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Simple chaotic 3D flows with surfaces of equilibria

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Abstract Using a systematic computer search, twelve simple three-dimensional chaotic flows were found that have surfaces of equilibria. Although there are some four-dimensional systems with surfaces of equilibria, there is no such system in three-dimensional state space reported in the literature. Such systems are not difficult to design, but they can have some practical benefits. Study of chaotic flows with surfaces of equilibria provides a good reference for building systems with attractors that are protected from external influences, which can increase the safety of engineering.

Keywords Chaotic flows · Surface equilibrium · Hidden attractors

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1 Introduction

It is widely recognized that mathematically simple systems of nonlinear differential equations can exhibit chaos. With the advent of fast computers, it is now possible to explore the entire parameter space of these systems with the goal of finding parameters that result in some desired characteristics of the system [1].

Recent research has involved categorizing periodic and chaotic attractors as either self-excited or hidden [2–5]. A self-excited attractor has a basin of attraction that is associated with an unstable equilibrium, whereas a hidden attractor has a basin of attraction that does not intersect with small neighborhoods of any equilibrium points. The classical attractors of Lorenz, Rössler, Chua, Chen, Sprott systems (cases B to S) and other widely known attractors are those excited from unstable equilibria. From a computational point of view, this allows one to use a numerical method in which a trajectory started from a point on the unstable manifold in the neighborhood of an unstable equilibrium, reaches an attractor and identifies it [2]. Hidden attractors cannot be found by this method and are important

Table 1 Twelve simple chaotic flows with surface equilibrium

Case	Surface type	Equations	(a, b, c)	Equilibria	LEs	D_{KY}	(x_0, y_0, z_0)
ES_1	One plane	$\dot{x} = f \times y$	$a = 1.54$	$(0, y, z)$	0	2.0065	6
		$\dot{y} = f \times z$			−1.0869		0
		$\dot{z} = f \times \left(-x + ay^2 - xz\right)$					−1
		$f = x$					
ES_2	One plane	$\dot{x} = f \times y$	$a = 1$	$(0, y, z)$	0.0644	2.0778	0.15
		$\dot{y} = f \times (-x + az)$	$b = 3$		0		0
		$\dot{z} = f \times \left(by^2 - xz\right)$			−0.8279		0.8
		$f = x$					
ES_3	One plane	$\dot{x} = f \times \left(y^2 + axy\right)$	$a = 2$	$(0, y, z)$	0.0661	2.0397	0.87
		$\dot{y} = f \times (-z)$	$b = 1$	$\left(\sqrt{\frac{b}{a}}, -\sqrt{ab}, 0\right)$	0		0.4
		$\dot{z} = f \times (b + xy)$		$\left(-\sqrt{\frac{b}{a}}, \sqrt{ab}, 0\right)$	−1.664		0
		$f = x$			4		
ES_4	One plane	$\dot{x} = f \times (-y)$	$a = 2$	$(x, y, 0)$	0.0560	2.0516	0
		$\dot{y} = f \times (x + z)$	$b = 0.35$		0		0.46
		$\dot{z} = f \times \left(ay^2 + xz - b\right)$			−1.0855		0.7
		$f = z$					
ES_5	Two planes	$\dot{x} = f \times (-az)$	$a = 0.4$	$(0, y, z)$	0.1242	2.0677	1
		$\dot{y} = f \times \left(b + z^2 - xy\right)$	$b = 1$	$(x, 0, z)$	0		1.44
		$\dot{z} = f \times \left(x^2 - xy\right)$		$\left(\sqrt{b}, \sqrt{b}, 0\right)$	−1.8356		0
		$f = xy$		$\left(-\sqrt{b}, -\sqrt{b}, 0\right)$			
ES_6	Three planes	$\dot{x} = f \times (y + ayz)$	$a = 2$	$(0, y, z)$	0.0294	2.0725	1
		$\dot{y} = f \times \left(bz + y^2 + cz^2\right)$	$b = 8$	$(x, 0, z)$	0		−1.3
		$\dot{z} = f \times \left(x^2 - y^2\right)$	$c = 7$	$(x, y, 0)$	−0.4051		−1
		$f = xyz$		$(\pm 1.5, 1.5, -0.5)$			
				$(\pm 1.5, -1.5, -0.5)$			
ES_7	Sphere	$\dot{x} = f \times (ay)$	$a = 0.4$	$x^2 + y^2 + z^2 = 1$	0.0113	2.0119	0
		$\dot{y} = f \times (xz)$	$b = 6$	$(0, 0, 0)$	0		0.1
		$\dot{z} = f \times \left(-z - x^2 - byz\right)$			−0.9501		0
		$f = 1 - x^2 - y^2 - z^2$					
ES_8	Sphere	$\dot{x} = f \times \left(az + y^2\right)$	$a = 1$	$x^2 + y^2 + z^2 = 1$	0.0323	2.0338	0.24
		$\dot{y} = f \times \left(-y + bx^2\right)$	$b = 5$	$(0, 0, 0)$	0		0.2
		$\dot{z} = f \times (-xy)$			−0.9552		0
		$f = 1 - x^2 - y^2 - z^2$					
ES_9	Circular cylinder	$\dot{x} = f \times \left(y^2 - axy\right)$	$a = 5$	$x^2 + y^2 = 1$	0.0388	2.0321	0.06
		$\dot{y} = f \times (xz)$	$b = 7$	$(-0.0756, -0.3781, 0)$	0		0
		$\dot{z} = f \times \left(1 - by^2\right)$		$(+0.0756, +0.3781, 0)$	−1.2078		1
		$f = 1 - x^2 - y^2$					

Table 1 continued

Case	Surface type	Equations	(a, b, c)	Equilibria	LEs	D_{KY}	(x_0, y_0, z_0)
ES_{10}	Hyperbolic cylinder	$\dot{x} = f \times (a - z^2)$	$a = 0.1$	$y^2 - x^2 = 1$	0.0420	2.1883	0
		$\dot{y} = f \times (xz)$	$b = 1$	$(0, 0, -0.3162)$	0		-0.08
		$\dot{z} = f \times (y + bxz)$		$(0, 0, +0.3162)$	-0.2230		0
		$f = 1 + x^2 - y^2$					
ES_{11}	Paraboloid	$\dot{x} = f \times (yz)$	$a = 1$	$x^2 + y^2 = -z$	0.0283	2.0458	0.46
		$\dot{y} = f \times (x - axz)$	$b = 0.6$	$(0, y, 0)$	0		0
		$\dot{z} = f \times (x - bz^2)$		$(0.6, 0, 1)$	-0.6171		0.8
		$f = z + x^2 + y^2$					
ES_{12}	Saddle	$\dot{x} = f \times (yz)$	$a = 0.1$	$y^2 - x^2 = z$	0.0068	2.0135	1
		$\dot{y} = f \times (-ax)$	$b = 6$	$(0, 0, 0)$	0		0
		$\dot{z} = f \times (-z + by^2 + xz)$			-0.4998		1
		$f = z + x^2 - y^2$					

in engineering applications because they allow unexpected and potentially disastrous responses to perturbations in a structure such as a bridge or an airplane wing [6–10].

The chaotic attractors in dynamical systems without any equilibrium points [11–19], with only stable equilibria [20–23], or with curves of equilibria [24–28] are hidden attractors. That is the reason such systems are rarely found, and only recently such examples have been reported in the literature [29–37]. There is a similar definition for hidden attractors in chaotic maps [38,39]. Even more specific systems such as chaotic systems with multi-scroll attractors [40,41] can have hidden attractors [15].

In this paper, we introduce a new category of chaotic systems with hidden attractors: systems with surfaces of equilibria. Although in such systems the basin of attraction may intersect the equilibrium surface in some sections, there are usually uncountably many points on the surface that lie outside the basin of attraction of the chaotic attractor, and thus, it is impossible to identify the chaotic attractor for sure by choosing an arbitrary initial condition in the vicinity of the unstable equilibria. In other words, from a computational point of view these attractors are hidden. On the other hand, to the best of our knowledge, there are no 3D chaotic systems with surfaces of equilibria in the literature (there are papers on 4D chaotic systems with a plane of equilibria [42]). As described in the next sections, such systems are not difficult to construct, but they can have some practical benefits.

2 Simple chaotic flows with surfaces of equilibria

In the search for chaotic flows with surfaces of equilibria, we followed a simple procedure. Consider the general parametric form of quadratic three-dimensional flows:

$$\begin{aligned} \dot{x} &= Q_1(x, y, z) \\ \dot{y} &= Q_2(x, y, z) \\ \dot{z} &= Q_3(x, y, z) \end{aligned} \tag{1}$$

in which

$$\begin{aligned} Q_1 &= a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + a_6 z^2 + a_7 xy \\ &\quad + a_8 xz + a_9 yz + a_{10} \\ Q_2 &= a_{11} x + a_{12} y + a_{13} z + a_{14} x^2 + a_{15} y^2 + a_{16} z^2 \\ &\quad + a_{17} xy + a_{18} xz + a_{19} yz + a_{20} \\ Q_3 &= a_{21} x + a_{22} y + a_{23} z + a_{24} x^2 + a_{25} y^2 + a_{26} z^2 \\ &\quad + a_{27} xy + a_{28} xz + a_{29} yz + a_{30} \end{aligned} \tag{2}$$

In order to have a surface on which all the points are an equilibrium, there should be a multiplying factor such as $f(x, y, z)$ in all the equations, so that an equilibrium surface occurs whenever $f(x, y, z) = 0$. Thus, the equations to be examined are

$$\begin{aligned} \dot{x} &= f(x, y, z)\, Q_1 \\ \dot{y} &= f(x, y, z)\, Q_2 \\ \dot{z} &= f(x, y, z)\, Q_3 \end{aligned} \tag{3}$$

The simplest candidates for the surface $f(x, y, z)$ are simple planes (one plane such as $f = x$, two orthogonal planes such as $f = xy$ and three orthogonal

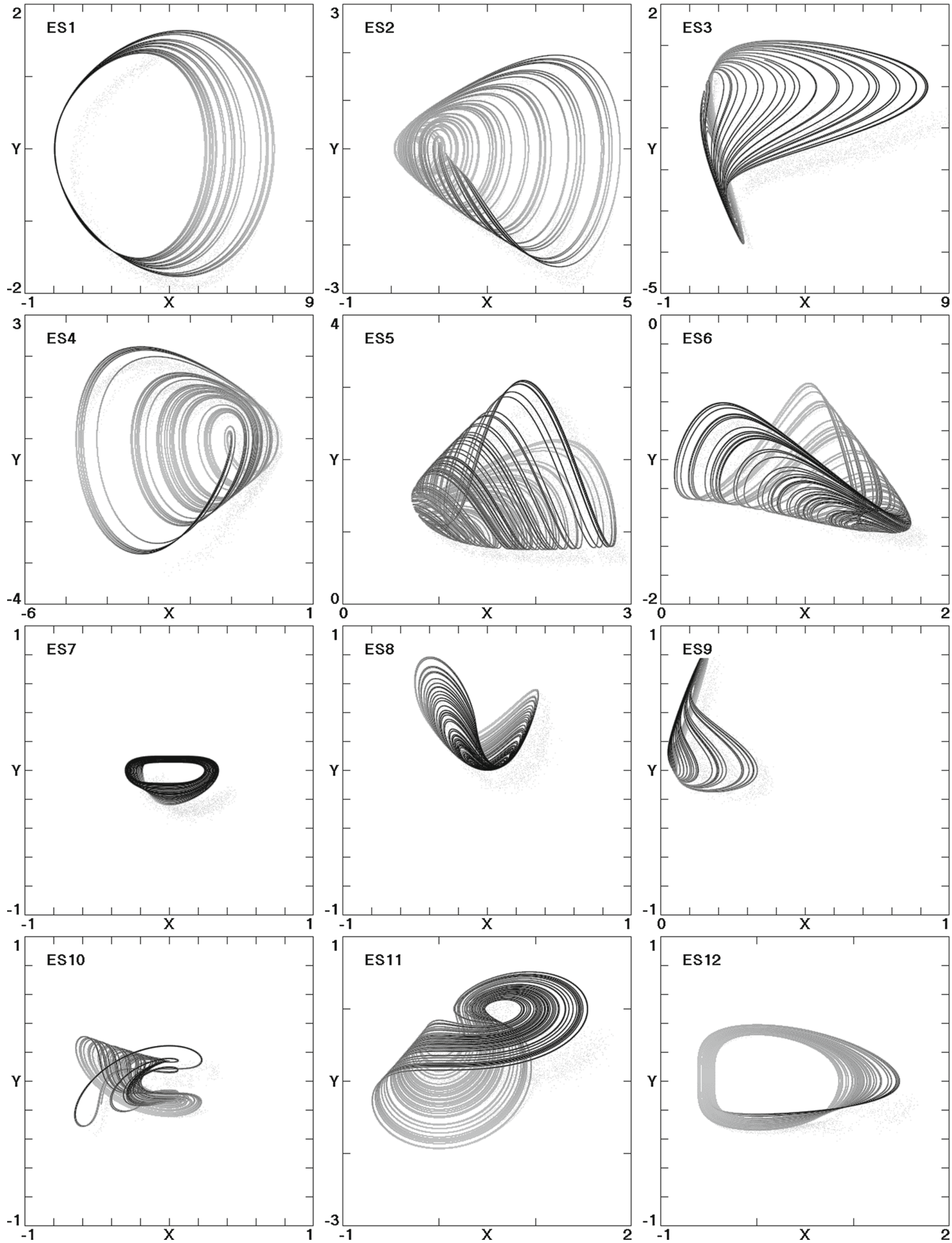

Fig. 1 State space plots of the cases in Table 1 projected onto the *xy* plane

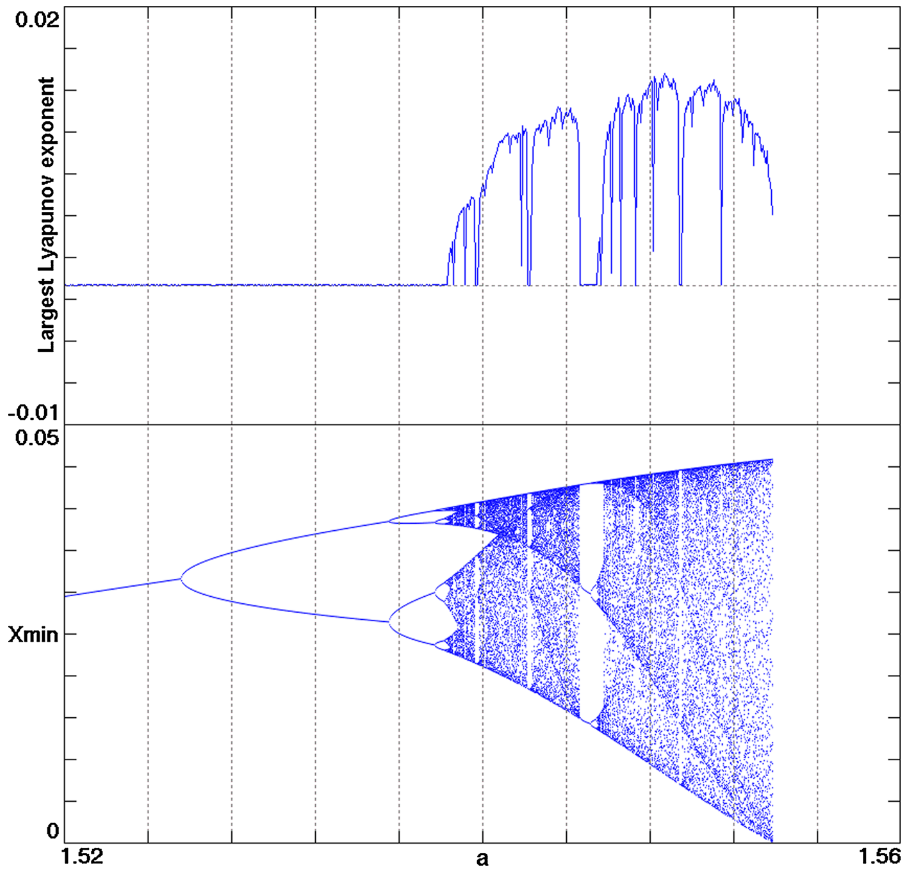

Fig. 2 Largest Lyapunov exponent and bifurcation diagram of case ES_1 showing a period-doubling route to chaos

planes such as $f = xyz$). Also standard quadrics (ellipsoids, hyperboloids and paraboloids) are proper candidates.

An exhaustive computer search was done, seeking elegant [1] dissipative cases for which the largest Lyapunov exponent is greater than 0.001. Cases ES_1–ES_{12} in Table 1 are twelve of the simplest cases found in this way. All these cases are dissipative with attractors projected onto the xy plane as shown in Fig. 1. The value of parameters, Lyapunov exponent spectra and Kaplan–Yorke dimensions are shown in Table 1 along with initial conditions that are close to the attractor. As is common for strange attractors from three-dimensional autonomous systems, the attractor dimension is only slightly greater than 2.0, the largest of which is ES_{10} with $D_{KY} = 2.1883$, although no effort was made to tune the parameters for maximum chaos. All the cases appear to approach chaos through a succession of period-doubling limit cycles, a typical example of which (ES_1) is shown in Fig. 2. with increasing a. As a increases further, the strange attractor is destroyed in a boundary crisis.

Figure 3 shows a cross section in the xz plane at $y = 0$ of the basin of attraction for the two attractors for the typical case ES_1. The basin of attraction of the chaotic attractor intersects the plane equilibrium almost nowhere except in a tiny area near $x = 0$, $z = 1$. Thus, the strange attractor is hidden in the sense that there are uncountable points on the equilibrium plane of which a tiny fraction intersect the basin of the chaotic attractor. In other words, for computational purposes, the attractor is hidden from the equilibria to some extent, and knowledge about the equilibrium plane will not guarantee that its location can be found.

As in the cases with a line of equilibrium points, the strange attractor can reside very close to the surface of equilibria. However, there is a fundamental difference between the two cases. In a three-dimensional system, the attractor can surround the equilibrium line, and often does, whereas a surface equilibrium divides

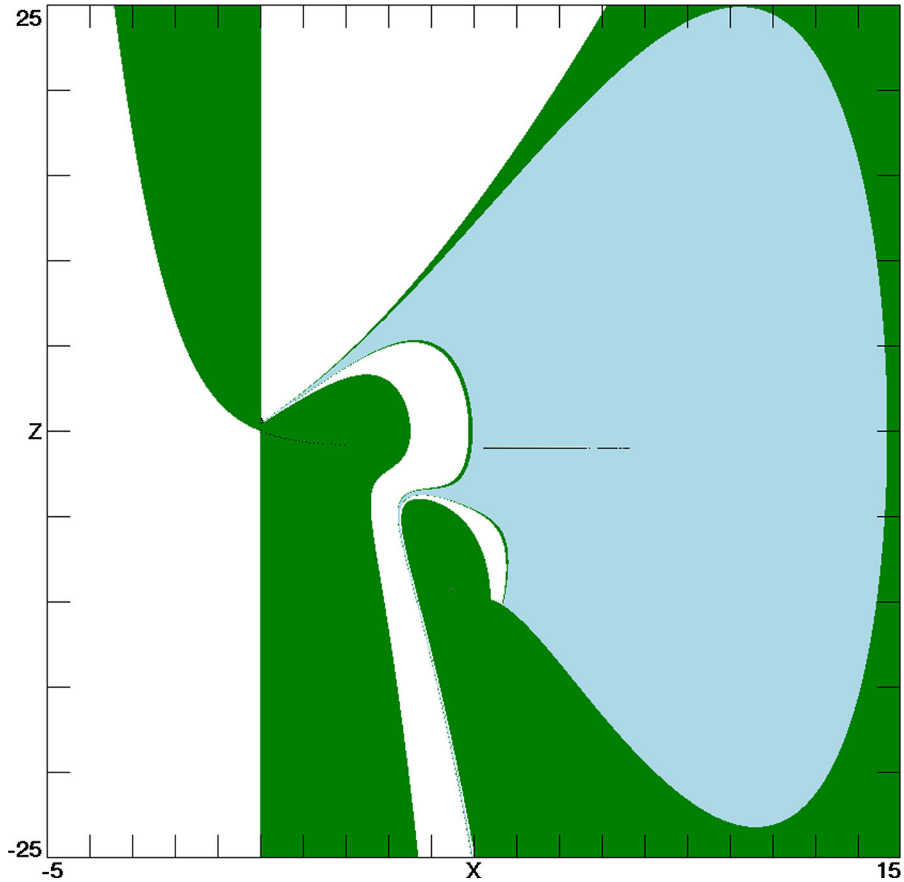

Fig. 3 Cross section of the basins of attraction of the two attractors in the xz plane at $y = 0$ for case ES_1. Initial conditions in the white region lead to unbounded orbits, those in the *light blue region* lead to the strange attractor whose cross section is in *black*, and those in the *green region* lead to the plane equilibrium

the space into two regions that the attractor cannot span because the normal component of the flow is zero at the surface. Our experience shows that the surface equilibrium can be and often is very close to the strange attractor, and that the strange attractor is little altered if the surface is removed by eliminating the factor $f(x, y, z)$ in the equations. Furthermore, it is possible to have separate strange attractors in the two regions (or more if there are multiple surfaces), although we do not show any such examples here.

There can be some practical uses for such systems. A surface of equilibrium (such as a sphere) can act like a protector shield for the strange attractor, causing no entrance and exit. No perturbation smaller than that radius of the sphere can cause unbounded solutions. With such closed surfaces, the attractor will be hidden in an egg, which makes finding it more difficult. This is important in secure communication uses of chaotic systems. In the next section, we investigate the feasibility of constructing a real electrical circuit based on the proposed systems.

3 Circuit implementation

Errors in numerical simulations and the long simulation time required to investigate the dynamics of chaotic systems have encouraged their physical implementation as an alternate means of study [43,44]. Moreover, the hardware implementation of theoretical chaotic models is important in engineering applications [43,45]. Until now, realizations using commercial amplifiers [1] and integrated circuits have been introduced [46].

In addition, the designed circuit is an effective way for discovering new dynamics of the theoretical model. An electronic circuit that emulates ES_7 is presented in this section. The state variables x, y and z of ES_7 are linearly scaled by changing the system ES_7 to

$$\begin{cases} \dot{X} = f \times (aY) \\ \dot{Y} = f \times \left(\frac{XZ}{10}\right) \\ \dot{Z} = f \times \left(-Z - \frac{X^2}{10} - b\frac{YZ}{10}\right) \end{cases} \quad (4)$$

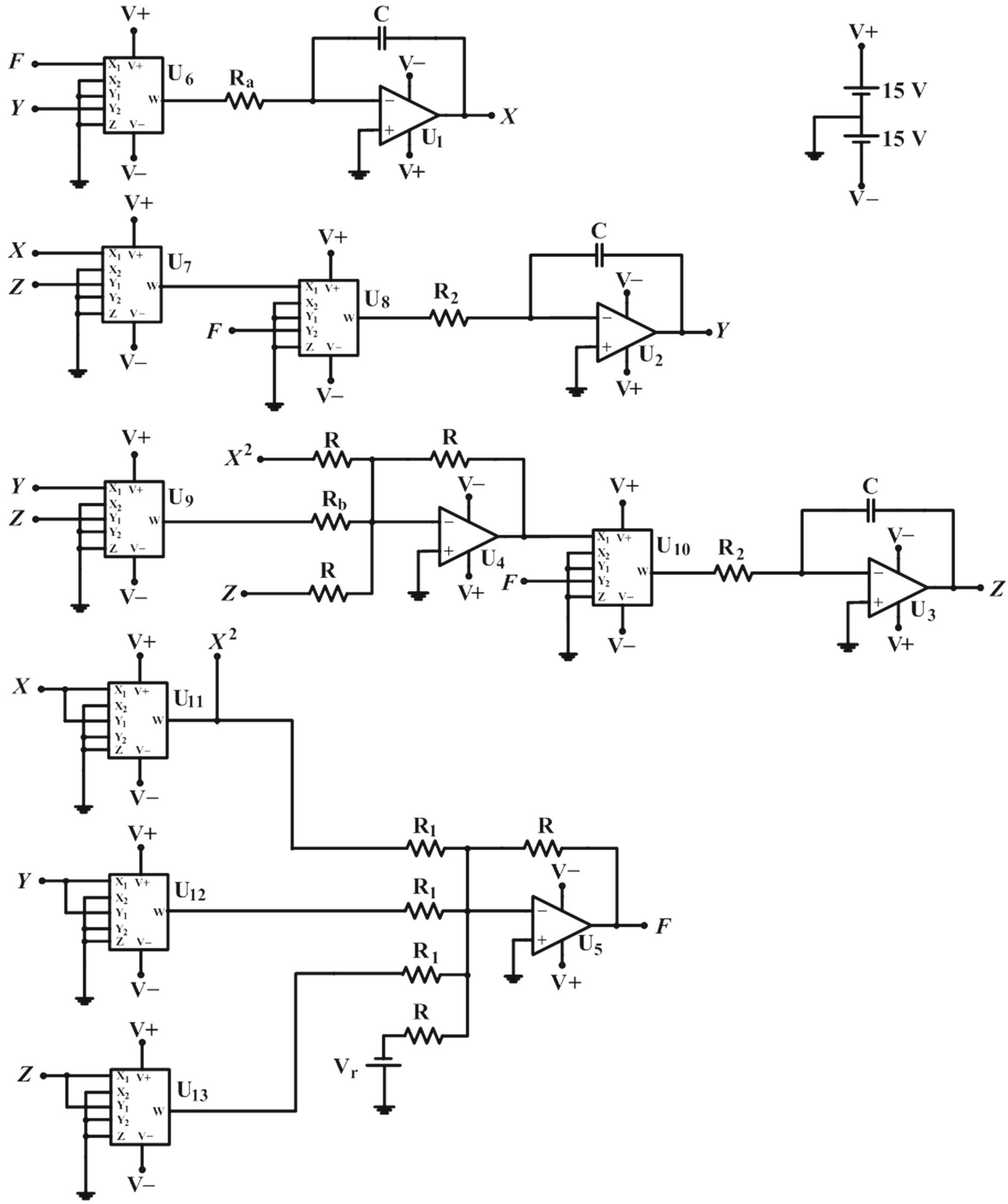

Fig. 4 Schematic of the designed circuit where F is the signal at the output of the adder U_5

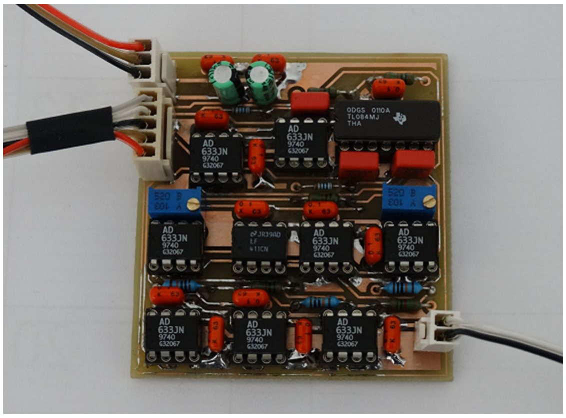

Fig. 5 Experimental realization of the system with a surface of equilibria (3)

with

$$f = r^2 - \frac{X^2}{100} - \frac{Y^2}{100} - \frac{Z^2}{100} \quad (5)$$

where $X = 10x$, $Y = 10y$, $Z = 10z$.

The schematic of the circuit and its experimental realization are shown in Figs. 4 and 5, respectively. This circuit has three integrators (U_1–U_3) and two adders (U_4, U_5), which are implemented with the operational amplifiers TL084 and LF411, as well as eight signal multipliers (U_6–U_{13}) by using the analog multiplier AD633. A series of capacitors for decoupling the power supplies for each of the TL084 and AD633 has been used.

By applying Kirchhoff's circuit laws, the circuital equations of the designed circuit are:

$$\begin{cases} \dot{X} = \frac{1}{RC}\left[F \times \left(\frac{R}{R_a 10V} Y\right)\right] \\ \dot{Y} = \frac{1}{RC}\left[F \times \left(\frac{10R}{R_2 10^2 V^2} \frac{XZ}{10}\right)\right] \\ \dot{Z} = \frac{1}{RC}\left[F \times \left(-\frac{R}{R_2 10V} Z - \frac{10R^2}{R_2 R 10^2 V^2} \frac{X^2}{10} - \frac{10R^2}{R_2 R_b 10^2 V^2} \frac{YZ}{10}\right)\right] \end{cases} \quad (6)$$

with

$$F = \frac{R}{R_1 10V}\left[-\frac{R_1 10V}{R} V_r - X^2 - Y^2 - Z^2\right] \quad (7)$$

where the variables X, Y and Z correspond to the voltages at the outputs of the integrators U_1, U_2 and U_3, while F is the signal at the output of the adder U_5. By normalizing the differential equations of system (6) for $\tau = t/RC$, this system is equivalent to ES_7, with $a = \frac{R}{10R_a}$, $b = \frac{R^2}{10R_2R_a}$, and $r^2 = -V_r$. The circuit components have been chosen as: $R = 10\,\mathrm{k\Omega}$, $R_1 = 100\,\mathrm{k\Omega}$, $R_2 = 1\,\mathrm{k\Omega}$, $R_a = 2.5\,\mathrm{k\Omega}$, $R_b = 1.666\,\mathrm{k\Omega}$, $C = 10\,\mathrm{nF}$ and $V_r = -4V_{DC}$, while the power supplies of all active devices are $\pm 15V_{DC}$. As a result, the parameters of system (6) are: $a = 0.4$, $b = 6$ and $r = 2$. The chaotic attractors captured from the oscilloscope are shown in Fig. 6. The circuit emulates well the proposed system.

4 Conclusion

In conclusion, it is apparent that simple chaotic systems with surfaces of equilibria that seemed to be rare may in fact be rather common. These systems belong to the newly introduced class of chaotic systems with hidden attractors and have not been previously described. Furthermore, the study of chaotic flows with surfaces of equilibria provides a good reference for building systems with attractors that are protected from external

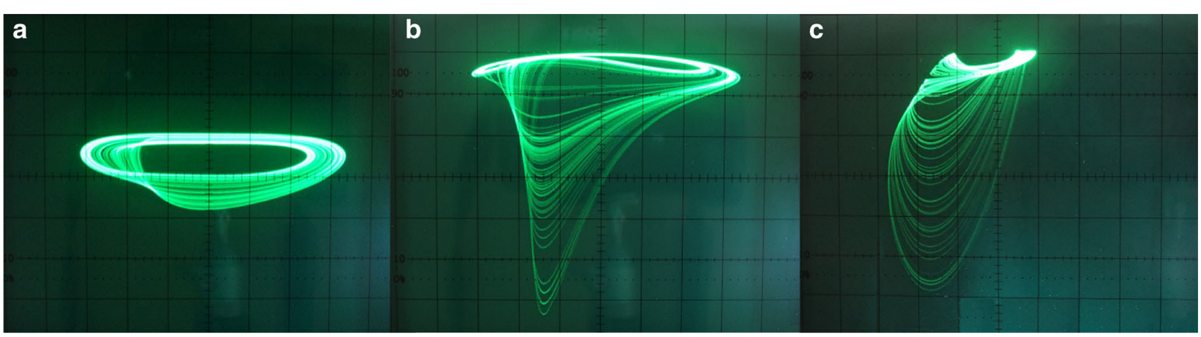

Fig. 6 Experimental chaotic attractors captured from the oscilloscope in **a** $X - Y$, **b** $X - Z$ and **c** $Y - Z$ phase planes (X: 1 V/div, Y: 2 V/div)

influences, which can increase the safety of engineering.

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