Chaos

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Cite as: Chaos **30**, 063144 (2020); https://doi.org/10.1063/5.0007668
Submitted: 17 March 2020 . Accepted: 03 June 2020 . Published Online: 22 June 2020

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Chaos ARTICLE scitation.org/journal/cha

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Chunbiao Li,[1,2,a)] Jiayu Sun,[1,2] Tianai Lu,[1,2] Julien Clinton Sprott,[3] and Zuohua Liu[4]

AFFILIATIONS

[1]Jiangsu Collaborative Innovation Center of Atmospheric Environment and Equipment Technology (CICAEET), Nanjing University of Information Science and Technology, Nanjing 210044, China
[2]Jiangsu Key Laboratory of Meteorological Observation and Information Processing, Nanjing University of Information Science and Technology, Nanjing 210044, China
[3]Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706, USA
[4]State Key Laboratory of Coal Mine Disaster Dynamics and Control, Chongqing University, Chongqing 400044, China

[a)]**Author to whom correspondence should be addressed:** goontry@126.com and chunbiaolee@nuist.edu.cn

ABSTRACT

There are complex chaotic manifolds in practical nonlinear dynamical systems, especially in nonlinear circuits and chemical engineering. Any system attractor has its own geometric and physical properties, such as granularity, orientation, and spatiotemporal distribution. Polarity balance plays an important role in the solution of a dynamical system including symmetrization, attractor merging, and attractor self-reproducing. The absolute value function and the signum function manage and control the polarity balance, strictly regulating the attractor distribution by switching the polarity balances. Attractor self-reproducing is an attractive regime for constructing the desired multistability, where the coexisting attractors at different positions can be extracted by a selected initial value. Polarity balance is the key factor for attractor self-reproducing, where the offset boosting of an attractor needs an available polarity controller to restore the imbalanced polarity.

Three types of polarity controllers can be applied for restoring the polarity balance in a dynamical system, namely, the variable inversion, the polarity controller from the signum function and absolute value operation, and the polarity converter from a general absolute value function. Offset boosting in any dimension may disrupt the polarity balance in the attractor self-reproducing process. A new polarity balance can be restored by all types of polarity controllers; in particular, the polarity imbalance raised by the polarity converter can be restored by itself.

I. INTRODUCTION

In a multistable dynamical system, the types of those coexisting attractors may be the same or different and even the number of those coexisting attractors can be greater or less. Two[1,2] or three[3] different types of coexisting attractors and infinite uncountable different attractors[4–8] or countable similar attractors[9–15] can be found. Coexisting attractors reside in their own basins of attraction in phase space.[16–18] The basin boundary can have a fractal structure[16,17] or smoothly separated zones.[18] In self-reproducing systems, the attractors shrink their original basins and reside in a limited zone of attraction, saving space for other coexisting replicas.[18] In this case, the original attractor reproduces itself and arranges the replica in the phase space by offset boosting.[18–20] In this process, the polarity of the replica may change in one[21] or more dimensions.[21] In particular, the reproduced attractors may be exactly the same without polarity revision in any direction or obtain polarity reverse in some directions. Thus, to reproduce the inherent attractors in a system, it is important to keep the polarity balance of the system structure. If the function for self-reproducing revises the polarity of a variable, it calls a return in other ways. These chaotic systems of conditional symmetry maintain a polarity balance by revising the polarity of some of the variables when a polarity break arises from the process of offset boosting.

Symmetric chaotic systems can have a symmetric pair of coexisting attractors[22–26] that maintain the polarity balance due to their special structure rather than offset boosting. Self-reproducing systems with an infinite number of attractors[27,28] easily obtain a polarity balance from offset boosting by a suitable choice of the period of space. Meanwhile, for doubling coexisting attractors,[11] a plug-in signum function is necessary for restoring the polarity broken by the offset boosting. Furthermore, the polarity balance can be retrieved by the absolute value function when the offset boosting brings extra polarity revision. In this paper, possible polarity revisions induced by offset boosting are discussed in Sec. II, and the polarity balance recovered from an internal polarity modification and from an extra signum function is discussed in Sec. III. The polarity balance recovered by an absolute value operation is demonstrated in Sec. IV. A discussion of polarity balance is included in the last section.

II. OFFSET-BOOSTING-INDUCED POLARITY REVISIONS

Any variable in a dynamical system includes two classes of information: amplitude and polarity. Correspondingly, there are two opposite polarity adapters: one is the signum function that maintains the polarity while the other is the absolute value function that removes the polarity, and, therefore, a system variable can be written as $x = |x|\mathrm{sgn}(x)$. Based on these two basic operations, those dynamical systems can be revised for amplitude control,[29] linearization,[30] or even self-reproducing.[11,18] Two typical opposite polarity adapters can be applied for keeping the polarity balance of a system structure when it produces two or more coexisting attractors with different orientations. For better demonstration, it is easy to see that symmetric systems can give symmetric pairs of coexisting attractors without any polarity adapter owing to its own polarity balance. It is the introducing of polarity adapter that transforms the asymmetric Rössler system to be symmetric ones.[31]

Therefore, any polarity-associated revision in a dynamical system should maintain the balance of polarity and the above two adapters make it possible. For constructing self-reproducing systems, the polarity restoring comes not only from the inner polarity operation but also from offset boosting. As shown in Fig. 1, two classes of polarity adapters revise the polarity directly while the other two typical operations from the trigonometric function and absolute value function change the polarity in the manner of a slide polarity converter, where the polarity reversal is caused by offset boosting. Different segments in the function give two opposite polarities, as shown in Fig. 2. Here, we take the four functions as follows: $f(x) = |x| - 5$, $g(x) = 5 - |x|$, $h(x) = \sin(x)$, and $r(x) = \cos(x)$. Polarity switching (PS) can clearly be seen, as indicated by the positive slope and negative slope. In the corresponding regions of the function, it poses a positive sign or a negative sign into the system which brings different polarity interferences. In fact, it is found that the polarity adapter and slide polarity converter are widely applied in chaotic systems together to obtain conditional symmetry or attractor self-reproducing. The newly introduced polarity reverse from the offset boosting can be balanced by the signum function for attractor doubling.[31] In general, the polarity adapter and slide polarity converter can realize polarity compensation for each other in the process of attractor self-reproducing.

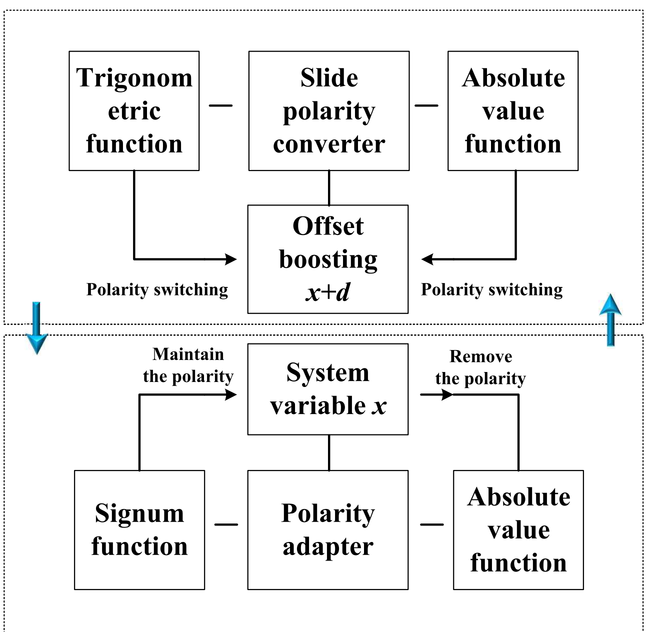

FIG. 1. Polarity balance in a dynamical system.

III. POLARITY BALANCE RECONSTRUCTION BY THE SLIDE POLARITY CONVERTER

The Rössler system has a unique structure for polarity revision,

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = b - cz + xz. \end{cases} \tag{1}$$

When $a = b = 0.2$ and $c = 5.7$, the system has a chaotic attractor with Lyapunov exponents (LEs) of (0.0714, 0, −5.3943) and a Kaplan–Yorke dimension of $D_{KY} = 2.0132$. System (1) has two equilibrium points, (0.0070, −0.0351, 0.0351) and (5.6930, −28.4649, 28.4649) with corresponding eigenvalues (−5.6870, $0.0970 \pm 0.9952i$) and (0.1930, $-0.000005 \pm 5.4280i$), which are spiral saddle points.[31] The application of a polarity adapter, namely, the absolute value function and signum function, revises the polarity balance freely, giving three regimes of symmetry: reflection symmetry, rotational symmetry, and inversion symmetry.[31]

Furthermore, such flexible polarity freedom leaves another possibility for hosting conditional symmetry, where the polarity adjustment induced by the slide polarity converter can be retrieved by internal polarity modification. Let us consider the following derived version of system (1),

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = b + xz - c|z|. \end{cases} \tag{2}$$

When $a = b = 0.2$ and $c = 5.7$, system (2) has a chaotic attractor with Lyapunov exponents (LEs) of (0.0711, 0, −5.3945) and a Kaplan–Yorke dimension of $D_{KY} = 2.0132$. The polarity distribution is revised by the polarity adapter of the absolute value function $|z|$, which produces a new possibility of polarity balance when a slide polarity converter is used. Suppose there is offset boosting in the z dimension realized by $F(z) = |z| - 15$,

$$\begin{cases} \dot{x} = -y - F_1(z), \\ \dot{y} = x + ay, \\ \dot{z} = b + xF_2(z) - c|F_3(z)|. \end{cases} \tag{3}$$

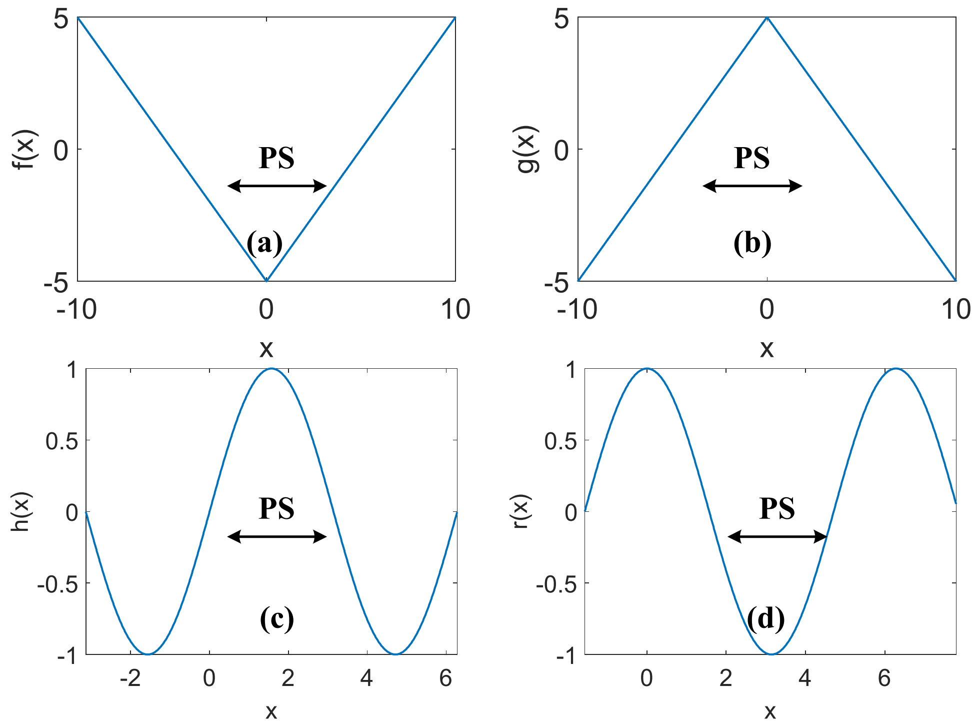

FIG. 2. Polarity switching (PS) in the slide polarity converter: (a) $f(x)$, (b) $g(x)$, (c) $h(x)$, and (d) $r(x)$.

System (3) provides conditional rotational symmetry according to the dimensions x and y, while the variable z returns the polarity balance from the slide polarity converter $F(z)$. As predicted, when $a = b = 0.2$, $c = 5.7$, and $F_1(z) = F_2(z) = F_3(z) = F(z) = |z| - 15$, system (3) exhibits coexisting chaotic oscillations of conditional symmetry with approximate Lyapunov exponents (LEs) of (0.0700, 0, −5.4009) and a Kaplan–Yorke dimension of $D_{KY} = 2.0130$, as shown in Fig. 3. Basins of attraction of the coexisting attractors are shown in Fig. 4. Note that unlike other chaotic cases of conditional symmetry, the basin of one attractor is included in another basin from a specific projection of $z = 0$.[21] Black lines are cross sections of the attractors, and white regions are unbounded orbits. The red attractor in Fig. 3 never intersects the $z = 0$ plane, and that is why it does not appear in black on Fig. 4.

The specific fractal asymmetric basins of attraction also indicate an unusual bifurcation. When the parameter b varies in [0.1, 0.4], system (3) undergoes two independent different bifurcations from two separate initial conditions, as shown in Fig. 5. Coexisting different chaotic attractors or other combinations of coexistence including chaos and limit cycles are identified from the asymmetric plots. This is mainly because that the polarity balance depends on the slide polarity converter and the reversal of two other variables as well. Furthermore, any other mismatch in the slide polarity converter leads to asymmetric coexistence. For $F_1(z) = |z| - 14$, $F_2(z) = F_3(z) = |z| - 15$, the coexisting asymmetric attractors are shown in Fig. 6. In this case, two attractors have different Lyapunov exponents and Kaplan–Yorke dimensions, which are $LE_1 = (0.0481, 0, -5.5382)$, $D_{KY1} = 2.0087$ and $LE_2 = (0.0059,$

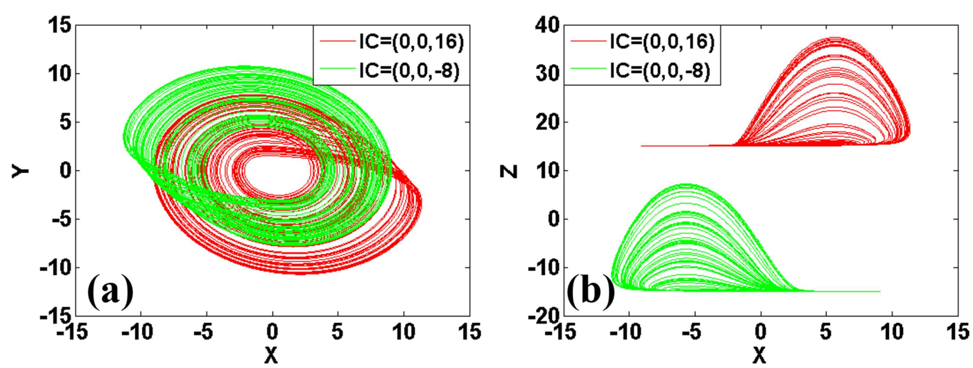

FIG. 3. Coexisting attractors of system (3) with conditional rotational symmetry when $a = b = 0.2$, $c = 5.7$, and $F_1(z) = F_2(z) = F_3(z) = |z| - 15$ in the: (a) x–y plane and (b) x–z plane.

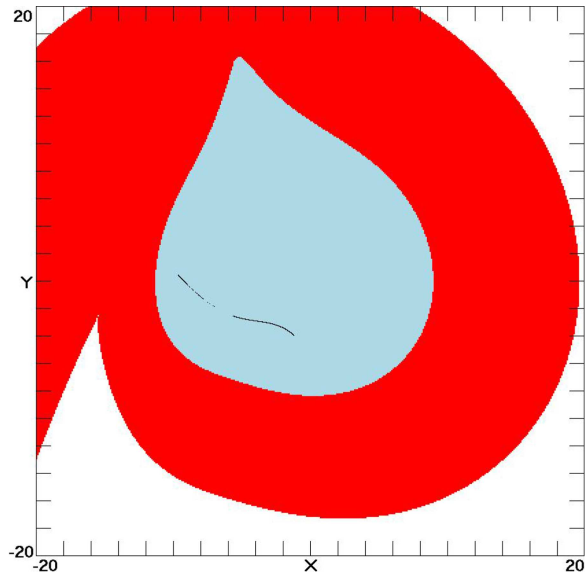

FIG. 4. Basins of attraction of the coexisting attractors on the x–y plane ($z = 0$) in system (3) with $a = b = 0.2$, $c = 5.7$, and $F_1(z) = F_2(z) = F_3(z) = |z| - 15$.

0, −5.1412), $D_{KY2} = 2.0011$ for the green and red attractors, respectively.

IV. POLARITY BALANCE RECONSTRUCTION BY JOINT MODIFICATION

A. Polarity balance from a slide polarity converter and a signum function

As demonstrated in Ref. 11, the change in the polarity induced by a slide polarity converter can be canceled by the extra signum function. By this means, Rössler system (1) can also be transformed to have different regimes of symmetry. Without loss of generality, we can transform system (3) into reflection symmetry according to the dimensions x or z, rotational symmetry according to x and y, and inversion symmetry, and the corresponding equations are Eqs. (4)–(7). Here, the introduced slide polarity converters are $f(x) = |x| - d_1$, $g(y) = |y| - d_2$, $h(z) = |z| - d_3$, $d_1 = 10$, $d_2 = 12$, and $d_3 = 0$. When $a = b = 0.2$ and $c = 5.7$, all of the attractors are doubled in their own patterns of symmetry in the derived systems, as shown in Fig. 7. Here, three different constants are chosen for separating doubled attractors. Since the original attractor stays in positive z-space, there is no requirement of offset boosting for attractor doubling. To compare different regimes of symmetry, basins of

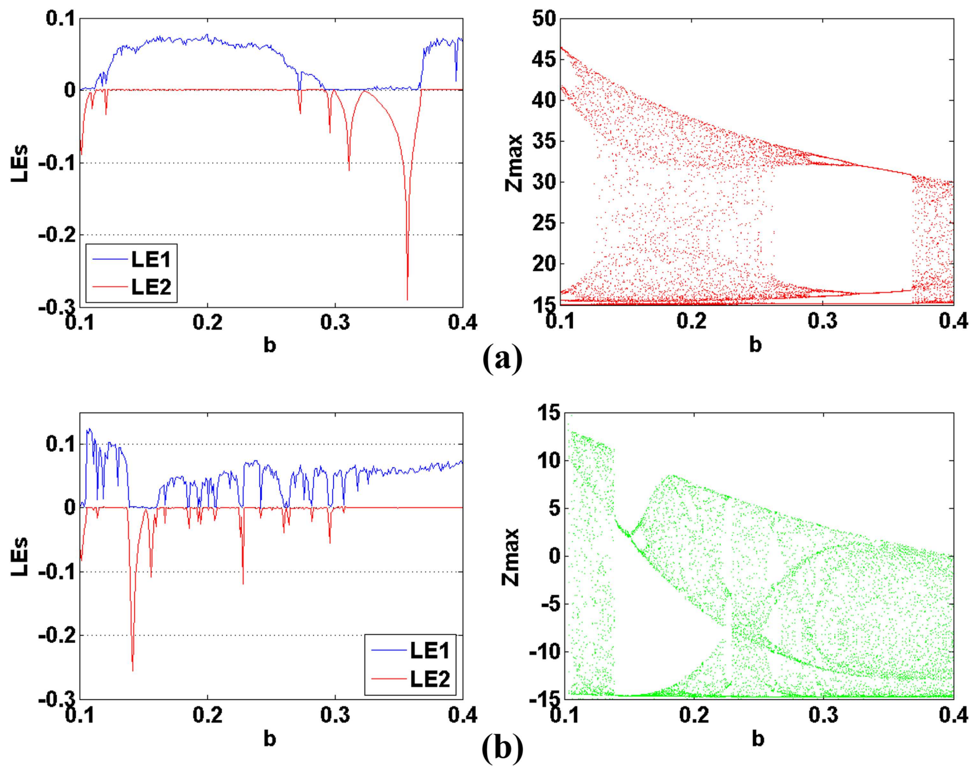

FIG. 5. Lyapunov exponents and bifurcation diagram of system (3) with $a = 0.2$, $c = 5.7$, and $F_1(z) = F_2(z) = F_3(z) = |z| - 15$, when b varies in [0.1, 0.4]: (a) IC = (0, 0, 16) and (b) IC = (0, 0, −8).

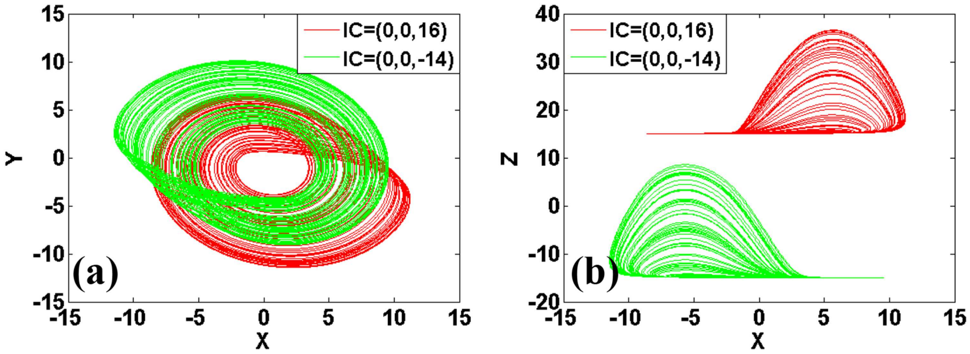

FIG. 6. Asymmetric coexisting attractors of system (3) with $a = b = 0.2$, $c = 5.7$, and $F_1(z) = |z| - 14$, $F_2(z) = F_3(z) = |z| - 15$ in the: (a) x–y plane and (b) x–z plane.

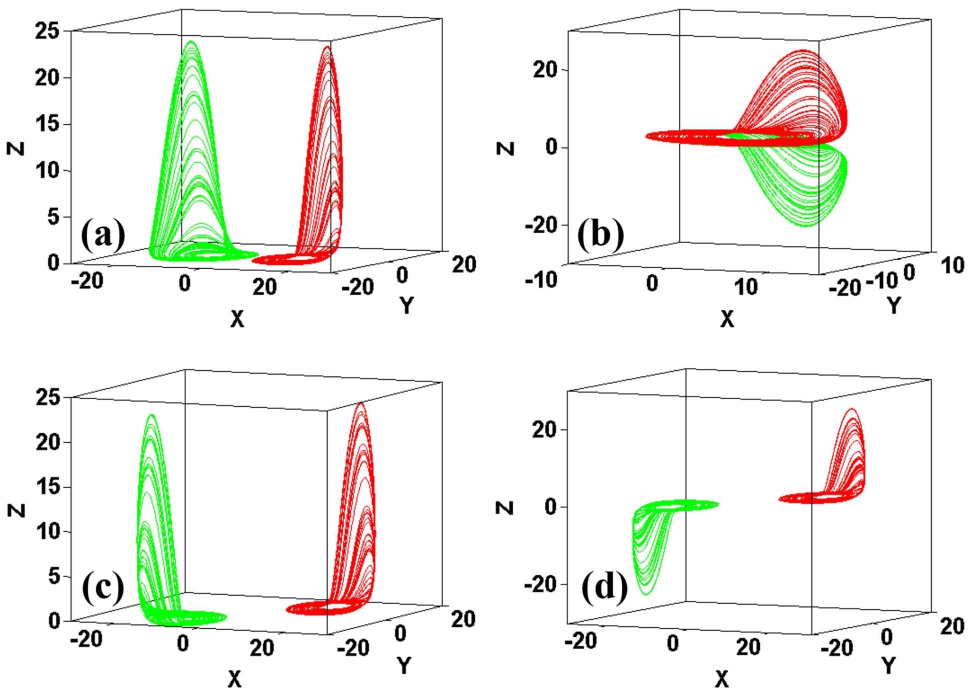

FIG. 7. Coexisting attractors in Eqs. (4)–(7) with $(a, b, c) = (0.2, 0.2, 5.7)$: (a) in Eq. (4), IC = (10.003, 0.02, 0.02) is red, and IC = (−10.003, 0.02, 0.02) is green; (b) in Eq. (5), IC = (0.003, 0.02, 0.02) is red, and IC = (0.003, 0.02, −0.02) is green; (c) in Eq. (6), IC = (10.003, 12.02, 0.02) is red, and IC = (−10.003, −12.02, 0.02) is green; and (d) in Eq. (7), IC = (10.003, 12.02, 0.02) is red, and IC = (−10.003, −12.02, −0.02) is green.

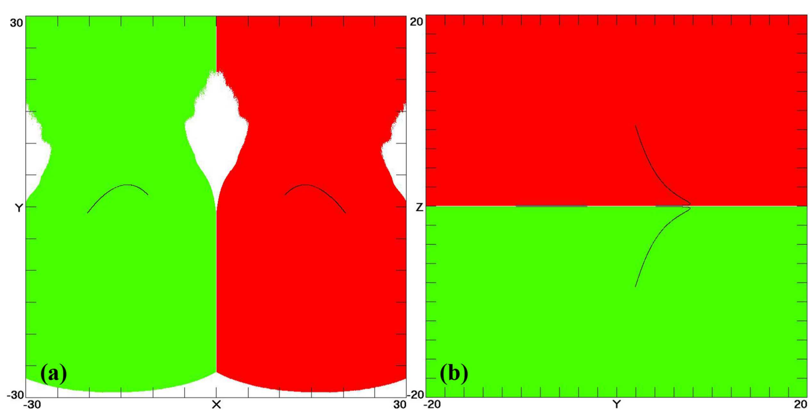

FIG. 8. Basins of attraction for the coexisting attractors: (a) the x–y plane ($z = 12$) for system (4) with $a = b = 0.2$, $c = 5.7$, and $f(x) = |x| - 10$ and (b) the y–z plane ($x = 0$) for system (5) with $a = b = 0.2$, $c = 5.7$, and $h(z) = |z|$.

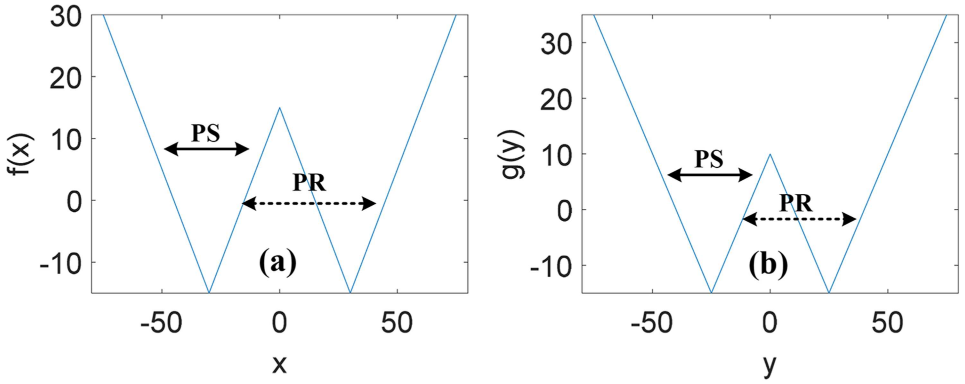

FIG. 9. Polarity switching and recovery in a nested slide polarity converter: (a) $f(x)$ and (b) $g(y)$.

attraction for systems (4) and (5) are shown in Fig. 8. All black lines are also cross sections of the attractors, and white regions are unbounded orbits. It is thus clear that the basins of attraction are consistent with the corresponding structure of symmetry and conditional symmetry. The symmetry nesting and composition can also be conducted according to the theory proposed in Ref. 11,

$$\begin{cases} \dot{x} = \mathrm{sgn}(x)(-y - z), \\ \dot{y} = f(x) + ay, \\ \dot{z} = b - cz + f(x)z, \end{cases} \tag{4}$$

$$\begin{cases} \dot{x} = -y - h(z), \\ \dot{y} = x + ay, \\ \dot{z} = \mathrm{sgn}(z)(b - ch(z) + xh(z)), \end{cases} \tag{5}$$

$$\begin{cases} \dot{x} = \mathrm{sgn}(x)(-g(y) - z), \\ \dot{y} = \mathrm{sgn}(y)(f(x) + ag(y)), \\ \dot{z} = b - cz + f(x)z, \end{cases} \tag{6}$$

$$\begin{cases} \dot{x} = \mathrm{sgn}(x)(-g(y) - h(z)), \\ \dot{y} = \mathrm{sgn}(y)(f(x) + ag(y)), \\ \dot{z} = \mathrm{sgn}(z)(b - ch(z) + f(x)h(z)). \end{cases} \tag{7}$$

B. Polarity balance from the mutually nested slide polarity converters

However, the polarity reversal produced by the slide polarity converter can be shielded by itself. Let us consider a slide polarity converter with the dimension x, $f(x) = |x| - a$, which raises the polarity switching (PS) and breaks the polarity balance, which in turn calls for extra internal or external polarity recovery (PR). Polarity recovery comes from three paths: original variable inversion, polarity adapter, or slide polarity converter. The first two cases have been demonstrated in the above sections, where the newly produced polarity imbalance from offset boosting recovers immediately. In fact, the creator of polarity imbalance can also be a solver. Considering the nested slide polarity converter, $f(x) = ||x| - 30| - 15$ and

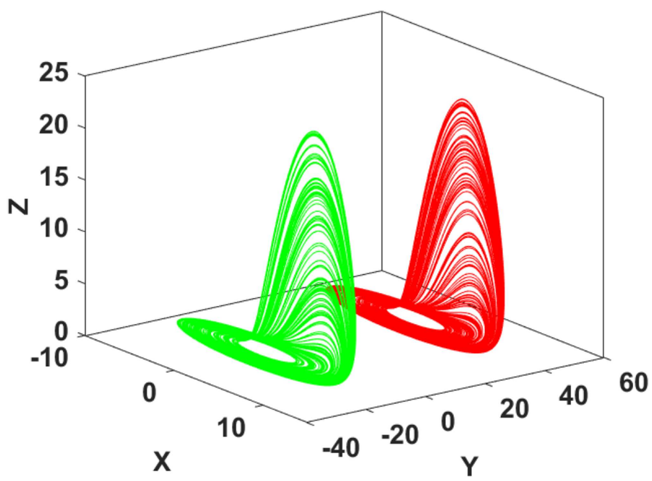

FIG. 10. Coexisting attractors in system (8) with $a = 0.2$, $b = 0.2$, $c = 5.7$, and $g(y) = ||y| - 25| - 15$, where IC = (0.003, 40.02, 0.02) is red and IC = (0.003, −9.98, 0.02) is green.

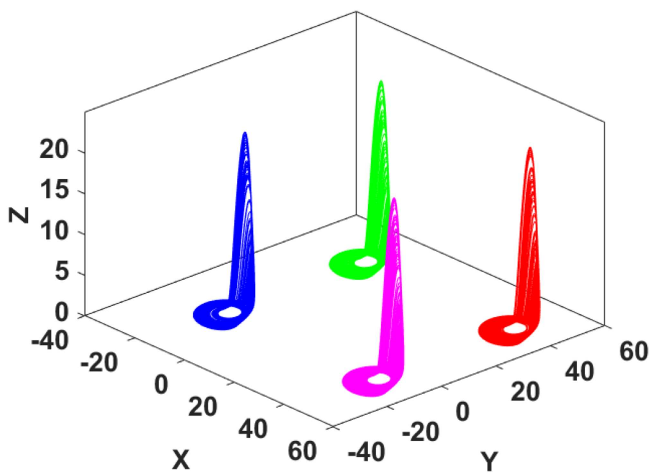

FIG. 11. Coexisting attractors for system (9) with $a = 0.2$, $b = 0.2$, $c = 5.7$, and $g(y) = ||y| - 25| - 15$, $f(x) = ||x| - 30| - 15$, where (45.003, 40.02, 0.02) is red, (−14.997, 40.02, 0.02) is green, (−14.997, −9.98, 0.02) is blue, and (45.003, −9.98, 0.02) is pink.

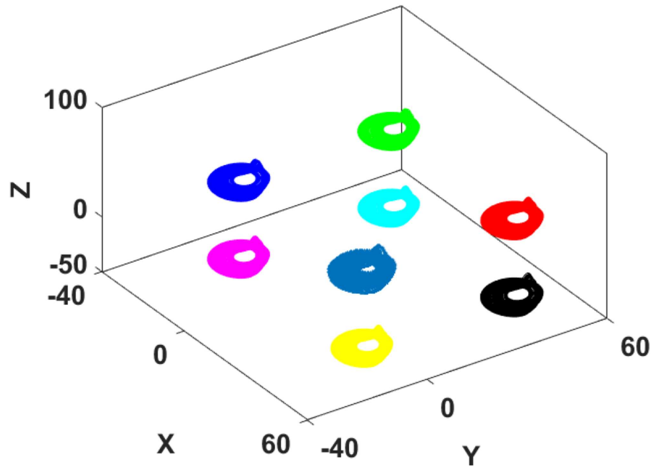

FIG. 12. Coexisting attractors in system (10) with $a = 0.2$, $b = 0.2$, $c = 5.7$, and $g(y) = ||y| - 25| - 15$, $f(x) = ||x| - 30| - 15$, and $h(z) = ||z| - 35| - 5$, where $(45.003, 40.02, 40.02)$ is red, $(-14.997, 40.02, -29.98)$ is cyan, $(-14.997, 40.02, 40.02)$ is green, $(45.003, -9.98, -29.98)$ is yellow, $(-14.997, -9.98, 40.02)$ is blue, $(45.003, 40.02, -29.98)$ is black, $(-14.997, -9.98, -29.98)$ is pink, and $(45.003, -9.98, 40.02)$ is azure blue.

$g(y) = ||y| - 25| - 15$, the process of polarity switching and recovery can be clearly seen in Fig. 9.

Generally speaking, a substitution is associated with all the same variables in an equation, while local substitution produces the substitution with a function on the right-hand side of an equation. Like the local substitution for producing conditional symmetry, in the following, local substitution can also be applied for producing coexisting attractors. Taking a local substitution with $f(x) = ||x| - 30| - 15$, $g(y) = ||y| - 25| - 15$, and $h(z) = ||z| - 35| - 5$, the offset-boosted coexisting attractors appear in systems (8)–(10) when new balances of polarity are established, as shown in Figs. 10–12,

$$\begin{cases} \dot{x} = -g(y) - z, \\ \dot{y} = x + ag(y), \\ \dot{z} = b - cz + xz, \end{cases} \tag{8}$$

$$\begin{cases} \dot{x} = -g(y) - z, \\ \dot{y} = f(x) + ag(y), \\ \dot{z} = b - cz + f(x)z, \end{cases} \tag{9}$$

$$\begin{cases} \dot{x} = -g(y) - h(z), \\ \dot{y} = f(x) + ag(y), \\ \dot{z} = b - ch(z) + f(x)h(z). \end{cases} \tag{10}$$

The basins of attraction for coexisting attractors in systems (9) and (10) are shown in Fig. 13. For system (10), only four of the eight attractors projected onto the x–y plane are visible. The other four are hidden behind those with the same x and y but different z. In Fig. 13(b), the basins for four of the eight attractors are plotted on the hemisphere given by $z = 5.02 + \sqrt{2750 - (x - 15.003)^2 - (y - 15.02)^2}$ using only the positive square root. The other hemisphere is similar. Black lines are those of cross sections of the captured attractors, and white regions mean unbounded orbits. As predicted, the strategy of polarity balance determines the structure of the basin of attraction. Comparing Figs. 4 and 13 with Fig. 8, it is true that the nested slide polarity converter results in an intertwined fractal structure rather than a simple folded structure. But the basin structure remains elegant symmetry, and therefore it does not destroy the coexisting doubled bifurcations. As shown in Fig. 14, for system (8) when $a = 0.2$, $c = 5.7$, $g(y) = ||y| - 25| - 15$, and b varies in $[0.1, 0.4]$, two identical bifurcations show up independently, which proves that any of other solutions in parameter space are exactly doubled by the nested slide polarity converter.

In fact, the nesting of the slide polarity converter can be introduced in any dimensions. As shown in Fig. 15, for system (8) when $g(y) = ||||y| - 70| - 22| - 25| - 15$, four coexisting attractors appear in dimension y. In some chaotic systems, the nesting of the slide polarity converter interacts with the internal polarity reversal, producing pairs of attractors with conditional symmetry. When $F_1(z) = F_2(z) = F_3(z) = F(z) = ||z| - 100| - 50$ are chosen in system (3), four coexisting attractors with conditional symmetry are produced, as shown in Fig. 16.

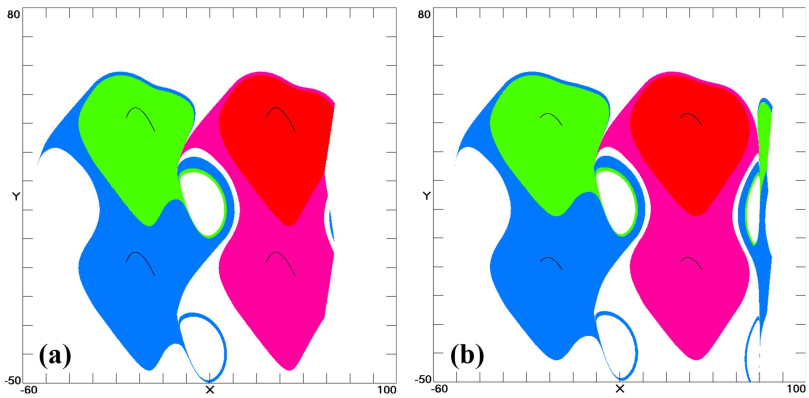

FIG. 13. Basins of attraction for the coexisting attractors on the x–y plane: (a) cross section $z = 5$ for system (8) with $a = b = 0.2$, $c = 5.7$, $f(x) = ||x| - 30| - 15$, and $g(y) = ||y| - 25| - 15$, (b) cross section $z = 5.02 + \sqrt{2750 - (x - 15.003)^2 - (y - 15.02)^2}$ for system (9) with $a = b = 0.2$, $c = 5.7$, $f(x) = ||x| - 30| - 15$, $g(y) = ||y| - 25| - 15$, and $h(z) = ||z| - 35| - 5$.

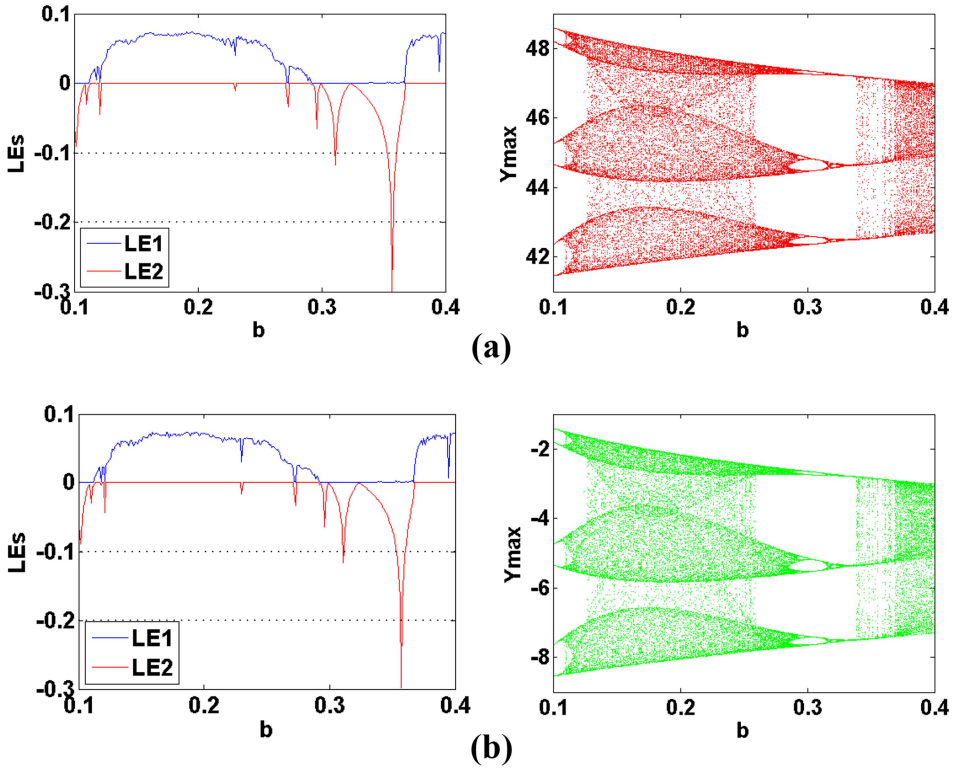

FIG. 14. Lyapunov exponents and bifurcation diagram of system (8) with $a = 0.2$, $c = 5.7$, and $g(y) = ||y| - 25| - 15$, when b varies in [0.1, 0.4]: (a) IC = (0.003, 40.02, 0.02) and (b) IC = (0.003, −9.98, 0.02).

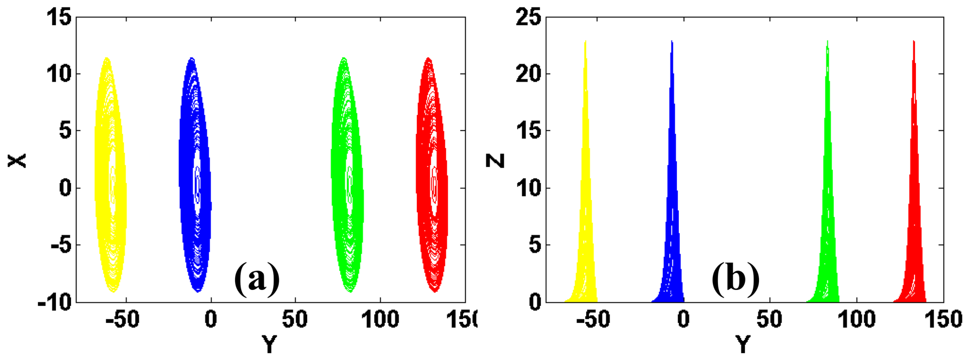

FIG. 15. Coexisting attractors of system (8) with $a = b = 0.2$, $c = 5.7$, and $g(y) = ||||y| - 70| - 22| - 25| - 15$, where (0.003, 132.02, 0.02) is red, (0.003, 82.02, 0.02) is green, (0.003, −7.98, 0.02) is blue, and (0.003, −57.98, 0.02) is yellow in the: (a) y–x plane and (b) y–z plane.

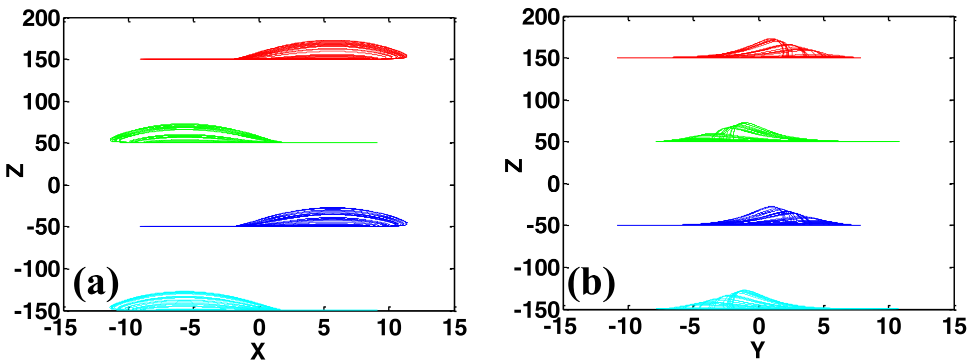

FIG. 16. Coexisting pairs of chaotic attractors with rotational conditional symmetry in system (3) $a = b = 0.2$, $c = 5.7$, and $F_1(z) = F_2(z) = F_3(z) = ||z| - 100| - 50$ in the: (a) x–z plane and (b) y–z plane.

V. CONCLUSIONS AND DISCUSSION

The chaotic Rössler system provides a unique structure for symmetry switching based on its flexible polarity balance. Internal polarity revision and extra polarity compensation (from a signum function) or shielding (by an absolute value function) provide two dominant approaches for balancing disrupted polarity. For a symmetrical structure, there is more flexibility for attractor doubling since the polarity balance is restored by the reversal of variable, which can further resort to the polarity revision or compensation instead. Therefore, the self-reproduced chaotic attractors are of conditional symmetry or become exactly the same under the polarity balance. Different approaches repeat the attractors, dividing the basins of attraction, which provides sufficient choices when an attractor with desired orientation is needed for chaos-based security communication. Also note that when any of the parameters change or the original model loses chaos, the polarity balance is still necessary for attractor self-reproduction. In this case, the coexisting reproduced attractors are periodic or fix points.

ACKNOWLEDGMENTS

This work was supported financially by the National Natural Science Foundation of China (Grant Nos. 61871230 and 51974045), the Natural Science Foundation of Jiangsu Province (Grant No. BK20181410), and a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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