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Are Perpetual Points Sufficient for Locating Hidden Attractors?

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Perpetual Points (PPs) have been introduced as an interesting new topic in nonlinear dynamics, and there is a conjecture that these points can be used to find hidden attractors. This note demonstrates some examples where PPs cannot locate their hidden attractors.

Keywords: Perpetual point; hidden attractors; limit cycles.

1. Introduction

Recently, a new category of attractors that are called hidden attractors are of special interest [Leonov & Kuznetsov, 2014; Leonov *et al.*, 2014; Leonov *et al.*, 2011a; Leonov *et al.*, 2015b; Leonov *et al.*, 2011b; Leonov *et al.*, 2012; Leonov *et al.*, 2015a; Leonov & Kuznetsov, 2013a, 2013b; Bragin *et al.*, 2011; Kuznetsov *et al.*, 2011, 2010; Kuznetsov, 2016]. Hidden attractors are not associated with saddle points or unstable equilibria, and thus they can be found only by a numerical search through the space of initial conditions to find those within their basin of attraction. Many new chaotic attractors have been discovered in this category, such as flows without any equilibrium points, with only stable equilibria, or with a line of equilibrium points [Jafari & Sprott, 2013, 2015; Jafari *et al.*, 2013; Jafari *et al.*, 2015b; Kingni *et al.*, 2014; Lao *et al.*, 2014; Pham *et al.*, 2014a; Pham *et al.*, 2014b; Pham *et al.*, 2014c; Pham *et al.*, 2014d; Pham *et al.*, 2015; Shahzad *et al.*, 2015; Tahir *et al.*, 2015; Pham *et al.*, 2016c; Goudarzi *et al.*, 2016; Pham *et al.*, 2017; Kingni *et al.*, 2017; Pham *et al.*, 2016b; Panahi *et al.*, 2016; Pham *et al.*, 2016a; Barati *et al.*, 2016; Pham *et al.*, 2016d]. Multistability and coexisting attractors are other topics that have received increasing attention in nonlinear dynamics [Angeli *et al.*, 2004; Pisarchik & Feudel, 2014; Blazejczyk-Okolewska & Kapitaniak, 1996, 1998; Kapitaniak, 1985; Maistrenko *et al.*, 1997; Silchenko *et al.*, 1999]. The importance of these dynamical categories is their sensitivity to perturbations and initial conditions. On the other hand, knowledge of the basin and their classification are especially important for hidden attractors since the basin does not contain the small neighborhood of any equilibrium points [Sprott & Xiong, 2015].

One important structural feature in nonlinear dynamics is the fixed point [Ott, 2002; Strogatz, 2014]. Recently, Perpetual Points (PPs) were

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introduced as another structural feature [Prasad, 2015; Dudkowski *et al.*, 2015]. It has been claimed that the existence of these points can confirm whether a system is dissipative [Prasad, 2015], but that claim is false for some systems [Jafari *et al.*, 2015a]. Also, there is a conjecture that PPs can be used to locate hidden attractors [Dudkowski *et al.*, 2015; Dudkowski *et al.*, 2016]. On the other hand, multistable systems can have more attractors than the number of perpetual points, and there are systems with attractors but no perpetual points. Thus the conjecture cannot be true in general [Dudkowski *et al.*, 2016]. Furthermore, Dudkowski *et al.* [2016] discussed the chaotic system proposed in [Hoover *et al.*, 2015] which has neither fixed points nor PPs, although this system is conservative and has chaotic sea rather than a strange attractor. Thus the conjecture needs more analysis to be well understood because there is not any mathematical proof of the conjecture. In this note, we show some examples where PPs are not useful in finding hidden attractors.

2. Efficiency of Perpetual Points in Locating Hidden Attractors

Consider a general dynamical system

$$\begin{aligned} v_1 &= \dot{x}_1 = f_1(x_1, x_2, \ldots, x_n) \\ v_2 &= \dot{x}_2 = f_2(x_1, x_2, \ldots, x_n) \\ &\vdots \\ v_n &= \dot{x}_n = f_n(x_1, x_2, \ldots, x_n) \end{aligned} \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are dynamical states, $v_1, v_2, \ldots, v_n$ are the time derivatives of the states (velocities) and $f_1(X), f_2(X), \ldots, f_n(X)$ are the evolution equations. It is well known that the fixed points (FPs) of the above system are points $(x_1^*, x_2^*, \ldots, x_n^*)$ at which the time derivatives of all states are zero. Analysis of the FPs plays an essential role in dynamical systems [Ott, 2002; Strogatz, 2014; Prasad, 2015; Dudkowski *et al.*, 2015; Jafari *et al.*, 2015a].

Since acceleration is the second time derivative of the state, we obtain

$$\begin{aligned} a_1 &= \ddot{x}_1 = \dot{x}_1\frac{\partial f_1}{\partial x_1} + \dot{x}_2\frac{\partial f_1}{\partial x_2} + \cdots + \dot{x}_n\frac{\partial f_1}{\partial x_n} \\ &= v_1\frac{\partial f_1}{\partial x_1} + v_2\frac{\partial f_1}{\partial x_2} + \cdots + v_n\frac{\partial f_1}{\partial x_n} \\ a_2 &= \ddot{x}_2 = \dot{x}_1\frac{\partial f_2}{\partial x_1} + \dot{x}_2\frac{\partial f_2}{\partial x_2} + \cdots + \dot{x}_n\frac{\partial f_2}{\partial x_n} \\ &= v_1\frac{\partial f_2}{\partial x_1} + v_2\frac{\partial f_2}{\partial x_2} + \cdots + v_n\frac{\partial f_2}{\partial x_n} \\ &\vdots \\ a_n &= \ddot{x}_n = \dot{x}_1\frac{\partial f_n}{\partial x_1} + \dot{x}_2\frac{\partial f_n}{\partial x_2} + \cdots + \dot{x}_n\frac{\partial f_n}{\partial x_n} \\ &= v_1\frac{\partial f_n}{\partial x_1} + v_2\frac{\partial f_n}{\partial x_2} + \cdots + v_n\frac{\partial f_n}{\partial x_n} \end{aligned} \tag{2}$$

where $a_1, a_2, \ldots, a_n$ are the second derivatives of the states. For all the FPs, the accelerations are zero, since $v_1, v_2, \ldots, v_n$ are zero. By definition [Prasad, 2015], Perpetual Points (PPs) are points like $(x_1^*, x_2^*, \ldots, x_n^*)$ at which all the accelerations are zero but the velocities are not [Prasad, 2015].

There is a conjecture in [Prasad, 2015] that suggests that in systems with hidden attractors (in which FPs do not intersect with the basin of attraction of the attractor), PPs can be considered instead (PPs intersect with the basin of attraction of that hidden attractor).

In the remainder of this paper, we show some examples where PPs cannot locate hidden attractors.

2.1. *Example one*

Consider the NE4 system which is proposed in [Jafari *et al.*, 2013]. This system has no equilibrium and is simple in the sense that it has only six terms and a single quadratic nonlinearity.

$$\begin{aligned} \dot{x} &= -0.1y + 1 \\ \dot{y} &= x + z \\ \dot{z} &= xz - 3y. \end{aligned} \tag{3}$$

The system has a hidden chaotic attractor with initial conditions $(-8.2, 0, -5)$. The system is dissipative with Lyapunov exponents $(0.0235, 0, -8.480)$. To find the strange attractor basin size, the probability P that an initial condition at a distance r from the D-dimensional strange attractor lies within the basin of the attractor can be calculated [Sprott & Xiong, 2015]. The probability P for this attractor is approximately $P = 1.2/r^{\gamma}$, $\gamma = 2.0$ in the limit of large r. This means that its basin is Class 3 and extends to infinity in some directions, but occupies an ever decreasing fraction of the state space with a fractal dimension of $D - \gamma = 1.0$ on the largest scale. This system has no equilibrium as one can check by setting to zero the right-hand side of the equations. Applying Eq. (2) gives

$$\begin{aligned} \ddot{x} &= -0.1\dot{y} = -0.1(x+z) = 0 \\ \ddot{y} &= \dot{x} + \dot{z} = -3.1y + 1 + xz = 0 \\ \ddot{z} &= \dot{x}z + x\dot{z} - 3\dot{y} = x^2z - 3xy - 0.1yz + z = 0. \end{aligned} \tag{4}$$

Setting the right-hand sides of these equations to zero gives one real and two complex solutions. Thus system (3) has one PP

$$\left(0, \frac{10}{31}, 0\right). \tag{5}$$

The conjecture that PPs intersect with the basin of attraction of at least one attractor is investigated in the neighborhood of the PP. Three perpendicular cross-sections of the basin of attraction passing through the PP can be seen in Figs. 1(a)–1(c). Initial conditions in the white region lead to unbounded orbits, and those in the light blue region lead to a chaotic attractor whose cross-section is shown in black. Figures 1(d)–1(f) show the zoomed versions of the previous sections around the PP. The chaotic attractor of this system cannot be detected by FP or PP.

2.2. *Example two*

Consider the SE11 system which is proposed in [Molaie *et al.*, 2013],

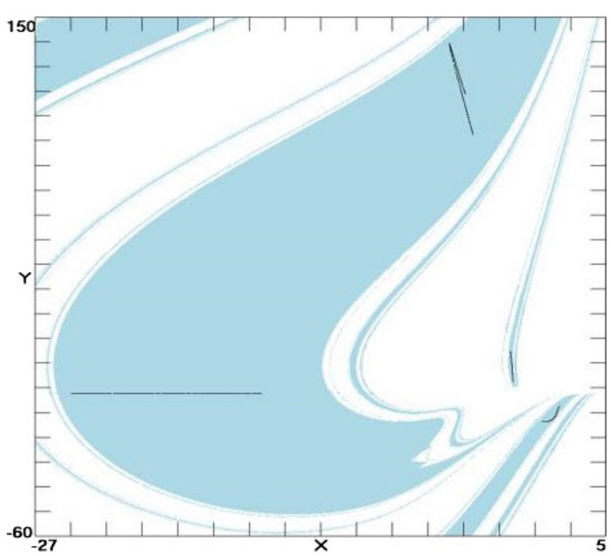
$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + yz \\ \dot{z} &= -y - 12z + x^2 + 9xz - 1. \end{aligned} \tag{6}$$

The system is an example of a rare chaotic flow with only one stable equilibrium $(0, 0, -1/12)$ with eigenvalues $(-12, -0.0417 \pm 0.9991i)$. This system has a hidden chaotic attractor with initial conditions $(-2, 0, 0.1)$. It is dissipative with Lyapunov exponents $(0.0801, 0, -14.1917)$.

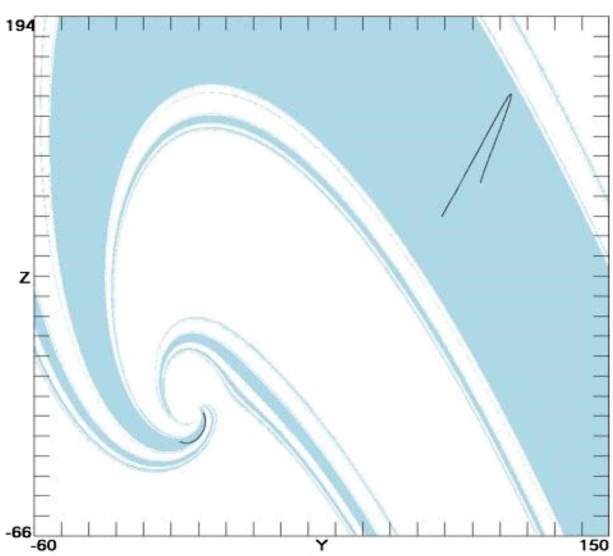
By applying Eq. (2), we obtain the following equations:

$$\begin{aligned} \ddot{x} &= \dot{y} = -x + yz = 0 \\ \ddot{y} &= -\dot{x} + \dot{y}z + y\dot{z} \\ &= 9xyz - y^2 - 12yz + x^2y - 2y = 0 \\ \ddot{z} &= -\dot{y} - 12\dot{z} + 2x\dot{x} + 9\dot{x}z + 9x\dot{z} \\ &= 9x^3 + 81x^2z - 12x^2 - 7xy - 216xz \\ &\quad + 9yz - 9x + 12y + 144z + 12 = 0. \end{aligned} \tag{7}$$

Setting the right-hand sides of these equations to zero shows that these equations have four solutions.



(a)



(b)

Fig. 1. Basin of attraction for system (3) in orthogonal planes passing through the PP. Initial conditions in the white region lead to unbounded orbits, and those in the light blue region lead to a chaotic attractor whose cross-section is shown in black. (a)–(c) show the basin of attraction through the PP and (d)–(f) show the zoomed figures around the PP. The black dots in (d)–(f) show the location of the PP.

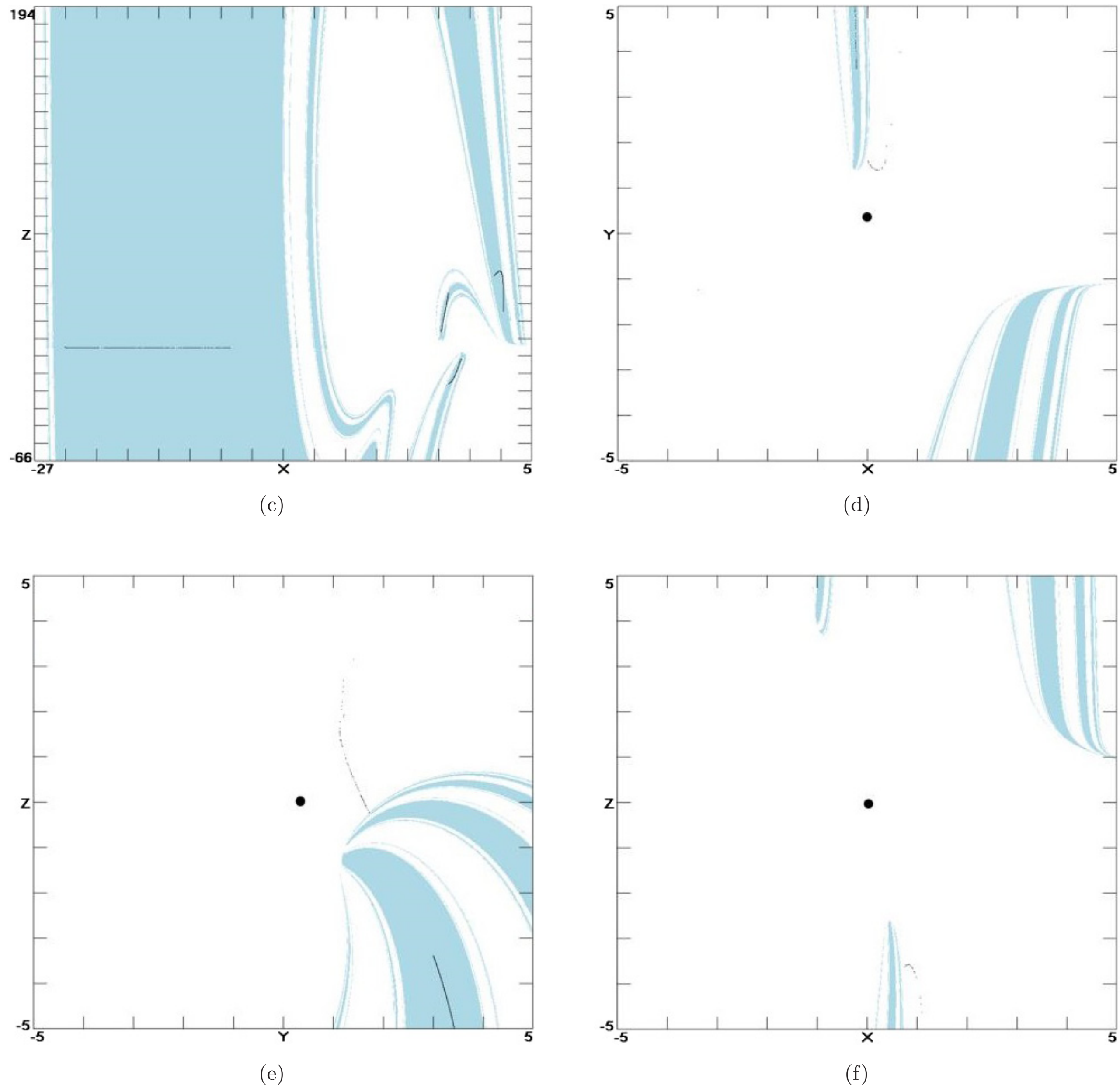


Fig. 1. (*Continued*)

In addition to two complex solutions and one fixed point, this system has the PP

$$(-11.8883, -9.5047, 1.2508). \tag{8}$$

Figure 2 shows the intersection of the basin of attraction with the plane $z = -0.11222x - 1/12$ that contains both the FP and the PP. The PP of this system is far from the basin of either attractor and lies in the region of unbounded orbits. Initial conditions in the white region lead to unbounded orbits, those in light blue region lead to a chaotic attractor whose cross-section is shown in black and those in red region lead to the stable FP shown as a black point. Thus the strange attractor of this system cannot be detected by FP or PP.

2.3. *Example three*

Consider the following (2D) dissipative system which is discussed in [Kahn & Zarmi, 2014],

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x - \sin(y). \end{aligned} \tag{9}$$

Setting the right-hand side of these equations to zero shows that the system has an equilibrium at $(0, 0)$ with eigenvalues $(-1 \pm \sqrt{3}i)/2$.

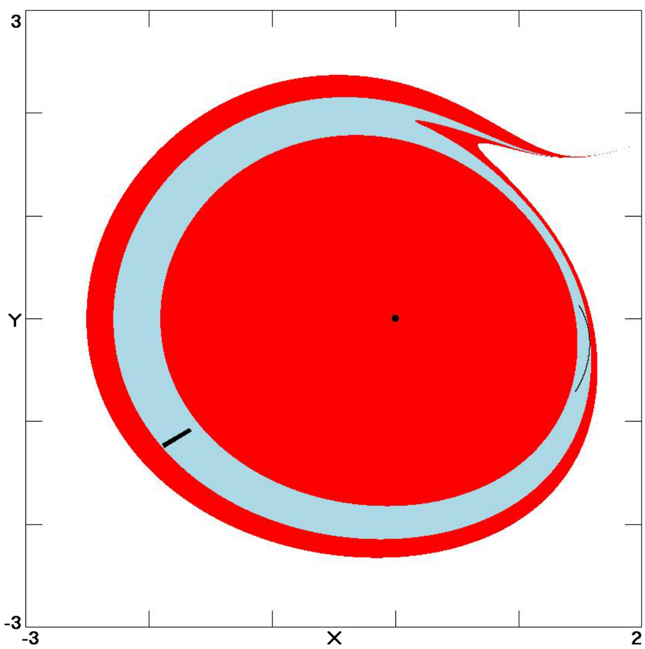


Fig. 2. Cross-section of the basins of attraction for system (6) in the plane $z = -0.11222x - 1/12$ that contains both the FP and the PP. Initial conditions in the white region lead to unbounded orbits, those in light blue region lead to a chaotic attractor whose cross-section is shown in black and those in red region lead to the stable FP shown as a black point.

Applying Eq. (2) to the system gives

$$\begin{aligned} \ddot{x} &= \dot{y} = -x - \sin(y) = 0 \\ \ddot{y} &= -\dot{x} - \cos(y)\dot{y} = -y = 0. \end{aligned} \tag{10}$$

Since the only solution of these equations is the fixed point $(0, 0)$ itself, the system does not have any perpetual points. However the system has an infinite number of limit cycles coexisting with one stable equilibrium, four of which are shown in Fig. 3. All the limit cycles are hidden since there is not any systematic way to choose initial conditions, and also there is not any PP to find the hidden attractors.

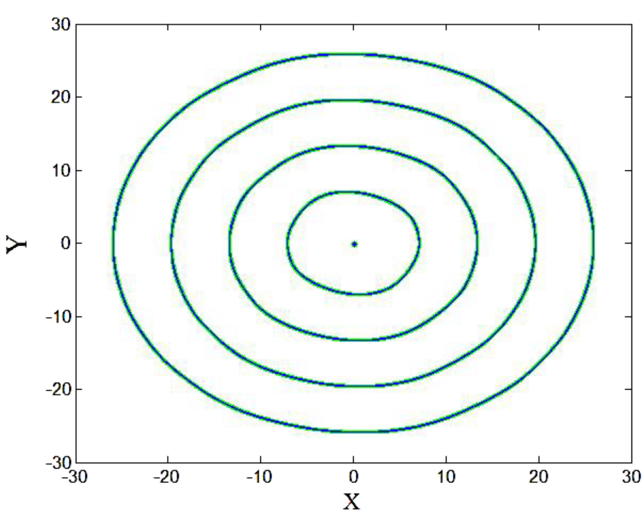


Fig. 3. Some limit cycles of system (9) for initial conditions $(x_0, 0)$ where x_0 is in the range $(0, 25)$.

3. Conclusion

Hidden attractors and perpetual points are new topics in nonlinear dynamics. Recently, a conjecture has been proposed that there is a connection between PPs and hidden attractors. The conjecture cannot help to find all hidden attractors and only gives some possibilities in the research of dynamical systems. This note demonstrates some examples in which PPs cannot locate hidden attractors.

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