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A New Category of Three-Dimensional Chaotic Flows with Identical Eigenvalues

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In this paper, some new three-dimensional chaotic systems are proposed. The special property of these autonomous systems is their identical eigenvalues. The systems are designed based on the general form of quadratic jerk systems with 10 terms, and some systems with stable equilibria. Using a systematic computer search, 12 simple chaotic systems with identical eigenvalues were found. We believe that systems with identical eigenvalues are described here for the first time. These simple systems are listed in this paper, and their dynamical properties are investigated.

Keywords: Identical eigenvalue; chaotic system; bifurcation diagram.

1. Introduction

The theory of nonlinear dynamics and chaotic systems has been an important topic for many years [Shao-Bo *et al.*, 2013; Kuznetsov *et al.*, 2018; Chen *et al.*, 2018]. The main relation between the governing equations of chaotic systems with their strange attractors is not completely known. For a long time, scientists thought that the existence of chaotic attractors is related to a saddle equilibrium point. Many famous chaotic systems such as the Lorenz system [Lorenz, 1963] and the Rössler system [Rössler, 1976] have this condition. However now there are many counterexamples for that belief [Jafari *et al.*, 2013]. Chaotic dynamics has many applications, such as image encryption [Cao *et al.*, 2018; Zhu & Sun, 2018]. Dynamical attractors have been categorized as self-excited and hidden [Leonov & Kuznetsov, 2013; Danca *et al.*, 2017; Wei *et al.*, 2014; Wei *et al.*, 2015c]. The basin of attraction for a self-excited attractor intersects with at least one unstable equilibrium point, while in the hidden attractors it does not [Wei *et al.*, 2015b; Danca *et al.*, 2019]. Complexity analysis of chaotic systems is an interesting topic [He *et al.*, 2018; Wei *et al.*, 2017; Danca, 2018].

Stability of equilibrium points of a dynamical system has been investigated using characteristic equations and eigenvalues which are calculated at

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each equilibrium point. Negative eigenvalues reveal the stable manifolds in a small neighborhood of an equilibrium point, while positive ones show the existence of unstable manifolds [Hilborn *et al.*, 2000; Chen & Dong, 1998]. Recently chaotic systems with special structural features have been proposed [Wei *et al.*, 2015a; Wei & Zhang, 2014; Wei *et al.*, 2015d]. It is now known that strange attractors can occur in flows without equilibrium points, with stable equilibria [Wei & Yang, 2011; Wang & Chen, 2012], and with an infinite number of equilibria [Wang & Chen, 2013; Jafari & Sprott, 2013; Jafari & Sprott, 2015]. Chaotic flows with new structural and topological properties are regularly proposed [Wang *et al.*, 2014; He *et al.*, 2016; Li *et al.*, 2017; He *et al.*, 2019b]. Chaotic systems have been a hot topic in the modeling of dynamical systems [Peng *et al.*, 2018; He *et al.*, 2019a; Danca & Fečkan, 2019]. Control of chaotic dynamics has been studied in [Chen *et al.*, 2019].

Many works have been completed on analyzing novel chaotic flows with special structural features. In this paper, we define a new category of chaotic systems with identical eigenvalues. There have been many studies of chaotic systems with different types of equilibria, but systems with identical eigenvalues have not been studied. Three general structures are designed with special conditions, and their chaotic attractors are described in this paper. In Sec. 2, the systems are designed, and their conditions are investigated. Based on the designed systems, 12 simple chaotic flows are proposed with identical eigenvalues. Dynamical properties of one of those systems are investigated. The paper is concluded in Sec. 3. These works help us to go one step ahead in investigating chaotic dynamics.

2. Proposed Systems

To design chaotic systems with identical eigenvalues, we consider three general structures as shown in Eqs. (1)–(3). The first structure is a general form of the quadratic jerk system with 10 terms. Jerk system is an equation of the form $\dddot{x} = f(\ddot{x}, \dot{x}, x)$. Calling the type of equations jerk is inspired by $\dddot{x}$ of the mechanical system which is called jerk where x is displacement [Sprott, 2010]. The second and third structures are general forms of systems with stable equilibria (SE15, SE19 and SE16 which were proposed in [Molaie *et al.*, 2013]). To simplify the calculations, we consider Eq. (2) in two special categories, $b_1 = -1$ and $b_2 = 1$, and $b_1 = 1$ and $b_2 = -1$. We have determined that the systems are not simply equivalent through some change of variables. In other words, all of the proposed systems have been checked to not simply be equivalent to other chaotic flows or with each other through some change of variables.

$$\begin{aligned}\dot{x} &= y,\\ \dot{y} &= z,\\ \dot{z} &= a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2\\ &\quad + a_7xy + a_8xz + a_9yz + a_{10},\end{aligned}\tag{1}$$

$$\begin{aligned}\dot{x} &= -z,\\ \dot{y} &= b_1x + b_2z,\\ \dot{z} &= a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2\\ &\quad + a_7xy + a_8xz + a_9yz + a_{10},\end{aligned}\tag{2}$$

$$\begin{aligned}\dot{x} &= z,\\ \dot{y} &= z - y,\\ \dot{z} &= a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2\\ &\quad + a_7xy + a_8xz + a_9yz + a_{10}.\end{aligned}\tag{3}$$

To make the three eigenvalues equal, it is necessary to apply appropriate constraints. First, we calculate the equilibrium points of the systems. For example, in the special case $b_1 = -1$ and $b_2 = 1$ of Eq. (2), they are computed as follows,

$$\begin{aligned}\dot{x} &= 0 \Rightarrow z = 0,\\ \dot{y} &= 0 \Rightarrow x = 0,\\ \dot{z} &= 0 \Rightarrow y = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_5a_{10}}}{2a_5}.\end{aligned}\tag{4}$$

Then, eigenvalues of the equilibrium points for the system are determined by setting the determinant of the matrix $\lambda I - J$ to zero, where J is the Jacobian matrix. This gives a cubic characteristic equation for each equilibrium point. Equation (4) shows that the system has two equilibrium points if $a_2^2 - 4a_5a_{10} \geq 0$ and $a_5 \neq 0$. The characteristic equation for the equilibrium point $(0, \frac{(-a_2+\sqrt{a_2^2-4a_5a_{10}})}{2a_5}, 0)$ is

$$\begin{aligned}&\lambda^3 - (a_3 - a_9m)\lambda^2 + (a_1 + a_7m - \sqrt{a_2^2 - 4a_5a_{10}})\lambda\\ &\quad - \sqrt{a_2^2 - 4a_2a_{10}} = 0,\end{aligned}\tag{5}$$

where $m = \frac{-a_2+\sqrt{a_2^2-4a_5a_{10}}}{2a_5}$. For the chaotic system to have identical eigenvalues, the three solutions of Eq. (5) should be equal. Thus we determine conditions such that Eq. (5) has three equal solutions as follows:

$$
\begin{aligned}
a_1 &= 3\left(\frac{a_3}{3} + a_9\frac{-a_2+\sqrt{a_2^2-4a_5a_{10}}}{6a_5}\right)^2 \\
&\quad + \sqrt{a_2^2-4a_5a_{10}} \\
&\quad - a_7\left(\frac{-a_2+\sqrt{a_2^2-4a_5a_{10}}}{2a_5}\right), \qquad (6)\\
a_3 &= -a_9\left(\frac{-a_2-\sqrt{a_2^2-4a_5a_{10}}}{2a_5}\right) \\
&\quad - 3(-\sqrt{a_2^2-4a_5a_{10}})^{\frac{1}{3}}.
\end{aligned}
$$

Similar conditions have been obtained for the other equilibrium point of Eq. (2) and for the other structures in Eqs. (1) and (3). The resulting equations were searched for chaotic solutions with the corresponding constraints. In summary, the first structure was quadratic jerk system since it is a well-known structure in proposing new chaotic flows. We can use this structure because the conditions of parameters [similar to Eq. (6)] have a well-form with one parameter as a function of other parameters. The second and third structures are selected with the help of the general structures of the systems with stable equilibrium points. In other words, we have tested many systems and three structures with a well-defined condition for parameters [such as Eq. (6)] selected to be searched for finding chaotic solutions. So, we have used an exhaustive computer search in each structure to find the parameters and initial conditions in which the system shows chaotic dynamics and also satisfy the special conditions for the identical eigenvalues. Twelve of the simplest examples found in this way are listed in Table 1 as IE_1–IE_{12}. All the cases have their equilibrium at the origin. All except IE_{10}, IE_{11} and IE_{12} which are more complex than the others, have three zero eigenvalues. This means that in the first nine cases, the eigenvalues do not determine the stability of equilibrium point. Cases 10–12 have three positive identical eigenvalues, which means that the equilibrium point is unstable with three symmetric unstable manifolds. Positive Lyapunov exponents signify chaotic solutions.

Projections of the strange attractors for Cases 1–12 in the xy-plane are shown in Fig. 1. It can be seen that the topologies of attractors are very different. In the interest of brevity, we focus on System IE_{10}. Case 10 is one of the interesting systems with three equal nonzero eigenvalues and its bifurcation diagram is interesting. A bifurcation diagram and plot of the largest Lyapunov exponent for IE_{10} in Fig. 2 shows a period-doubling route to chaos. The positive largest Lyapunov exponent shows chaos in the interval $b \in [-0.2, -0.008]$. Both plots in Fig. 2 use initial conditions with backward continuation, which means that initial conditions for each value of b are the same as the final conditions for the previous larger value of b. The first initial conditions at $b = 2$ are taken as $(17.82, -61.74, -82.95)$. The other parameters of the system are constant with the values in Table 1. Lyapunov exponents of the system are calculated using Wolf's method [Wolf *et al.*, 1985] with the runtime 20 000. The figure shows that the dynamic of the system is very sensitive to variations of parameter b in its smaller values. Kaplan–Yorke dimension (D_{KY}) is a complex degree and conjectures the dimension of strange attractor [Kaplan & Yorke, 1979]. D_{KY} of strange attractors of systems in Table 1 are calculated and they are shown in Table 2.

To investigate the dynamical properties of IE_{10} by changing initial values, the basin of attraction of the system in the xy-plane and $z_0 = -82.95$ is plotted in Fig. 3. In this figure, red color shows initial values which result in chaotic attractors, the blue region shows initial values which generate periodic attractors, and the white region depicts initial values which result in unbounded orbits. Basin of attraction determines that the attractor is self-excited or hidden. To investigate the chaotic attractor of 12 proposed systems, 1000 initial values around the equilibrium point at the origin are selected, and the final state of each initial condition is studied. If there is no initial condition which result in finding the chaotic attractor, then the chaotic attractor is called hidden. Otherwise, it is called self-excited. Table 3 shows the results of these investigations. Also, the stability of origin in nine cases with zero eigenvalues is investigated numerically by tracking the final state of the systems with

Table 1. Twelve three-dimensional chaotic systems with identical eigenvalues.

Case	Equations	Parameters	Equilibrium	Eigenvalue	LEs	(x_0, y_0, z_0)
IE_1	$\dot{x} = y$	$a = 0.78$	0	0	0.0070	−48.73
	$\dot{y} = z$		0	0	0	−30.86
	$\dot{z} = x^2 - y^2 + axz$		0	0	−18.3203	63.52
IE_2	$\dot{x} = -z$	$a = -4$	0	0	0.1042	−55.86
	$\dot{y} = x - z$	$b = -4$	0	0	0	0.68
	$\dot{z} = y^2 + axy + byz$		0	0	−60.2527	9.87
IE_3	$\dot{x} = -z$	$a = 0.1$	0	0	0.0118	−2.64
	$\dot{y} = x - z$	$b = -0.5$	0	0	0	0.91
	$\dot{z} = -x^2 + ay^2 + byz$		0	0	−0.7025	−4.14
IE_4	$\dot{x} = -z$	$a = 6$	0	0	0.0634	82
	$\dot{y} = x - z$	$b = 9$	0	0	0	−19.2
	$\dot{z} = -y^2 + axy + byz$		0	0	−309.2322	−9.54
IE_5	$\dot{x} = z$	$a = 0.3$	0	0	0.0887	−39.56
	$\dot{y} = z - y$	$b = -1.5$	0	0	0	−2.85
	$\dot{z} = -y + z + ax^2 + bxy + cxz$	$c = 0.6$	0	0	−28.7676	−41.22
IE_6	$\dot{x} = -z$	$a = -4$	0	0	0.0703	−0.41
	$\dot{y} = -x + z$	$b = -5$	0	0	0	−1.35
	$\dot{z} = -y^2 + axy + byz$		0	0	−51.5592	2
IE_7	$\dot{x} = -z$	$a = -2.1$	0	0	0.0369	−10.93
	$\dot{y} = -x + z$	$b = 0.23$	0	0	0	−2.63
	$\dot{z} = ax^2 + by^2 + cyz$	$c = 0.4$	0	0	−0.7322	24.04
IE_8	$\dot{x} = z$	$a = -0.4364$	0	0	0.1018	−3.87
	$\dot{y} = z - y$	$b = 2$	0	0	0	−0.7
	$\dot{z} = -y + z + ax^2 + bxy + cxz$	$c = -0.7229$	0	0	−28.1415	2.31
IE_9	$\dot{x} = y$	$a = -0.29$	0	0	0.0480	32.39
	$\dot{y} = z$	$b = 0.53$	0	0	0	−8.9
	$\dot{z} = ax^2 + by^2 + cxz$	$c = -0.4$	0	0	−16.8649	24.66
IE_{10}	$\dot{x} = -z$	$a = 0.297$	0	0.3	0.0321	17.82
	$\dot{y} = -x + z$	$b = 0.027$	0	0.3	0	−61.74
	$\dot{z} = ax + by + cz - x^2 + dy^2 + zy$	$c = 0.9$	0	0.3	−1.5423	−82.95
		$d = 0.3$				
IE_{11}	$\dot{x} = -z$	$a = 0.31$	0	0.1	0.0385	−32.71
	$\dot{y} = -x + z$	$b = 0.001$	0	0.1	0	−0.21
	$\dot{z} = ax + by + cz + dx^2 + ey^2 + fzy$	$c = 0.3$	0	0.1	−1.0039	−8.04
		$d = -0.5$				
		$e = 0.05$				
		$f = 0.4$				
IE_{12}	$\dot{x} = -z$	$a = 0.128$	0	0.2	0.0280	−21.36
	$\dot{y} = -x + z$	$b = 0.008$	0	0.2	0	−18.43
	$\dot{z} = ax + by + cz + dx^2 + ey^2 + fzy$	$c = 0.6$	0	0.2	−1.5524	−11.03
		$d = -0.16$				
		$e = 0.01$				
		$f = 0.1$				

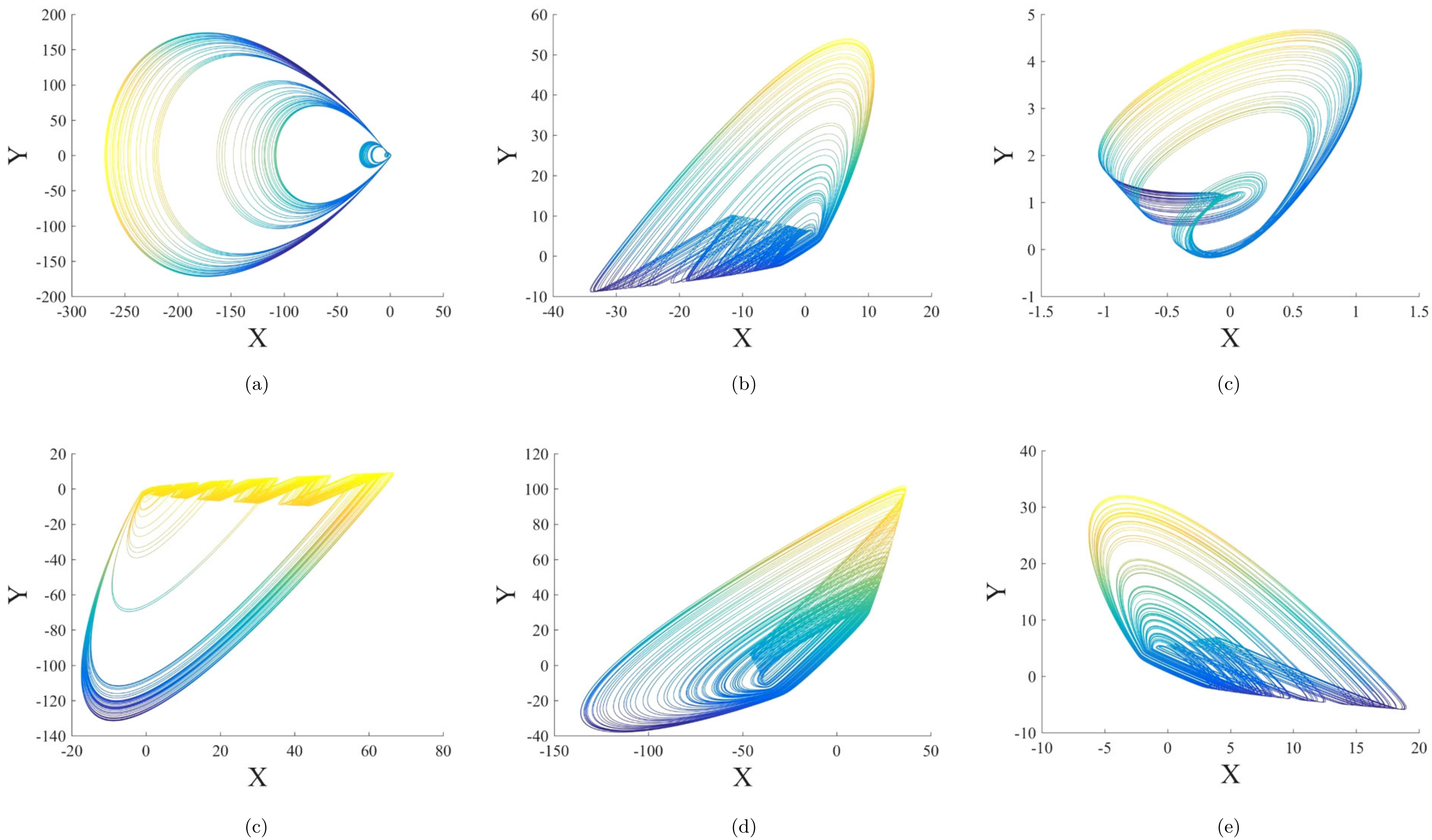


Fig. 1. Strange attractors for the 12 cases in Table 1 in the xy-plane with initial conditions given in the table. The strange attractor of: (a) IE_1, (b) IE_2, (c) IE_3, (d) IE_4, (e) IE_5, (f) IE_6, (g) IE_7, (h) IE_8, (i) IE_9, (j) IE_{10}, (k) IE_{11} and (l) IE_{12}.

(Continued)

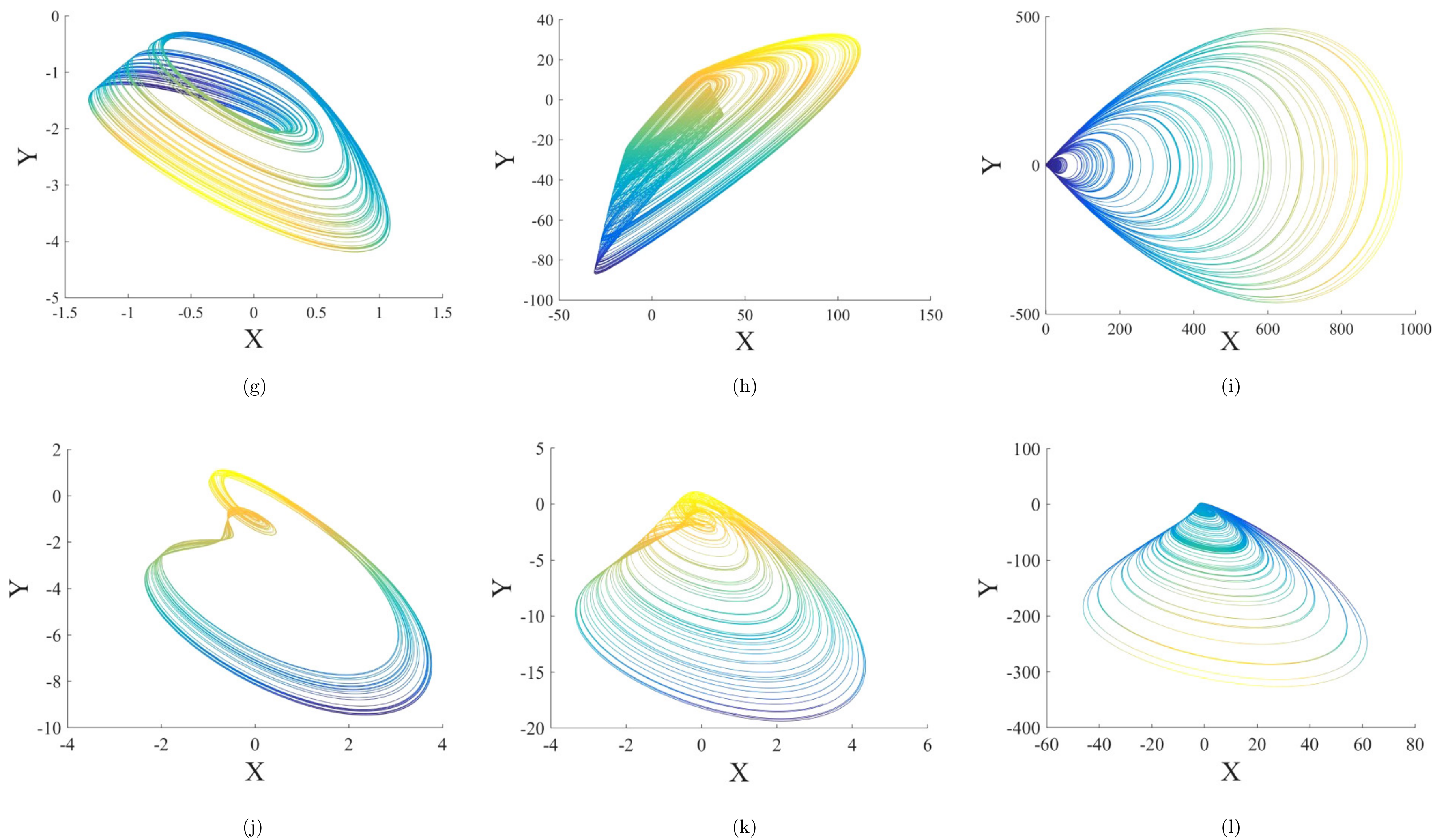
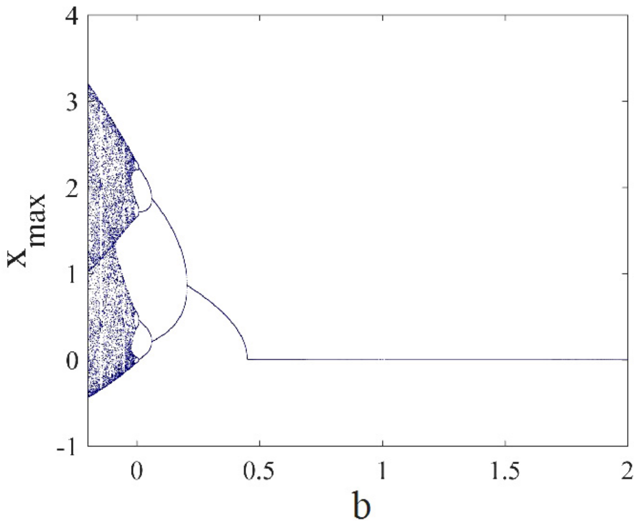
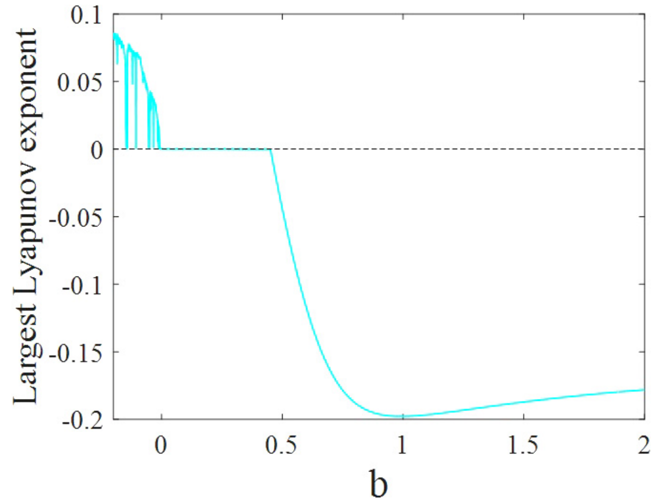


Fig. 1. (*Continued*)



(a)



(b)

Fig. 2. (a) Bifurcation diagram and (b) largest Lyapunov exponent of Case IE_{10} with respect to changing parameter b. Initial conditions are set with backward continuation, and the first initial conditions are (17.82, −61.74, −82.95).

Table 2. Twelve Kaplan–Yorke dimensions of strange attractors of systems in Table 1.

Case	IE_1	IE_2	IE_3	IE_4	IE_5	IE_6	IE_7	IE_8	IE_9	IE_{10}	IE_{11}	IE_{12}
D_{KY}	2.0004	2.0017	2.0168	2.0002	2.0031	2.0014	2.0504	2.0036	2.0028	2.0208	2.0384	2.0180

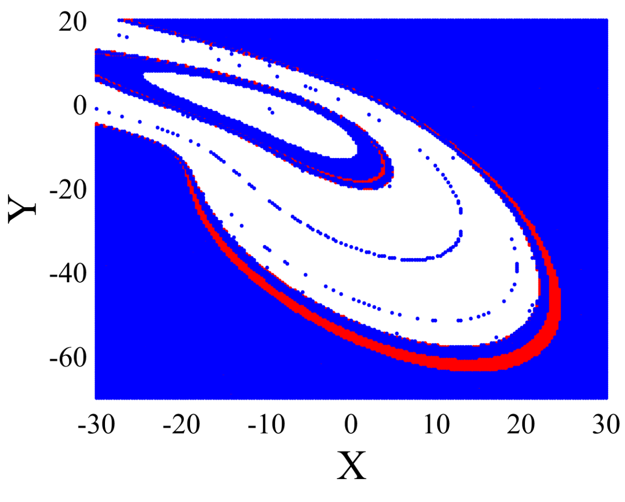


Fig. 3. Basin of attraction of System IE_{10} in xy-plane. Initial conditions in the red region lead to the chaotic attractors, initial conditions in the blue area lead to periodic attractors, and initial conditions in the white region lead to unbounded orbits.

Table 3. Categorization of the chaotic attractor of the proposed systems to being hidden or self-excited.

Case	IE_1	IE_2	IE_3	IE_4	IE_5	IE_6	IE_7	IE_8	IE_9	IE_{10}	IE_{11}	IE_{12}
Chaotic attractor: Hidden (H) or Self-excited (S)	H	S	H	S	H	S	H	H	H	H	H	S

initial values around the origin. The results show that in all nine cases, the origin is unstable.

3. Conclusion

In this paper, chaotic three-dimensional flows with three equal eigenvalues were proposed. Three structures were investigated, and the conditions were designed to give identical eigenvalues. Using a computer search, 12 chaotic systems were found with this special property. The origin was the equilibrium point of all the proposed systems. Nine of the systems have zero identical eigenvalues, while three of them have three positive and equal eigenvalues. Strange attractors of the systems have been presented. Dynamical properties of one of the systems (IE_{10}) were analyzed using a bifurcation diagram and plot of the largest Lyapunov exponent. Chaotic flows with such a property were proposed for the first time in this paper. It helps to go one step ahead in investigating chaotic dynamics.

Conflict of Interest

The authors declare no conflict of interest.

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