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Artificial Intelligence Study of the System JCS-08-13-2022

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The subject system of this paper is of no particular interest or importance. Rather the intent is to illustrate how artificial intelligence (AI) might someday find new dynamical systems with special properties, analyze their behavior, and prepare a publication reporting the results. In fact, this entire paper was written automatically without human intervention using a template and computer program developed by the author.

Keywords: Chaos; attractor; artificial intelligence.

1. Introduction

It is only a matter of time before most of the tasks currently performed by human researchers such as identifying interesting dynamical systems, analyzing their behavior, and preparing a paper for publication are relegated to intelligent machines. This paper describes and illustrates a tiny step in that direction.

Using software developed by the author over the past several decades [Sprott, 1993], it is possible to search a system of any number of dynamical equations with any number of parameters subject to any desired constraints for parameter values that give chaotic solutions and to simplify the resulting system by eliminating nonessential terms and setting as many of the remaining parameters to ±1 as possible, giving a system that might be considered "elegant." A companion program performs standard dynamical analyses of the resulting system and produces publication-quality figures.

The remaining step, illustrated in this paper by a particular example, is to collect the results of the analysis and the accompanying figures into a LaTeX template file, ready for publication without human intervention. The template is a modified version of the one provided by IJBC, although it could be easily adapted to any journal that accepts LaTeX manuscripts. Dozens of such papers could be quickly produced, although most would be highly repetitious and not worthy of publication. Perhaps an AI pre-review could be used to select those appropriate for submission!

To illustrate the idea, consider the dynamical system given by

$$\begin{aligned} \dot{x} &= a_1 x + a_2 y + a_3 z, \\ \dot{y} &= a_4 xy + a_5 xz + a_6 yz, \\ \dot{z} &= a_7 x^3 + a_8 y^3 + a_9 z^3, \end{aligned} \tag{1}$$

chosen by the author so as to avoid degeneracies when interchanging variables and sufficiently complicated to ensure a variety of new chaotic examples since most 3D autonomous systems with only quadratic nonlinearities have been previously identified [Sprott, 2010].

The program randomly tests millions of values for the parameters a_1 through a_9 and finds bounded cases where the largest Lyapunov exponent is greater than 0.001 and the sum of the Lyapunov exponents is negative. It then simplifies the parameters by setting as many of them to zero or ±1 as possible, although there are sometimes extraneous parameters in the interest of keeping their numerical values simple (few digits). The resulting system is then automatically analyzed by standard methods [Sprott, 2003] and described in this paper.

The preceding text along with the Conclusions and References are from the LaTeX template file that is easily edited. The body of the paper in the following sections, the Acknowledgments, and the figures were generated by the computer and will generally differ for each execution of the program.

2. New Chaotic System

After about 5×10^6 trials, of which 48 were chaotic, simplified parameters for Eq. (1) that give chaotic solutions are $a_1 = 0$, $a_2 = 0$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $a_6 = 0$, $a_7 = -0.1$, $a_8 = 1$, $a_9 = 0$. Thus Eq. (1) can be written more compactly as

$$\dot{x} = z, \quad \dot{y} = xy + xz, \quad \dot{z} = ax^3 + y^3, \qquad (2)$$

where $a = -0.1$.

Since it is easy to produce many such systems in this way, Eq. (2) will be labeled JCS-08-13-2022 where "JCS" is the author's initials followed by the date it was produced. It is probably a good idea to throttle the output of the AI program to one paper per day! Also it takes about a day for the program to do a thorough search, simplify the system, analyze its properties, and produce the figures. The program has been optimized for accuracy and quality rather than speed.

Note that with five terms, Eq. (2) should have one independent parameter through a linear rescaling of the three variables plus time, and so the dynamics is completely captured by the single parameter a, which could be put in any of the five terms, albeit with a different numerical value.

3. Equilibria

The system in Eq. (2) with $a = -0.1$ has a neutrally stable nonhyperbolic equilibrium at $(0, 0, 0)$ with eigenvalues $(0, 0, 0)$ and a Poincaré index of 0 in the $z = 0$ plane, and the attractor is self-excited. Despite the neutral linear stability, the equilibrium is nonlinearly unstable. The system has no symmetry.

4. Attractor

Figure 1 shows various views of the attractor for Eq. (2) with $a = -0.1$ and initial conditions

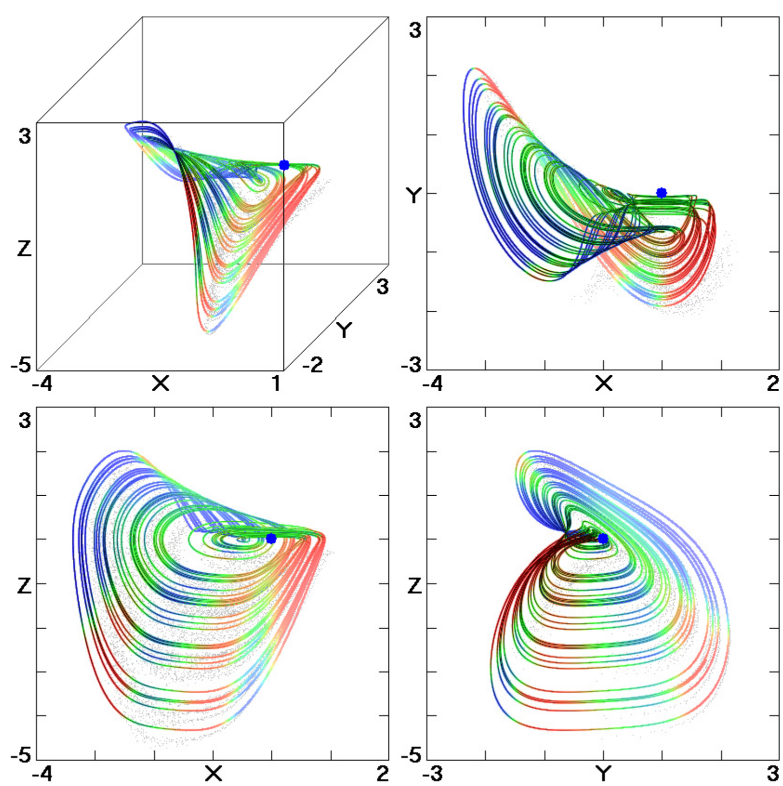

Fig. 1. Views of the attractor for Eq. (2) with $a = -0.1$ and initial conditions $(-1, 0, 0)$.

$(-1, 0, 0)$. The rainbow of colors shows the local value of the largest Lyapunov exponent with red indicating the most positive values (regions of worst predictability) and blue indicating the most negative values (regions of best predictability).

5. Time Series

Figure 2 shows the time series for the three variables along with the local value of the largest Lyapunov exponent (LL) for Eq. (2) with $a = -0.1$. Red color in the Lyapunov exponent indicates that the error is growing parallel to the orbit, while blue indicates growth perpendicular to the orbit. Note that the orbit passes through regions where the local Lyapunov exponent is strongly positive and other regions where it is strongly negative as is typical for a chaotic system and is also reflected by the colors in Fig. 1.

6. Lyapunov Exponents

The global Lyapunov exponents are determined by averaging the local Lyapunov exponents along the orbit. The values typically converge very slowly because of the large variation along the orbit, and an integration time of order 10^8 is required to obtain 4-digit accuracy.

The results of such a calculation for the system in Eq. (2) with $a = -0.1$ after a time of 2×10^6 are LE $= (0.0249, 0, -0.4182)$ with a Kaplan–Yorke dimension of 2.0595, where the last digit in the quoted values is only an approximation. The positive value of the largest Lyapunov exponent indicates that the system is chaotic, and the negative sum of the exponents (-0.3933) indicates that the system is dissipative with a strange attractor.

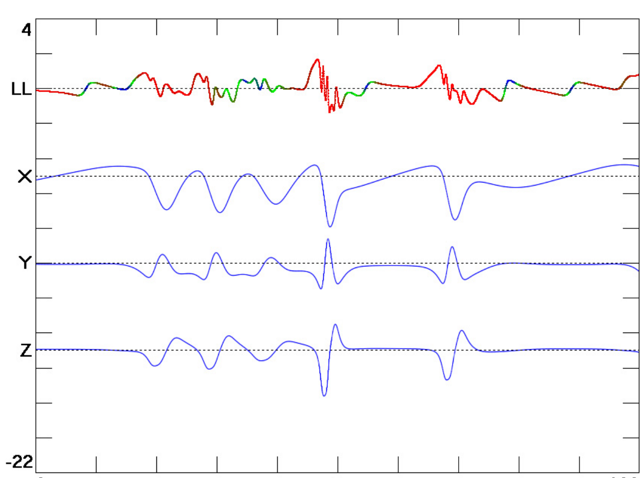

Fig. 2. Time series for the variables in Eq. (2) with $a = -0.1$ along with the local Lyapunov exponent (LL).

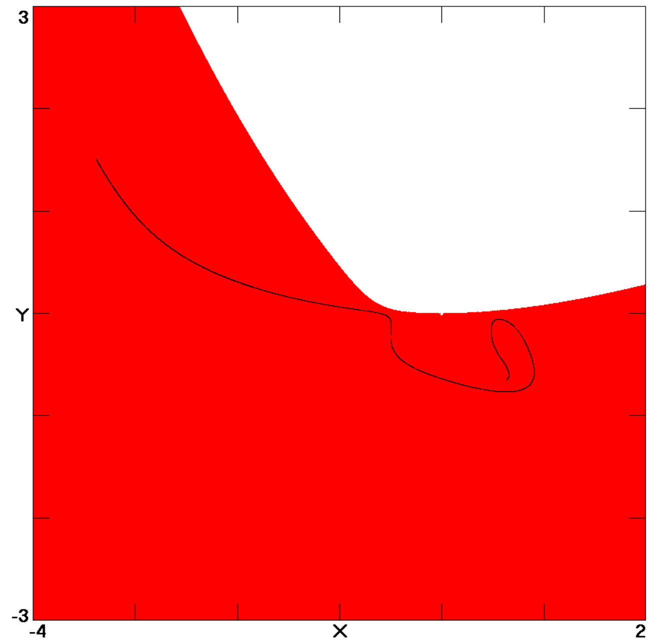

Fig. 3. Basin of attraction for Eq. (2) with $a = -0.1$ in the $z = 0$ plane.

7. Basin of Attraction

Figure 3 shows (in red) the basin of attraction for Eq. (2) with $a = -0.1$ in the $z = 0$ plane. Also shown (in black) is the cross-section of the attractor in the same plane.

8. Bifurcations

Figure 4 shows the bifurcation diagram for Eq. (2) as a function of the parameter a from $a = -0.2$ to $a = 0$. The initial condition was taken as $(-1, 0, 0)$ at $a = -0.2$ and was not changed as a slowly varied toward $a = 0$. Each of the 500 values of a was calculated for a time of 1×10^4.

The upper plot shows the three Lyapunov exponents. The middle plot shows the Kaplan–Yorke dimension, and the lower plot shows the local maxima of x. The chaotic region is in the vicinity of $a = -0.1$, and the route to chaos is clearly shown.

9. Robustness

One measure of the robustness of a chaotic system is the amount by which the parameters can be changed from their nominal values before the probability of chaos decreases to 50% [Sprott, 2022]. For

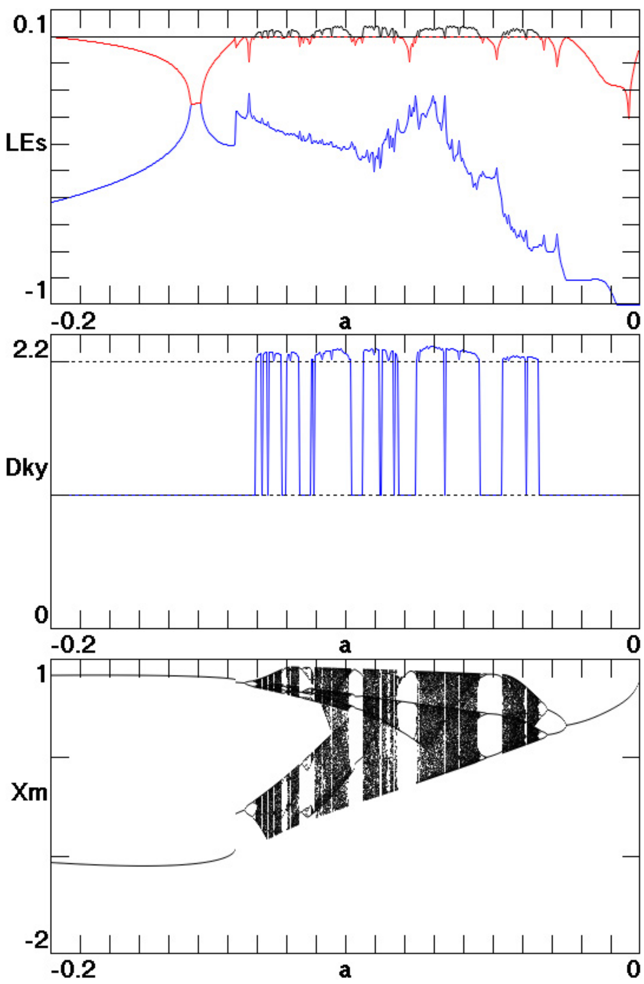

Fig. 4. Bifurcation diagram for Eq. (2) as a function of the parameter a.

the system in Eq. (2) with $a = -0.1$ and initial conditions $(-1, 0, 0)$, after 5001 trials, it is estimated that the parameter a can be changed by 48% before the chaos is more likely to be lost than not. Thus the system is somewhat robust. This result is consistent with the data in Fig. 4.

10. Conclusions

This paper provides a whimsical glimpse of one possible future in which artificial intelligence supplants most of the dynamical systems research currently done by humans. It represents a proof of principle. However, the program does not exhibit any meaningful intelligence but only carries out a series of tasks that it was programmed to do.

It also provides an example of a poorly motivated paper of the kind that is still regularly being submitted for publication [Sprott, 2011]. In particular, the only virtue of Eq. (2) is that it has never been studied before, and all the figures and results are presented without explanation or discussion.

Had this system been of general interest, it would be relatively easy to alter the Introduction, Conclusions, and References to reflect that fact, either by editing the template file or by editing the LaTeX file generated by the program. While it took vastly more effort to write the program that wrote this paper than just to write it directly, a tool is now available to facilitate further exploration and publications.

Please do not ask for a copy of the program that produced this paper. It is still under development and is not publicly available and probably never will be. For now there is no need to encourage additional poorly written papers describing new chaotic systems that do not advance our understanding of nonlinear dynamics.

Acknowledgments

The computation time required to produce this paper was 21:10:37 using the PowerBASIC Console Compiler running under Windows 10 on an Intel(R) Core(TM) i7-3930K CPU @ 3.20 GHz. The local Chaos and Complex Systems discussion group provided useful comments and suggestions.

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