Chaotic Analysis of Numerical Plasma Simulations

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Abstract:

Data generated by numerical simulations of plasma processes were examined for evidence of low dimensional chaos. Analysis included the generation of phase plots and Poincaré sections, and the determination of the correlation dimension and the largest Lyapunov exponent. Data from a simulation of ion convective cell turbulence showed evidence of a low dimension ranging from ~ 4.3 to 5.9, depending on the mode, for the case with no driving term. In the driven case the dimension was >8. Data from the DEBS code simulation of an RFP indicates a possible dimension <3, however more data is required to confirm this result.

Motivation:

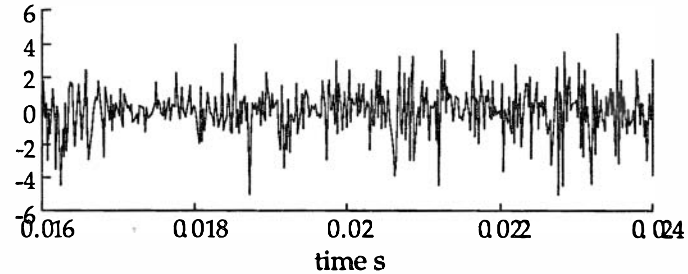
I. Numerical simulations have some advantages:

a) There are fewer modes, and possibly simpler dynamics.
b) The simulations are (in principle) reproducible.
c) One can generate an "infinite" amount of steady state data

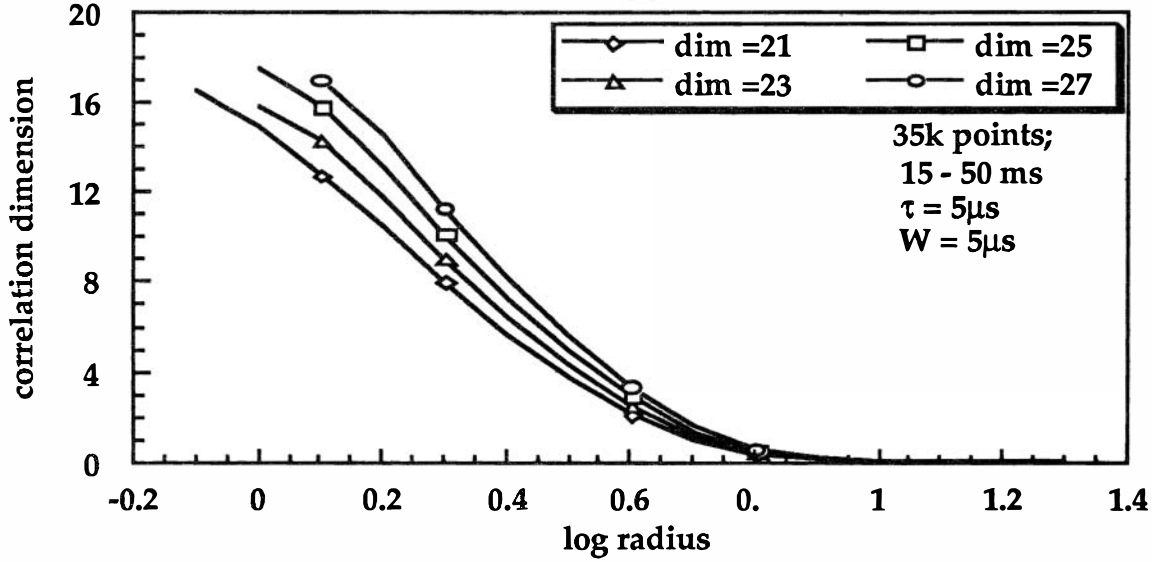
II. Evidence for chaotic behavior in numerical simulations would encourage a more thorough search for evidence in data from plasma experiments. Recall last year's results:

No Evidence of Low Dimensional Chaos in MST Experimental Data

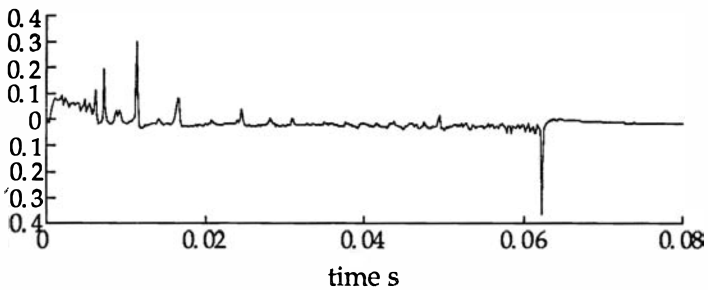
Edge Magnetic Fluctuations



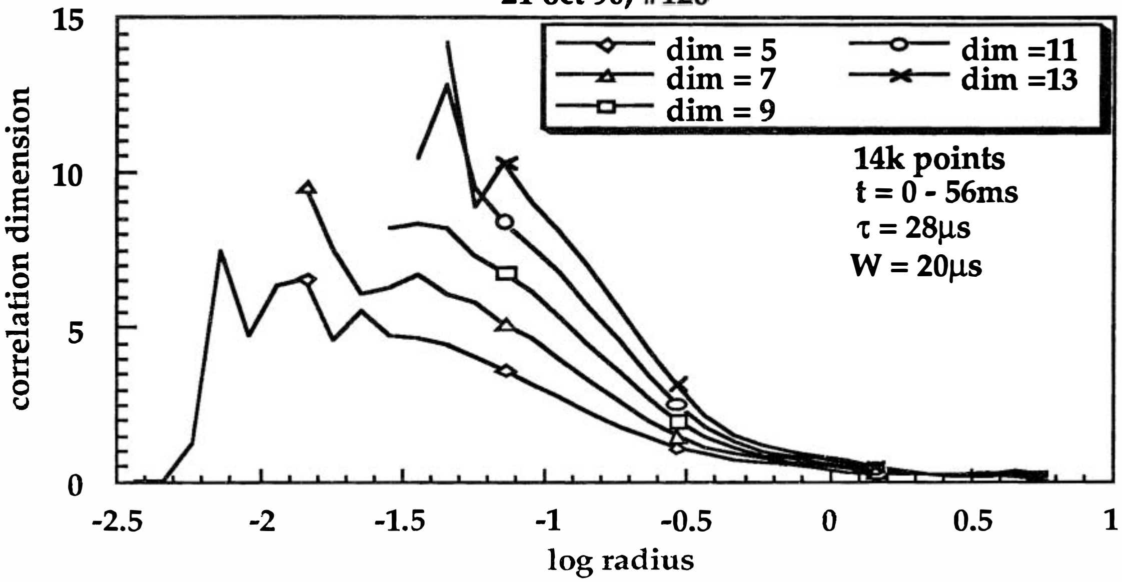
Correlation Dimension of Edge Toroidal Magnetic Fluctuations
@ various embeddings, 22 jul 90, #123



Toroidal Gap Volts



Correlation Dimension of Toroidal Gap Volts
21 oct 90, #120



Motivation for Chaos Studies:

Identifying low dimension chaos in a signal means that determinant processes govern the system.

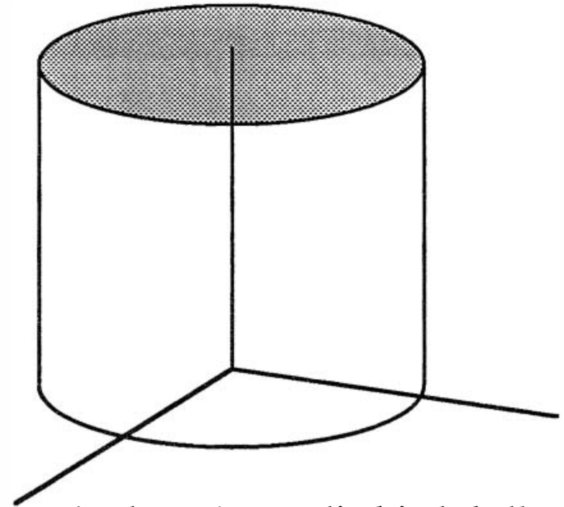
The fractal dimension of the system indicates the number of PDE's need to describe the system.

Time Lag Embeddings

a) The time series of a single quantity is sufficient to determine the chaotic nature of the system

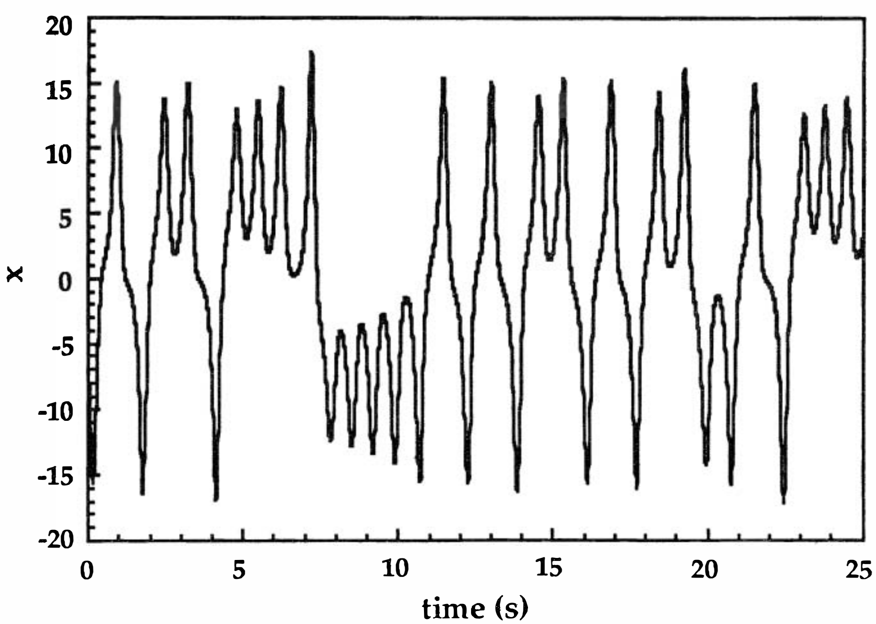
b) Reconstruct the attractor in an M dimension embedding space by creating time lag vectors:

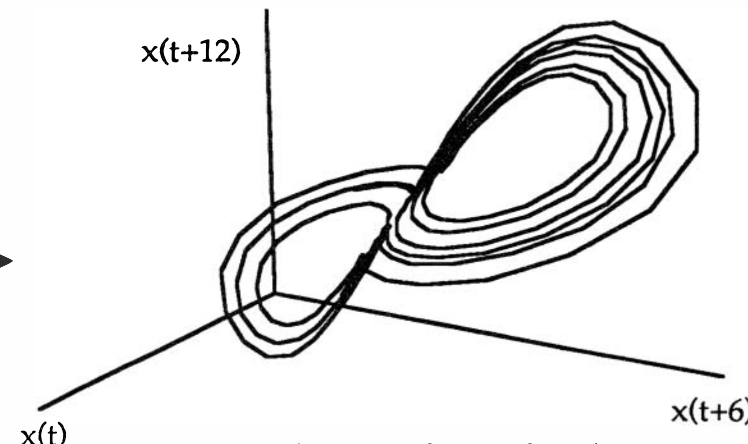
$$\underline{x}=[x(t),x(t+\tau), \ldots x(t+(M-1)\tau).$$



Analogy: A 2D cylindrical shell in a 3D embedding space

Time Series of the Lorenz Attractor





Lorenz Attractor in 3D time-lag Phase Space

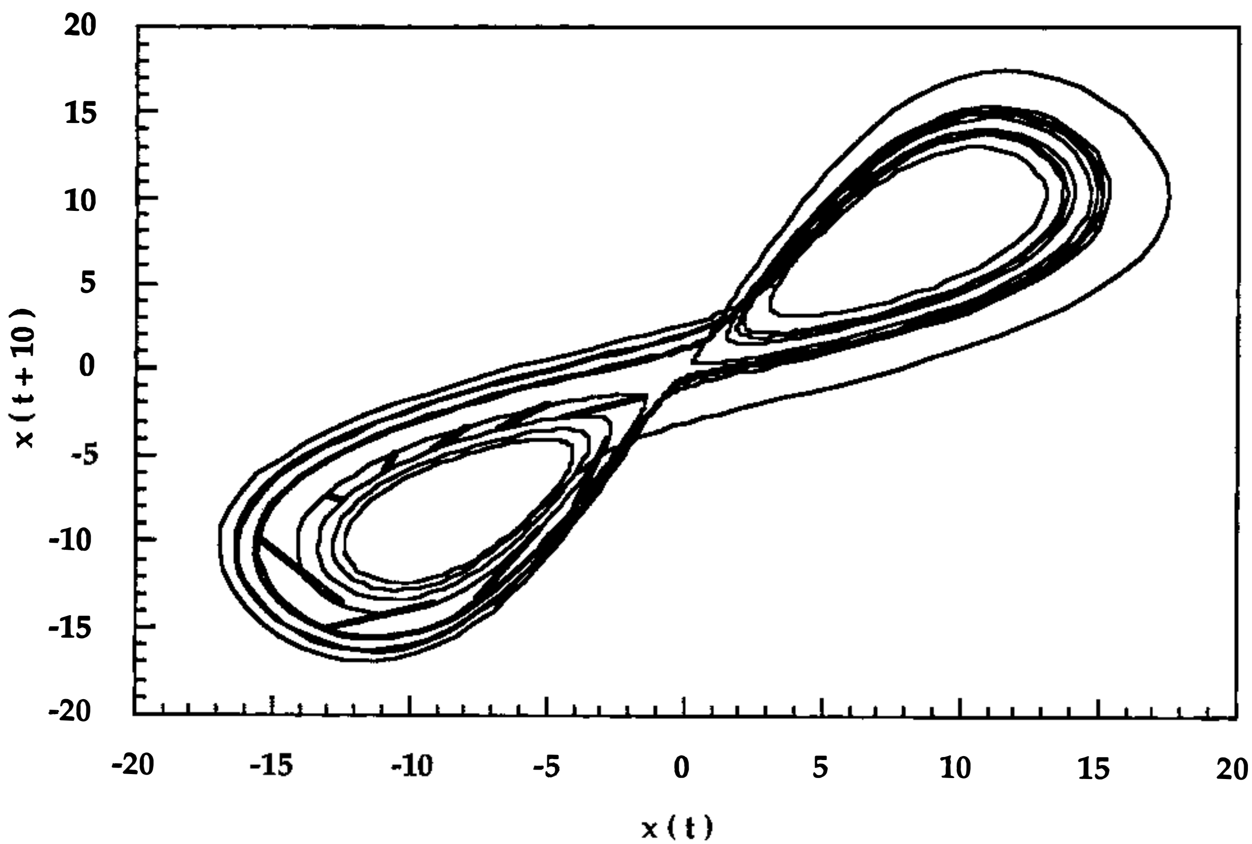
Lyapunov Exponent

a) The largest Lyapunov exponent measures the exponential divergence of nearby trajectories.

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln \left| \frac{df(x_k)}{dx_k} \right|$$

b) A positive Lyapunov exponent is indicative of chaos.

c) Using embedding coordinates, measure the divergence of two nearby points. Average this over all points in the attractor.



Correlation Dimension

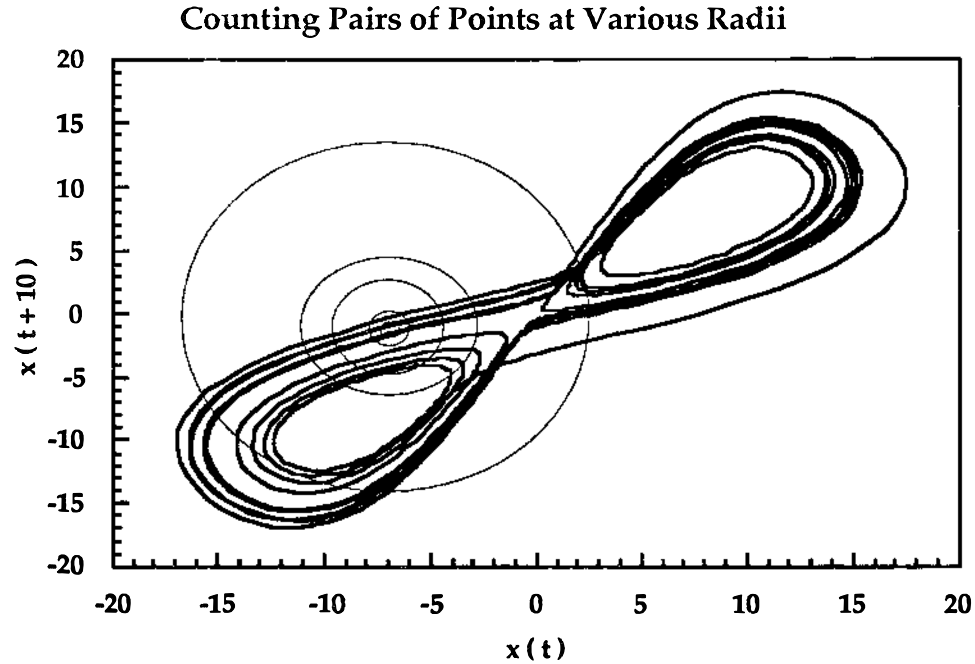
a) The **correlation dimension** is a measure of the nonuniformity of a chaotic attractor. It is closely related to the fractional dimension occupied by the chaotic attractor.

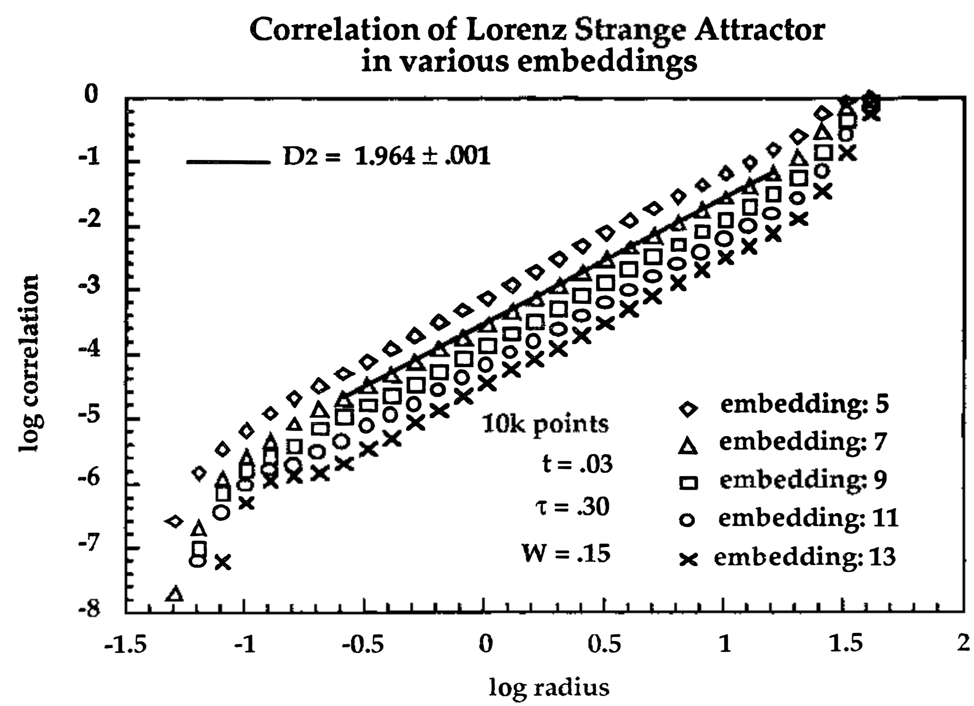
b) To estimate the correlation dimension, use embedding coordinates and compute the correlation integral by counting nearest neighbor pairs within a given radius.

$$\mathbf{C(r)} = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i\neq j} \quad 1 \quad \mathrm{j}$$

c) The slope of the log r - log C(r) plot is the correlation dimension D_2.

$$\mathbf{D}_2 = \lim_{r\to 0} \frac{\log C(r)}{\log r}$$





Trapped Ion Convective Cell Turbulence

The model simulates trapped ion modes which may be important in the core of tokamaks. The model is 2D, and energy is conserved.

$$\frac{\partial n_i}{\partial t} + v_D^* \frac{\partial n_i}{\partial y} + \gamma n_i + D \frac{\partial^2 n_i}{\partial y^2} - \frac{4DL}{\sqrt{\varepsilon}} \left(\nabla \frac{\partial n_i}{\partial y} \times \mathbf{z}\right) \cdot \nabla n = 0$$

n_i is the ion trapped particle fraction

ε is the aspect ratio

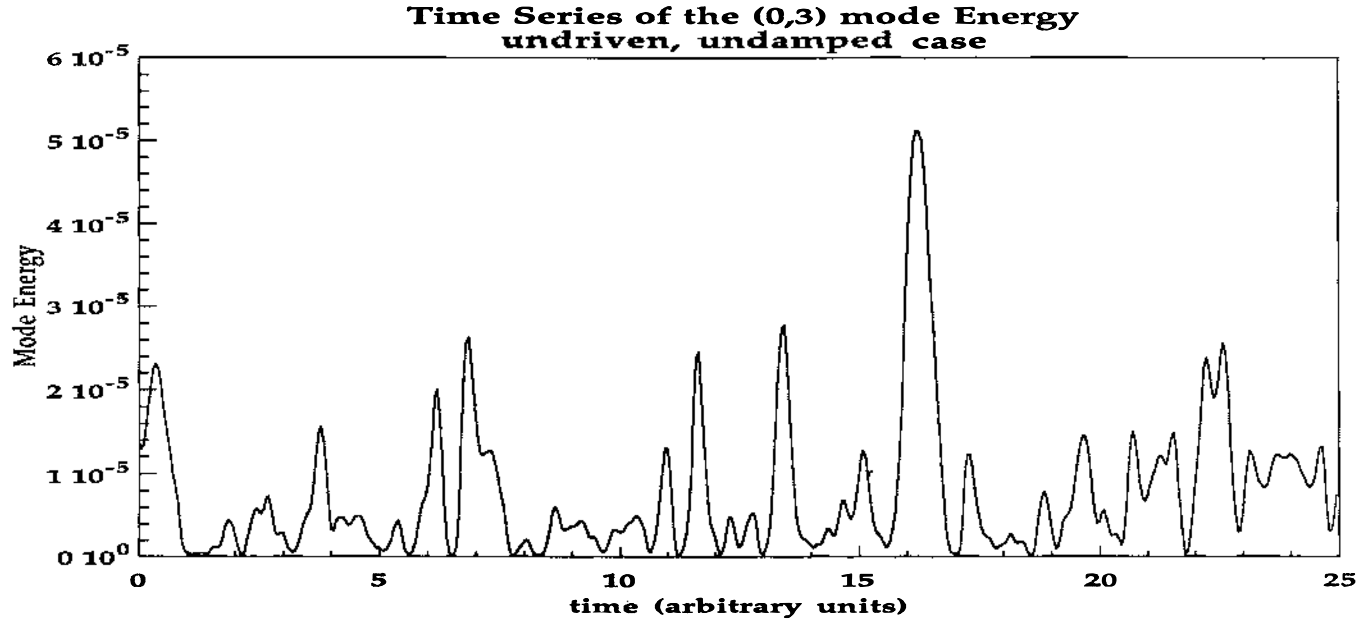
γ is the ion-ion collision frequency

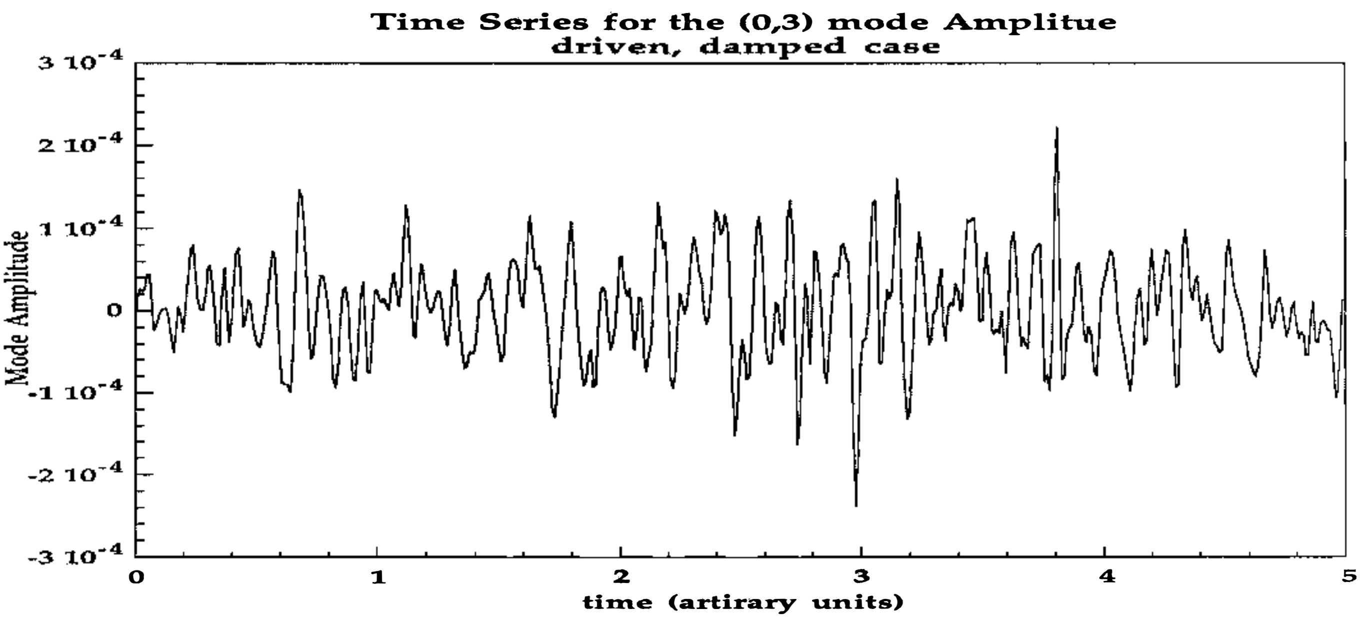
v^* is the diamagnetic drift velocity

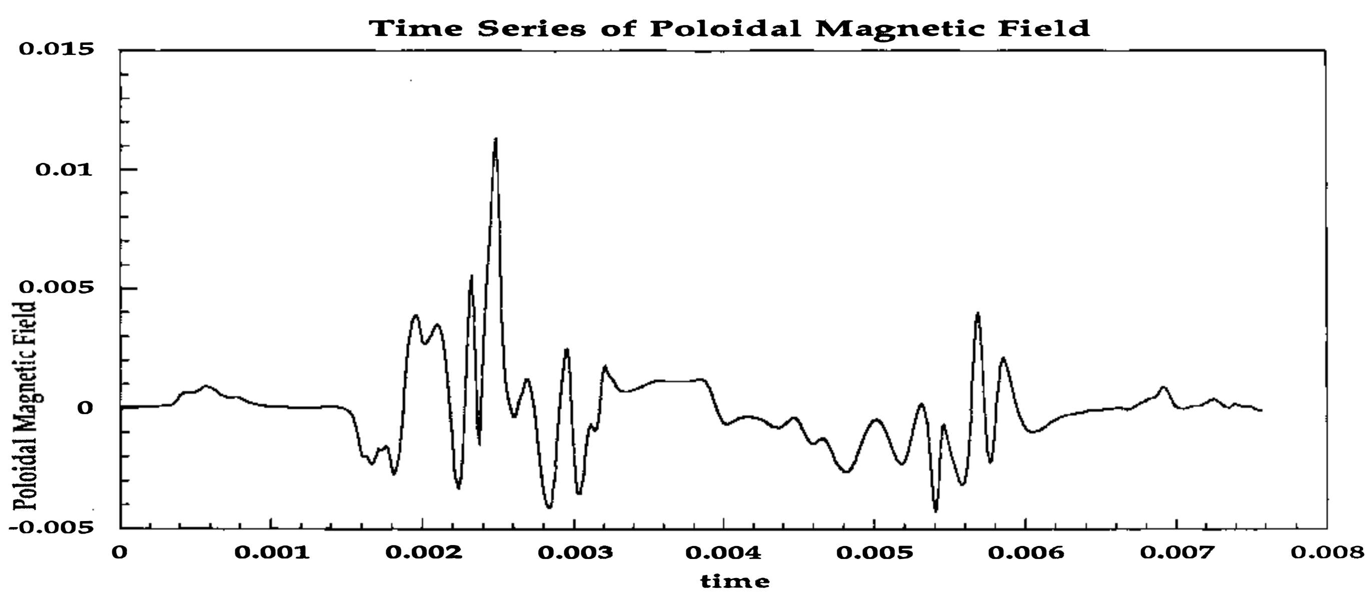
For a more thorough discussion of the model and underlying physics, see David Newman, 3T22.

For the cases below, a single nonlinearity is included: the dominant ExB nonlinearity.

The 2 cases presented, driven and undriven, are results from a 13x13 mode calculation. For a given mode, both the real part of the amplitude A and the energy $|A|^2$ were examined for evidence of chaos.


Time Series of the (0,3) mode Energy
undriven, undamped case
Mode Energy
6 10^-5
5 10^-5
4 10^-5
3 10^-5
2 10^-5
1 10^-5
0 10^0
0
5
10
15
20
25
time (arbitrary units)


Time Series for the (0,3) mode Amplitue
driven, damped case
Mode Amplitude
3 10^-4
2 10^-4
1 10^-4
0
-1 10^-4
-2 10^-4
-3 10^-4
0
1
2
3
4
5
time (artirary units)

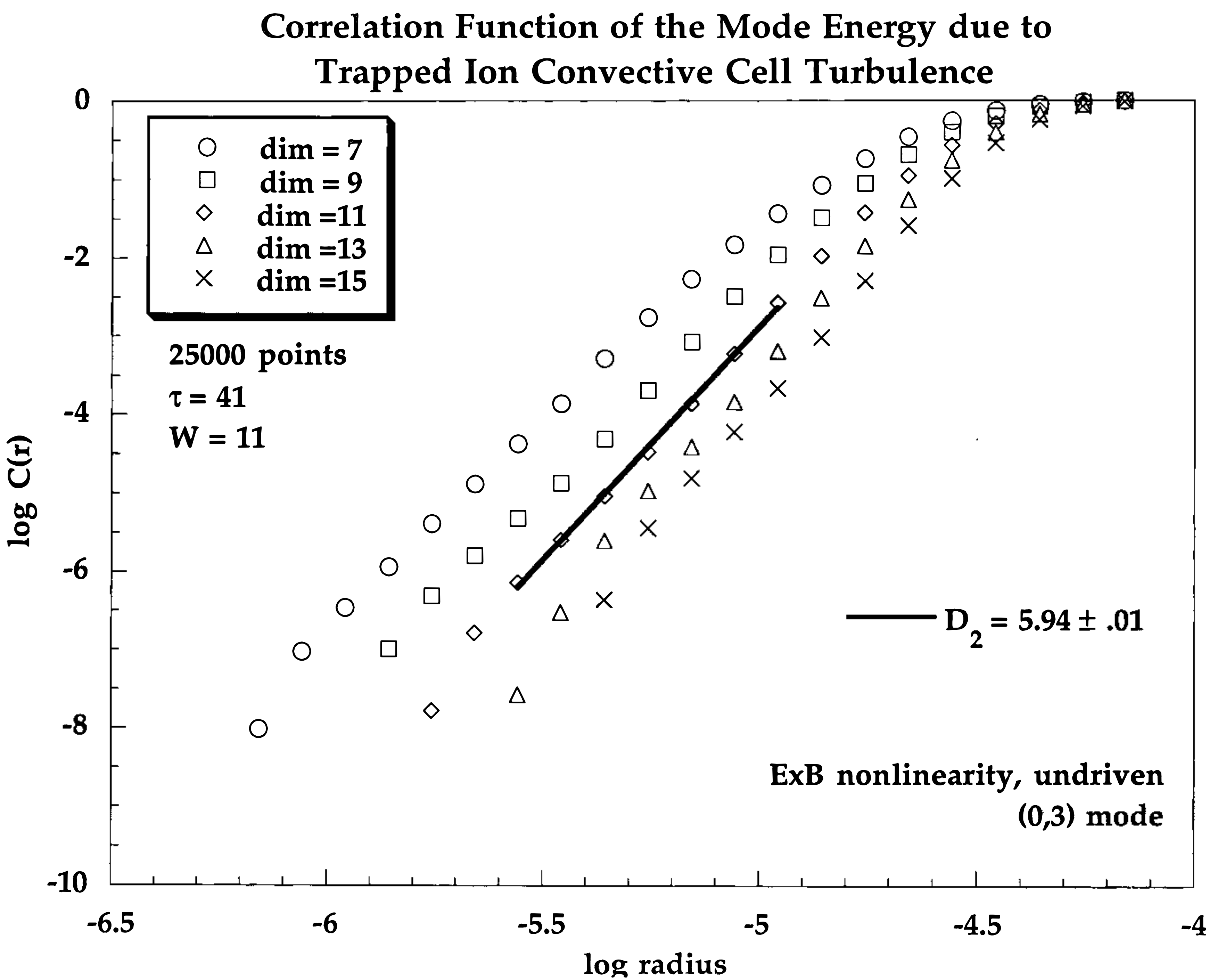

Time Series of Poloidal Magnetic Field
Poloidal Magnetic Field
0.015
0.01
0.005
0
-0.005
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
time

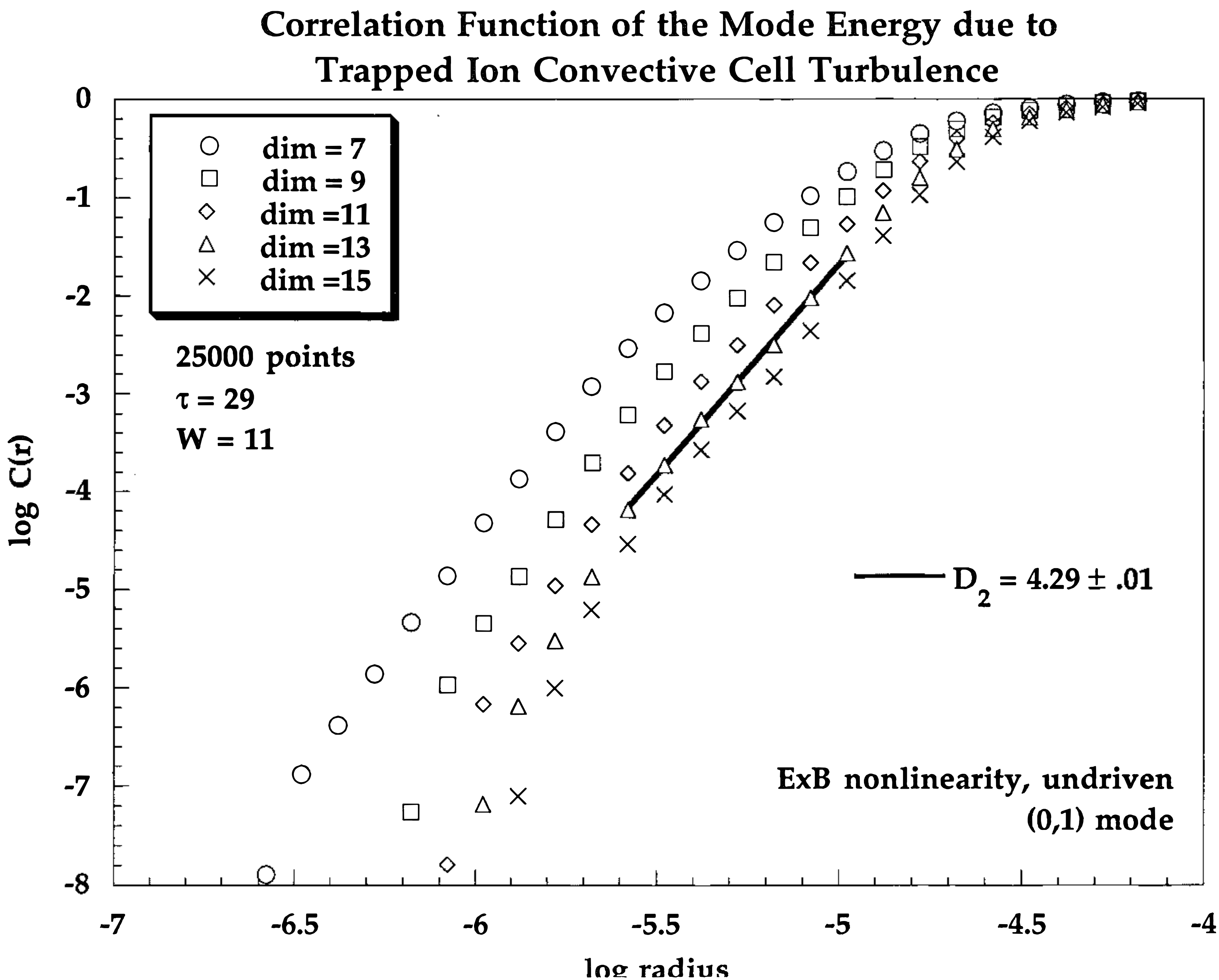
Case #1: Undriven, Undamped

In this case, the model had no driving or damping terms. Shown are the energies in the (0,1) and (0,3) modes.

Estimates of the correlation dimensions for the two modes are ~ 4.3 and 5.9, respectively.

An estimate of the largest Lyapunov exponent for the (0,1) mode is 0.28 ± 0.05 bits/time unit. Significant is only the fact that this number is positive with a small error.

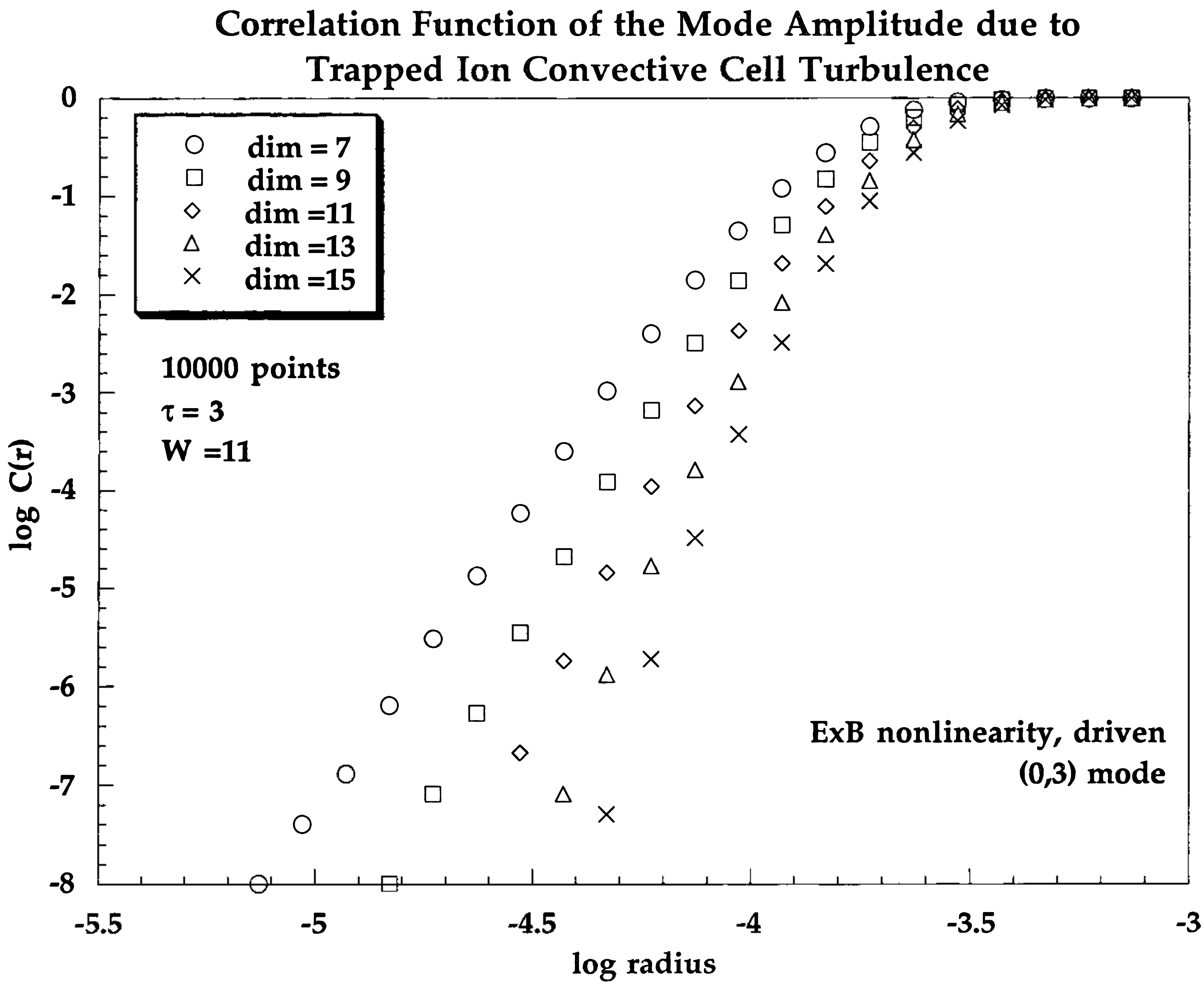

Correlation Function of the Mode Energy due to
Trapped Ion Convective Cell Turbulence
dim = 7
dim = 9
dim =11
dim =13
dim =15
25000 points
τ = 41
W = 11
D2 = 5.94 ± .01
ExB nonlinearity, undriven
(0,3) mode
log C(r)
log radius
0
-2
-4
-6
-8
-10
-6.5
-6
-5.5
-5
-4.5
-4


Correlation Function of the Mode Energy due to
Trapped Ion Convective Cell Turbulence
dim = 7
dim = 9
dim =11
dim =13
dim =15
25000 points
τ = 29
W = 11
D_2 = 4.29 ± .01
ExB nonlinearity, undriven
(0,1) mode
log C(r)
log radius
0
-1
-2
-3
-4
-5
-6
-7
-8
-7
-6.5
-6
-5.5
-5
-4.5
-4

Case #2: Driven, Damped

In this case, the model included a non-zero driving term and viscous damping. Shown is the amplitude of the (0,3) mode.

No Saturation (constant slope) region is evident in the log C(r) - log r plot, indicating that the correlation dimension must be greater than 7.


Correlation Function of the Mode Amplitude due to Trapped Ion Convective Cell Turbulence
dim = 7
dim = 9
dim =11
dim =13
dim =15
10000 points
τ = 3
W =11
ExB nonlinearity, driven (0,3) mode
log C(r)
log radius
0
-1
-2
-3
-4
-5
-6
-7
-8
-5.5
-5
-4.5
-4
-3.5
-3

Simulation of an RFP:

A Reversed Field Pinch plasma was simulated using the DEBS code. Current is kept constant while magnetic and electric fields are allowed to fluctuate. The simulation was run with the parameters listed to the right.

For more detail concerning the simulation, see Elizabeth Zita, 4Q26.

The time series used spans ~ 450 Alfvén times. The data record is far too short to draw any firm conclusions from. However, the following tentative statements can be made:

a) The "burstiness" in the signal may be indicative of intermittency.

b) A flattening of the slope is seen spanning about half a decade corresponding to a correlation dimension ~ 2.2.

Results are **highly** speculative. While the evidence suggests potential low dimensional chaos, a much longer time record (on the order 5x) is needed to confirm this.

The Resistive MHD equations reduce to:

$$d\mathbf{A}/dt = S\,\mathbf{v}\mathrm{x}\mathbf{B} - \eta\mathbf{J}$$

$$\rho\, d/dt = -S\rho\, \mathbf{v}.\nabla\mathbf{v} + S\,\mathrm{x} + \upsilon\nabla^2$$

$$\mathbf{B} = \nabla\mathrm{x}A,\quad \mathbf{E} = -d\mathbf{A}/dt,\quad \mathbf{J} = \nabla\mathrm{x}\mathbf{B},\quad S = v_A\tau_R/a$$

normalized with $v_A = B_0/\sqrt{4\pi\eta_0},\ \tau_R = 4\pi a^2/c^2\eta_0$

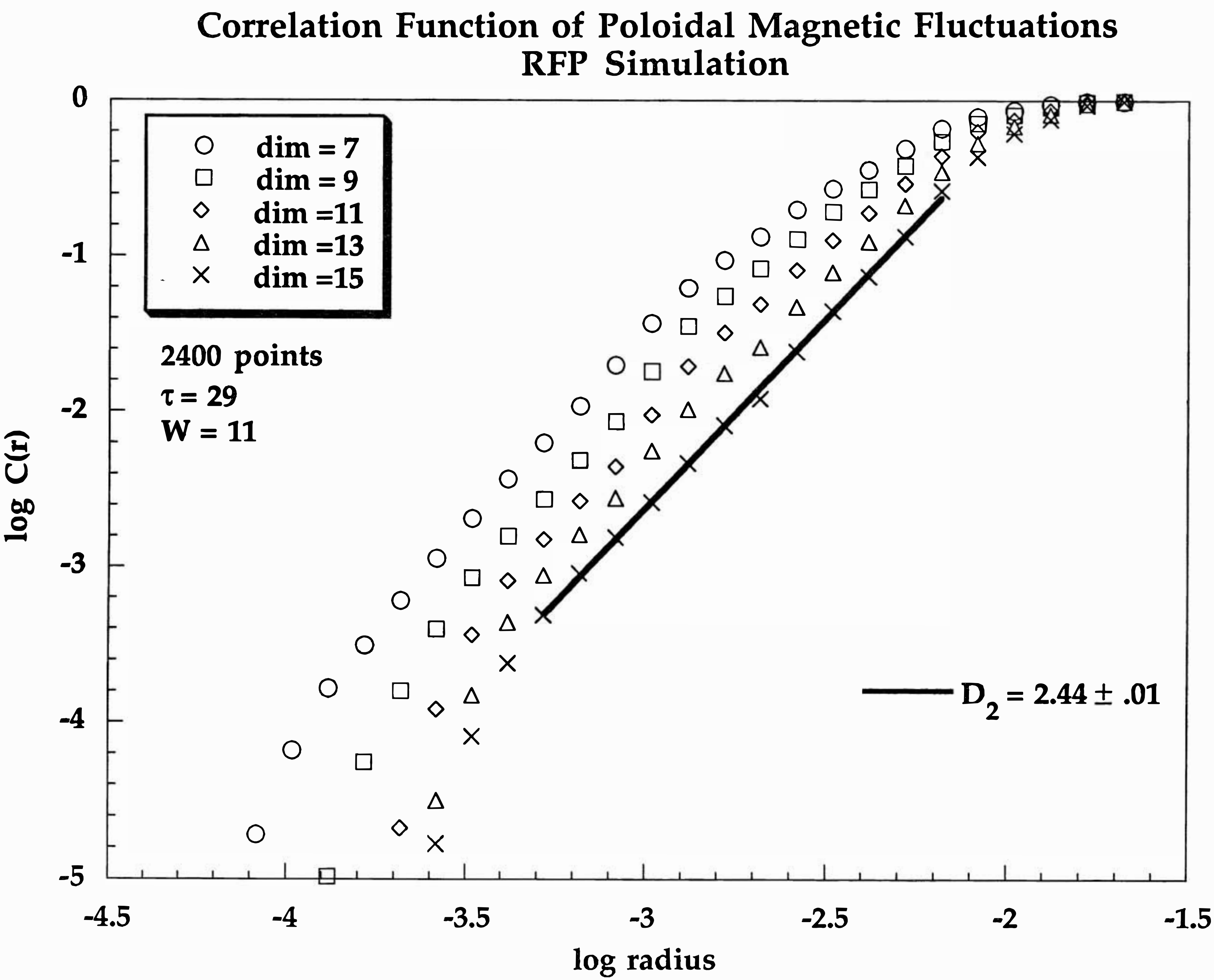
Assume fields of form $\mathbf{B} = B(r)\, e^{\omega t - i(m\theta + nkz)}$

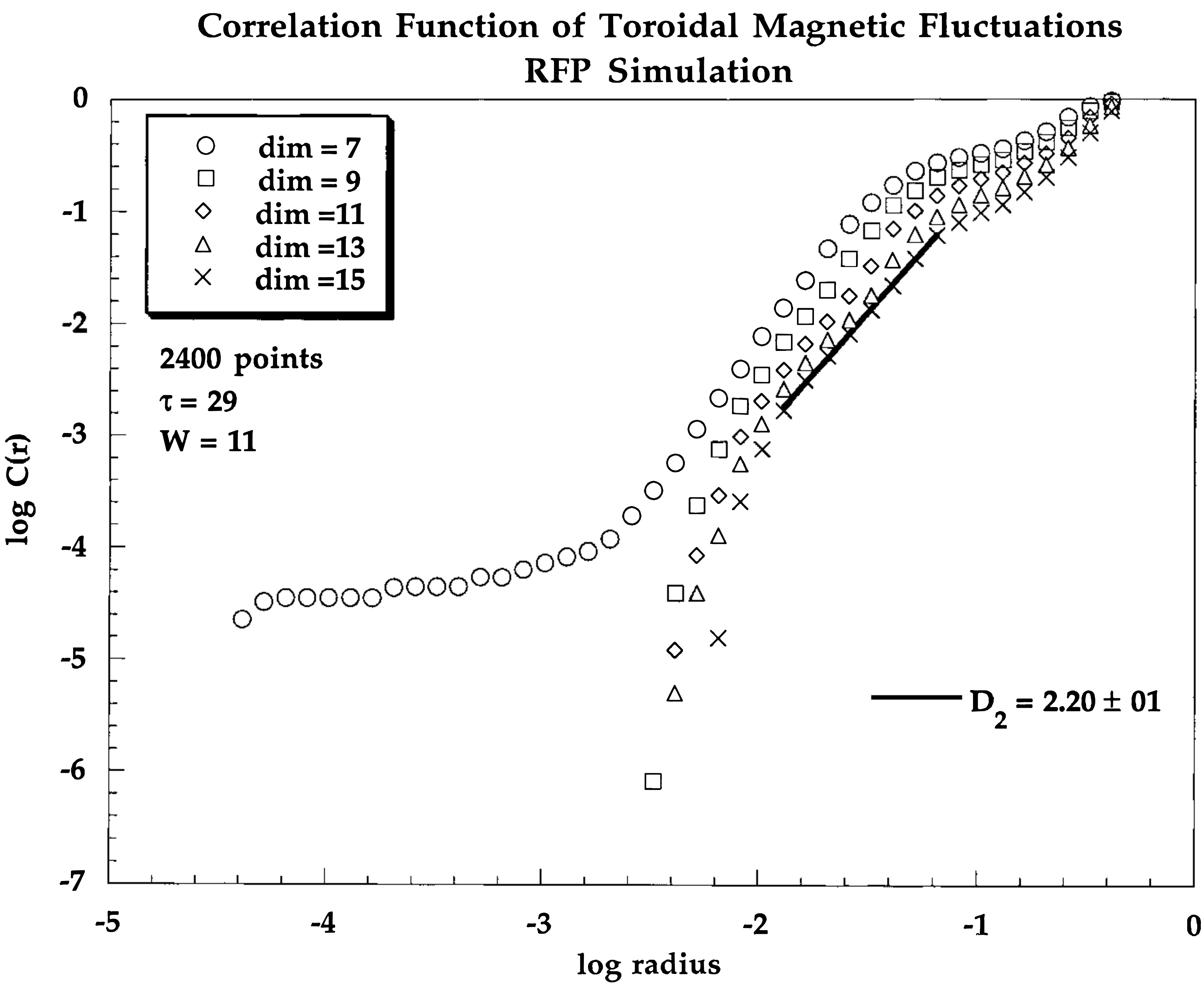
Parameters:

Magnetic Reynolds number $S=6\mathrm{x}10^3$

constant pinch parameter $\Theta=1.59$

also constant: toroidal flux, resistivity, pressure


Correlation Function of Poloidal Magnetic Fluctuations
RFP Simulation
dim = 7
dim = 9
dim =11
dim =13
dim =15
2400 points
τ = 29
W = 11
D2 = 2.44 ± .01
log C(r)
log radius
0
-1
-2
-3
-4
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5



Correlation Function of Toroidal Magnetic Fluctuations
RFP Simulation
dim = 7
dim = 9
dim =11
dim =13
dim =15
2400 points
τ = 29
W = 11
$D_2 = 2.20 \pm 01$
log C(r)
log radius
0
-1
-2
-3
-4
-5
-6
-7
-5
-4
-3
-2
-1
0

Conclusions:

a) There is strong evidence of low dimensional chaos in the undriven case of trapped ion mode turbulence. Various modes evince correlation dimensions ranging from ~ 4.3 to 5.9.

b) The driven-damped case shows no evidence of chaos below a dimension of 8.

c) The RFP simulation does show *tentative* evidence indicating low dimensional chaos, however further investigation with more data is necessary before any firm conclusions can be drawn.

Future Work:

a) Continue studies of trapped ion mode turbulence to determine whether or not the size of the k space influences the correlation dimension.

b) Examine the 3 different regimes of trapped ion mode turbulence - transient, steady state and sloshing - to determine any effect on dimension.

c) Continue generating the time series for the RFP case to better establish whether or not low dimensional chaos exists.

d) Using the equations of the DEBS code, analytically determine the spectrum of Lyapunov exponents.