2
3
4
5

Fig. 6

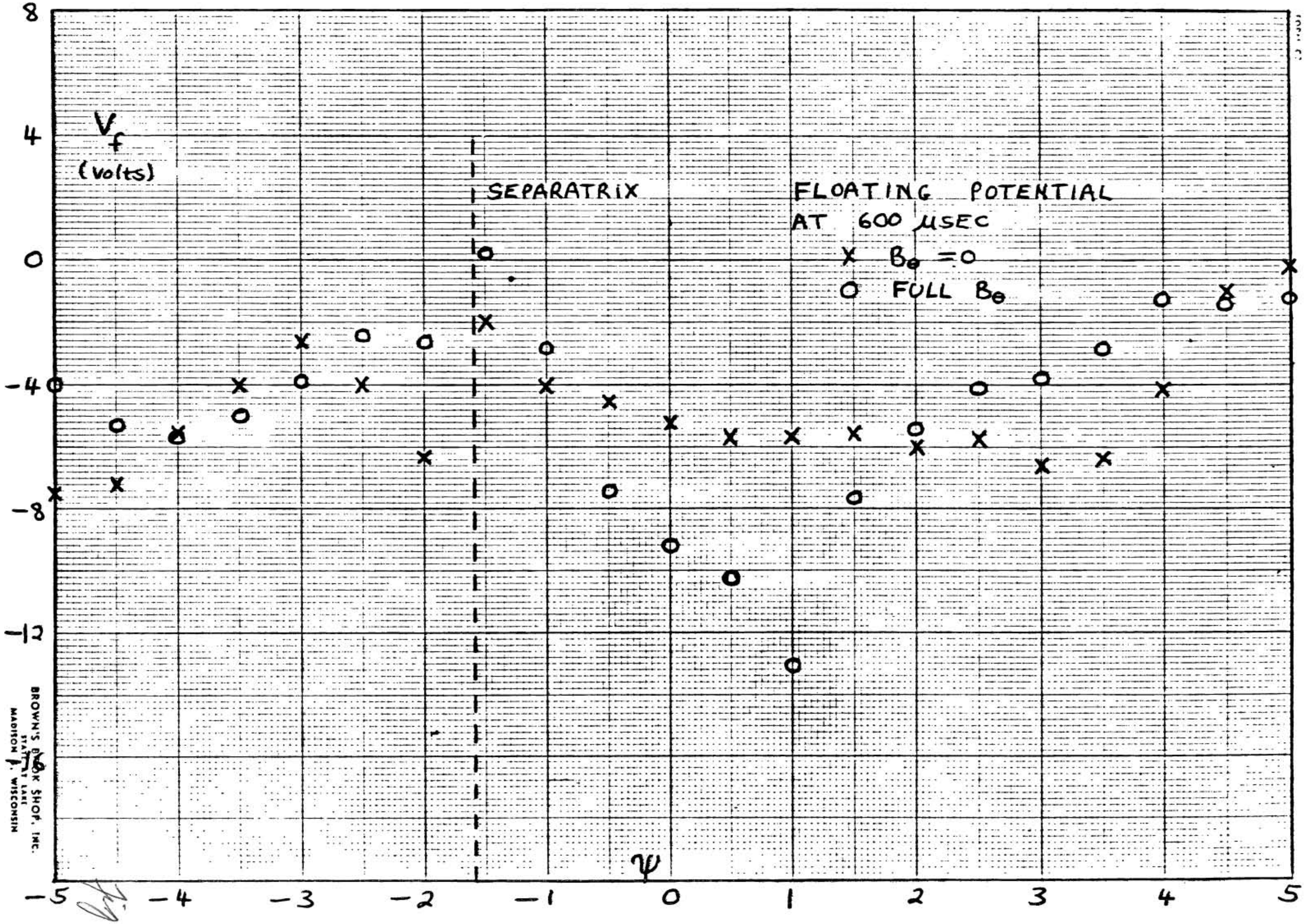
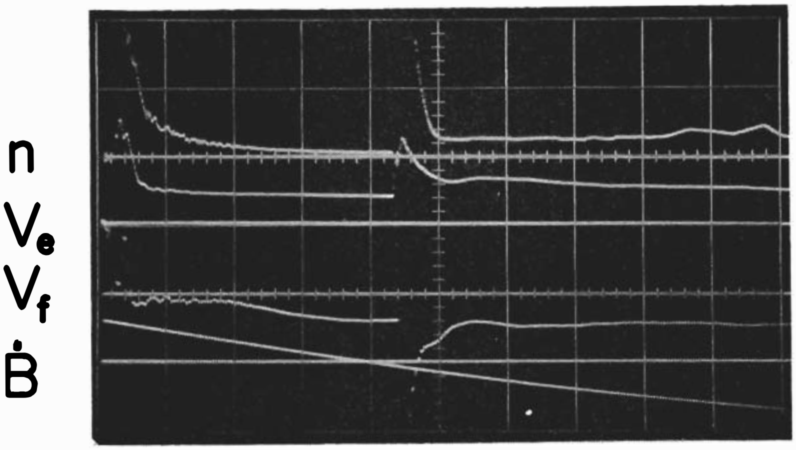

FLOATING POTENTIAL
AT 600 μSEC
X $B_\theta = 0$
O FULL B_θ
SEPARATRIX
V_f
(volts)
8
4
0
-4
-8
-12
ψ
-5
-4
-3
-2
-1
0
1
2
3
4
5

fig. 7

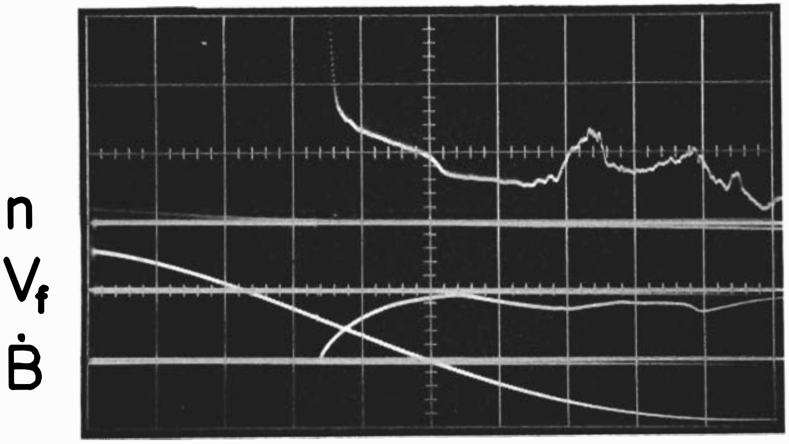
P-2, $\rho=5$, $\phi=0$



200 μsec/Div

PLASMA HEATING

P-2 $\rho=0$



500 usec/Div

PLASMA PRODUCTION

Figure 8