Extensions in Dynamic Models of Happiness: Effect of Memory

Abstract: Due to its more dynamical richness, fractional calculus is used to derive a dynamic model of happiness in response to real life events and under different mental conditions. Previous models are extended by using fractional derivatives whose order may vary in time. This extension helps to describe memory-dependent behaviors. Since the proposed model is an extension of the former models, it includes all of their merits. As simulations show, the new model can be utilized to describe the effect of short term memory on happiness when memory decays or is lost suddenly, similar to what happens in electroconvulsive therapy.

Keywords: Dynamic Models of Happiness, Fractional calculus, Variable Order Dynamics, Memory, Electro-convulsive Therapy.

1. Introduction

One important goal of science in various branches is to improve the welfare and health of human beings. An important component of human welfare is happiness. More generally, happiness can be classified as a part of human emotions. Although happiness is not a well-defined concept and can be interpreted in various ways, recently some mathematical dynamic models have been suggested to describe it and show how it varies in time (Song et al. 2010 and Sprott 2005). Concepts like happiness or other general emotions are not easy to measure or even define properly. However, a mathematical description can give an overall view of them. Furthermore, when an acceptable dynamic for prediction of a person's happiness is derived, it can be used to improve psychiatric treatments for related illnesses such as depression and bipolar disorder. This mathematical description can succeed if the equations explain and predict some experimental observations (Sprott 2004, 2005, Song and Yang 2009 and Song et al. 2010).

First endeavors to describe happiness as an output of a mental dynamic system was done by Sprott (Sprott, 2005), where he proposed some nonlinear ordinary differential equations that could explain some psychiatric effects related to happiness and the influence of external stimuli like winning a lottery, addictive behavior, and other real life events. Later, Song extended the model to include fractional derivatives and claimed that the order can be somehow related to memory (Song and Yang, 2009 and Song et al., 2010). According to the definition of non-integer derivatives, it seems that the order of the derivative can describe the effect of past events on the present situation (Sabatier et al. 2007). Since modeling memory and its effect on emotion is a challenging task in psychiatry (Guenther 1988), this new viewpoint seems capable of getting one step closer to this goal.

The idea of using the order of a derivative to represent memory is promising, but there can be some modifications and extensions that allow the model to express more events and facts about mutual relations between memory and context.

To examine the validity of a suggested dynamic or equation, we usually consider it as a system with inputs and outputs and check whether it can faithfully represent real world events. Later we use this method to test each suggested model.

Modeling happiness as a part of human emotion can help us in the following manners:

1. To predict that how an individual responses to a stimulant like some bad news. It can help us to provide the circumstances for a person to avoid undesirable behavior when he/she faces with such events.

2. To control the emotional status in the case that an individual is suffering from a psychiatric disorder. Remedy process can be translated to control method. Control methods can be used to make a dynamical system behave in a desired manner and if there is a proper dynamic model to describe happiness we can utilize these methods to make a patient behave normally by taking drugs which can be translated to control signals, in turn.

3. To make it possible to simulate human's emotional behavior in humanoid robots. Humanoid is a human appearing creature, having the character resembling that of human which is able to show behaviors similar to human at the same conditions. Even responding similarly to a specific stimulus, making decisions the same as human and in brief being as reasonable and sentimental as human beings. So the emotion dynamics have a fundamental rule in creating the humanoid to make them sentimental.

Therefore, modeling as a tool for predicting an individual's behavior and an approach to enhance the remedy processes by using control techniques can help us improve the living standards and develop the happiness level.

To this aim, former works have proposed models which are found to be in some consistencies with the real world facts. These models are able to describe some psychiatric facts such as addictive behavior, facing an unexpected event and different memory impacts. There are still important research questions which a model should be able to cope with them. As an example, former models are not considered to be able to describe changes in happiness made because of changes in short time memory. This is a known fact in psychiatric researches to which is referred as the relation between context and memory (Guenther 1988). In fact, the main purpose of this paper is to answer these questions in some familiar cases.

In this paper, we first study the earlier models of happiness (Sprott 2005); then we investigate a more refined model using fractional differential equations (FDEs) (Song et al. 2010). Inspired by those models, we introduce a new model which is a generalization of an FDE in which the order of the derivative changes in time. We believe this new model is more flexible and efficient because its varying order can be interpreted as the effect of variations in memory. Finally, we discuss memory, its effects on happiness, and the way that non-integer differential equations help represent these effects.

The rest of the paper is organized as follows: In Section 2 we discuss what we expect from a system claimed to be an appropriate model for happiness. Then we study the previous efforts to model this emotion. Section 3 is basically about non-integer derivatives and non-integer order systems. The recent model of happiness based on non-integer order calculus will be discussed there. Section 4 contains the main contribution of this paper which is mainly about extension of such dynamics to time-varying order. Simulations show that this idea helps to describe some interesting medical and psychiatric observations about human beings and the way that the brain works in facing with some internal stimulants like forgetting and remembering progress and external effects such as Electro-convulsive Therapy process.

2. Happiness Models

2.1. Validation of an Emotion-Describing Model

In dynamic models of emotion, we look at an individual as a system. The input of this system is an external event, such as good or bad news or an internal event like biochemical influence of drugs. The system reacts to the input with a response. An appropriate model should respond to a specific stimulus in a way similar to the real system. In the literature, researchers model some typical real life phenomena as mathematical time-domain functions inputs to a proposed model. If the output is suitable and meaningful, then the model performs well and is acceptable (Sprott 2005 and Song et al. 2010). In other words, a model is valid when it describes different personalities and generally accepted facts about human behavior (e.g. in this case, human ability to remember and forget, and its effect on happiness).

The validation process in the former works was just to examine the models and show that the responses to inputs are similar to expectations for human beings. This similarity is mainly in the aspects of overall emotional behavior, and the effect of memory was sometimes considered. We believe our proposed model is an improvement in emotion modeling with respect to the effect of remembering and forgetting on happiness as will be shown in Section 4.

2.2. A Brief Review of Former Models

The dynamic models proposed by Sprott for happiness and love (Sprott 2004, 2005) represent some of the first attempts at modeling the time-dependence of human emotions. After examining different dynamics, he introduced Eq. (2.1) as one's happiness model:

$$\frac{d^3R}{dt^3} + a\frac{d^2R}{dt^2} + b(1 - R^2)\frac{dR}{dt} = F(t) \tag{2.1}$$

In this equation *R(t)* is a function whose time derivative indicates a person's happiness. In other words *R(t)* is the integral of the person's happiness. *F(t)* is the external stimulus that effects the happiness. Parameters *a* and *b* are factors that can vary from person to person and give flexibility to the possible dynamics. We call them personality parameters. In this model the effects of anticipation is also considered by including a third derivative of R, which can be considered as the effects of long term memory.

Afterward (Song and Yang. 2009) used an FDE to describe happiness. They used the same model but with fractional order. It adds more flexibility to Sprott's model because its derivatives are not restricted to integers (Song et al., 2010, Song and Yang, 2009 and Tabatabaei et al., 2012).

Song's paper is valuable because different personalities were considered, and following Sprott's work, some specific inputs were introduced as models of real life events which we will describe shortly.

3. Dynamics of Happiness with Non-Integer Order

As already mentioned, the order of the derivatives can describe the effect of the past on the present. The characteristics of non-integer-order derivatives and non-integer-order systems suggest their use to describe the effect of short term memory. Song referred to the order of the system as the IFM (Impact Factor of Memory) (Song et al. 2010). This suggestion opened the doors for modeling the effects of remembering and forgetting on happiness. It was implicitly stated that changing the order is different from changing the parameters *a* and *b*. It is an inherent characteristic that describing the dynamics exhibited by the brain in response to an emotional stimulus.

3.1 Definition of Non-Integer-Order Derivative

Fractional calculus has a history of over 300 years, and its applications have attracted much attention in recent decades for modeling and control in different branches of science and engineering (Petras, 2001, Petras 2011, Goldfain 2008, Iomin, 2006, Bouafoura and Braiek, 2010 and Khader, 2011). The following equation is the Caputo definition of the fractional derivative of order q (Guenther 1988).

$$
\begin{aligned}
&{}^{C}D_t^{q} f(t) = \frac{1}{\Gamma(n-q)} \int_0^t (t-\tau)^{n-q-1} \frac{d^n}{d\tau^n} f(\tau) d\tau \\
&n-1 < q \le n
\end{aligned}
\tag{3.1}
$$

This definition shows that the value of ${}^{C}D_t^{q} f$ at t is affected by the value of $f(\tau), \tau \in [0,t]$. This is the memory factor which fractional order systems offers as an advantage over classic integer-order systems. There are other definitions for the fractional derivative, and the issue is still being debated, with an effort to unify this definitions on one hand (Ortigueira 2011,2012) and extend them on the other (Samko and Ross 1993, Odzijewicz et al., 2011 and Valerio and da Costa 2011).

3.2 Fractional-Order Systems

A fractional-order system is a set of fractional differential equations with orders less than 1, defined by the state equations

$$
\begin{cases}
D_t^{q_1} x_1 = f_1(x_1, \ldots, x_n, u, t) \\
\vdots \\
D_t^{q_n} x_n = f_n(x_1, \ldots, x_n, u, t)
\end{cases}
.
\tag{3.2}
$$

If the q_i's are equal, the system is called commensurate; otherwise, the system is incommensurate. u is the input vector (Kilbas and Marzan, 2005). This system can be written as below, in which $\bar{q}$ is the order vector and x is the state vector.

$$
D^{\bar{q}} x = F(x, u, t) \tag{3.3}
$$

As an extension of fractional derivatives, Samko suggested a time-varying order of the derivatives (Samko and Ross, 1993). Eq. (3.4) shows the definition of a variable-order derivative in the sense of Caputo (Valerio and da Costa, 2011). It is apparent that the order is not restricted to be a rational number or even a constant, and thus the word 'fractional' seems inadequate. Therefore, we use the term 'non-integer' instead.

$$
\begin{aligned}
&{}^{C}D_t^{q(t)} f(t) = \frac{1}{\Gamma(1-q(t))} \int_0^t (t-\tau)^{n-q(t)-1} \frac{d}{d\tau} f(\tau) d\tau \\
&0 < q(t) \le 1, \forall t
\end{aligned}
\tag{3.4}
$$

3.3 Non-Integer Models of Happiness

Lei Song's fractional-order model for happiness in the sense of Caputo is shown in Eq. (3.5). If q=1, the model reduces to Sprott's model in Eq. (2.1). Here, u is the input, and parameters (a,b) represent the individual's personality. Using Caputo's definition enables us to use initial conditions directly instead of their fractional derivatives (Kilbas and Marzan, 2005).

$$\begin{cases} {}^{C}D_t^{q_1}x = y \\ {}^{C}D_t^{q_2}y = z \\ {}^{C}D_t^{q_3}z = -x - by(1-x^2) - az + u \end{cases} \quad , \begin{matrix} 0 < q_i \leq 1 \\ i = 1,2,3 \end{matrix} \tag{3.5}$$

The orders are often taken as equal, and we can consider $q_1 = q_2 = q_3 = q$. In this system, y is the individual's happiness. Obviously in this model happiness can be influenced by personality parameters and q_i 's. Song supposed that the orders indicate the effect of the past on the present value of happiness and called them IFM. Happiness of persons with different IFMs are compared in Fig. 1. In this Figure, the input is an impulse function, modeling an unexpected event.

Fig. 1. Impulse response with different IFMs when (a,b)=(2,1)

Obviously adding another parameter (non-integer order) increases the flexibility of Song's model. However, that is not the main advantage of this extension. In fact this idea opens a new horizon to model memory. As we show in this paper, arbitrary order makes it possible to describe memory as a function of time, which is a useful extension.

Although Song's model was an important step in generalizing the model to better match reality, it neglected the fact that memory itself can change over time. In the next section we consider a model including this effect.

4. Happiness Dynamic with Improvement in Memory Modeling

Here, we discuss an approach to include changing memory in happiness modeling. In Song's fractional model we allow q to change in time. This modification helps explain some facts about the relation between memory and happiness mentioned in (Guenther, 1988). In the next parts we show how this extension can efficiently improve the fractional model of happiness. Improvements are considered in two parts:

a) Effect of gradual decreasing of memory.

b) Effect of loss of memory.

Each part will be discussed respectively.

4.1 Effect of Gradual Weakening of Memory

Here weakening of memory means the reduction in the effect of a past stimulus on happiness because of forgetting the stimulus, and it does not refer to memory loss as people age. Thus we use a decreasing continuous function of time instead of a constant value for q. As mentioned before, q is the fractional order of the derivative. In this case we assume memory changes in time according to a descending exponential function $q(t)$ with a decay time τ :

$$q(t) = e^{-\frac{t}{\tau}}\mathbf{1}(t - t_0) \tag{4.1}$$

1(.) is the step function, described in Eq. (4.2).

$$\mathbf{1}(z) = \begin{cases} 1, z > 0 \\ 0, z \le 0 \end{cases} \tag{4.2}$$

t_0 is the time which the stimulus is applied. In fact we have considered that the order is zero before that any stimulus appears. When the order is zero, we have no dynamic. Zero-order leads to a stationary state.

We do not assume happiness rests at zero. In psychiatric studies, for each normal person, there is a happiness level called his/her 'mood' which is the steady state value of his/her happiness (Batson et al., 1992). From the definition (4.1), we have assumed that the individual is at rest in his mood until the stimulus occurs. Then the dynamic appears, and the order gradually decreases with time. After a sufficient time, the effect of the stimulus disappears, and the individual recovers to his/her mood, which was the initial condition of the dynamic system. In the constant-order system (3.5), the origin is the equilibrium point. The happiness function of stable persons will be attracted to zero, and mood is meaningless in that case. But with time-varying order, we can model mood as an initial (and also final) state.

Fig. 2 shows the response of Song's model (constant order) to an unexpected stimulus (impulse response) where q=0.97, (a,b)=(2,1) with initial condition $\begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix} = \begin{bmatrix} 0 & 0.1 & 0 \end{bmatrix}$. As mentioned before, the equilibrium at the origin in Eq. (3.5) makes the happiness function converge to zero. But for the variable-order system this is not true. An impulse response for the variable-order models with different rates of memory weakening does not cause a convergence to zero because the stimulus usually has a residual effect. Furthermore, the smooth increase after the initial undershoot as seen in the varying-memory model is more plausible. Note that a greater τ implies a faster weakening of memory and a faster convergence of happiness to the ultimate value, which agrees with (Gurnther, 1988). These results can be seen in Fig. 3.

Fig. 2. Impulse response for constant order. Stimulus is applied at $t_i = 3$

q=0.97, and the rest value (mood) before getting the stimulus is 0.1 (a,b)=(2,1)

Fig. 3. Impulse response to time-varying-order system, Stimulus is applied at ,$t_i = 3$ mood value is 0.1, and

$q(t) = e^{-\frac{t}{\tau}}$ for different values of τ , (a,b)=(2,1)

4.2 Effect of Memory Loss on Happiness

In treatment of severe depression and bipolar disorder, one of the most effective remedies is electroconvulsive therapy (ECT) which is reported to be effective in many cases (Rich and Black, 1995). In this method, electrical current is given to brain tissue, and a loss in short-term memory helps the patient feel much better and decreases depression symptoms. It is expected that the memory loss helps the patient restore a stable condition. Using a time-varying function representing memory makes it possible to simulate the experimental such observations. In this process, memory loss occurs during a short time interval.

Now suppose a person has unstable emotional behavior. As an example, Fig. 4 shows happiness of such a person with personality parameters (0.1,1) and a constant IFM q=0.97. Without treatment, the happiness function represents an undesired growth that could lead to insanity. The treatment is applied as ECT. The memory-time function can be some function like Eq. (4.3) which is a sigmoid as shown in Fig. 5.

$$q(t) = 0.97 - \frac{1}{1+\exp(-8.2(x-8.8))} \tag{4.3}$$

The shock is applied at t=5. We can see in Fig. 6 that the happiness time function will rest after that, and thus the mathematical model has provided a reasonable representation of reality for this medical process.

There is another observation that we can model with this approach. In Fig. 7 after the loss of memory at t=8.5, an impulse is fed to the system at t=12. However, as shown in Fig. 8, the system responds only very slightly. So we can see instantaneous memory loss not only removes oscillations in happiness but also makes the patient more resistant to the next stimulus. Such behavior is definitely consistent with the experience of such treatments (Feinstein et al., 2010 and Moorhead et al., 2007).

Fig. 4. Unstable emotional behaviour in absence of treatment

Fig. 5. Memory-Time function in ECT process

Fig. 6. Happiness-time function after applying ECT

Fig. 7. Two consecutive impulses applied to the system as a model for unexpected events. The first is applied before the ECT process and the second is applied after that.

Fig. 8. Response to stimulus for a patient under ECT process of treatment

The effect of the second impulse is magnified in the right-hand graph

Conclusion

Different models for happiness were reviewed briefly, and an improvement in the memory part of the model was suggested. We used the order of the derivatives in a fractional differential equation model as a tool to show memory impact. We found that a constant-order of derivative is insufficient to explain experimental observations. Considering the derivative order as a function of time gives the model more flexibility and generality and helps to describe many psychiatric observations about the relation between memory and happiness. As was clear in simulations, forgetting, loss of memory due to severe oscillations, and ECT are real effects which can be described using the time-varying order of the dynamic models.

In future works, clinical researches can be done to test the proposed model. Also, parameter estimation and system identification methods can be utilized for justifying the model and obtaining every specific individual's characteristics parameters. In addition, statistical researches can be done to calculate the pair of (a,b) for normal and abnormal persons. The proposed model also can be used to open the doors for control method as efficient approaches for the treatment of psychiatric disorders.

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Figures:

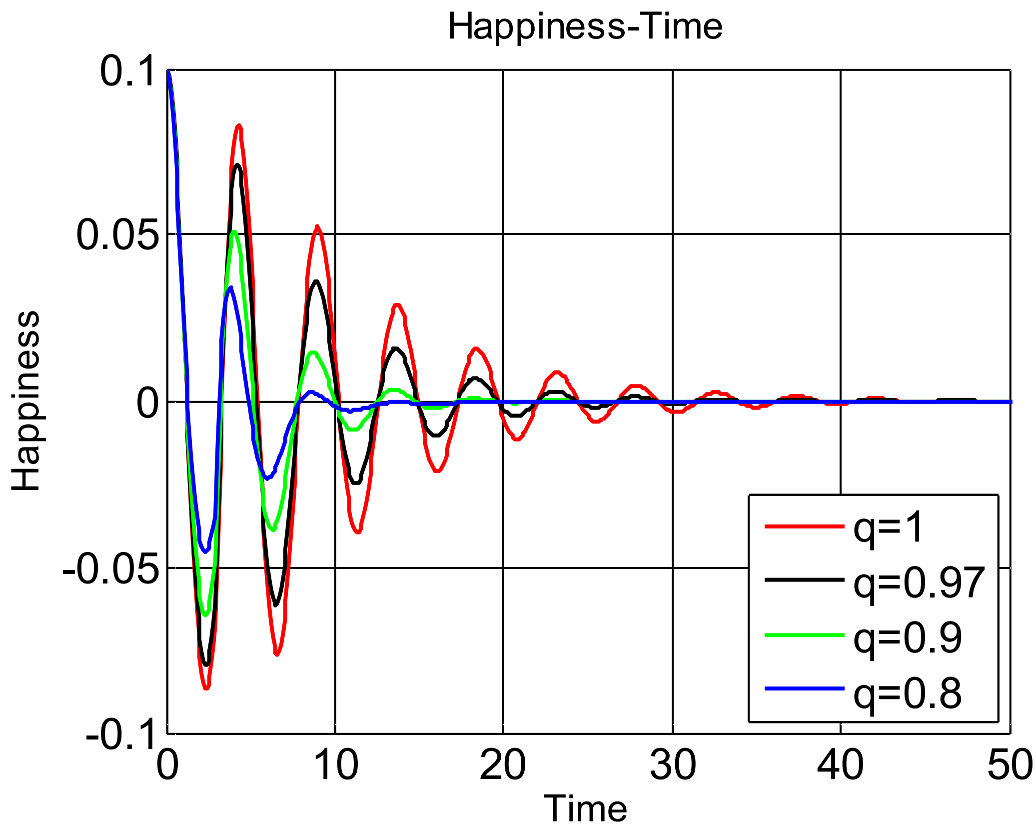
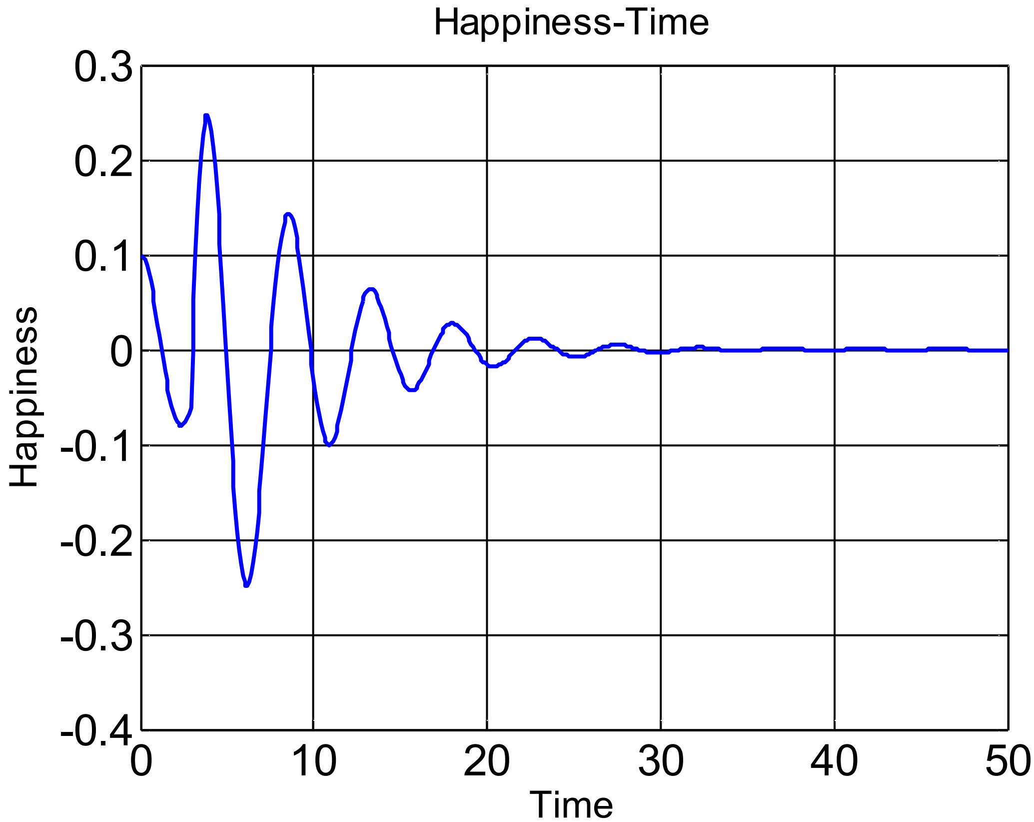


Fig. 1. Impulse response with different IFMs when $(a,b)=(2,1)$



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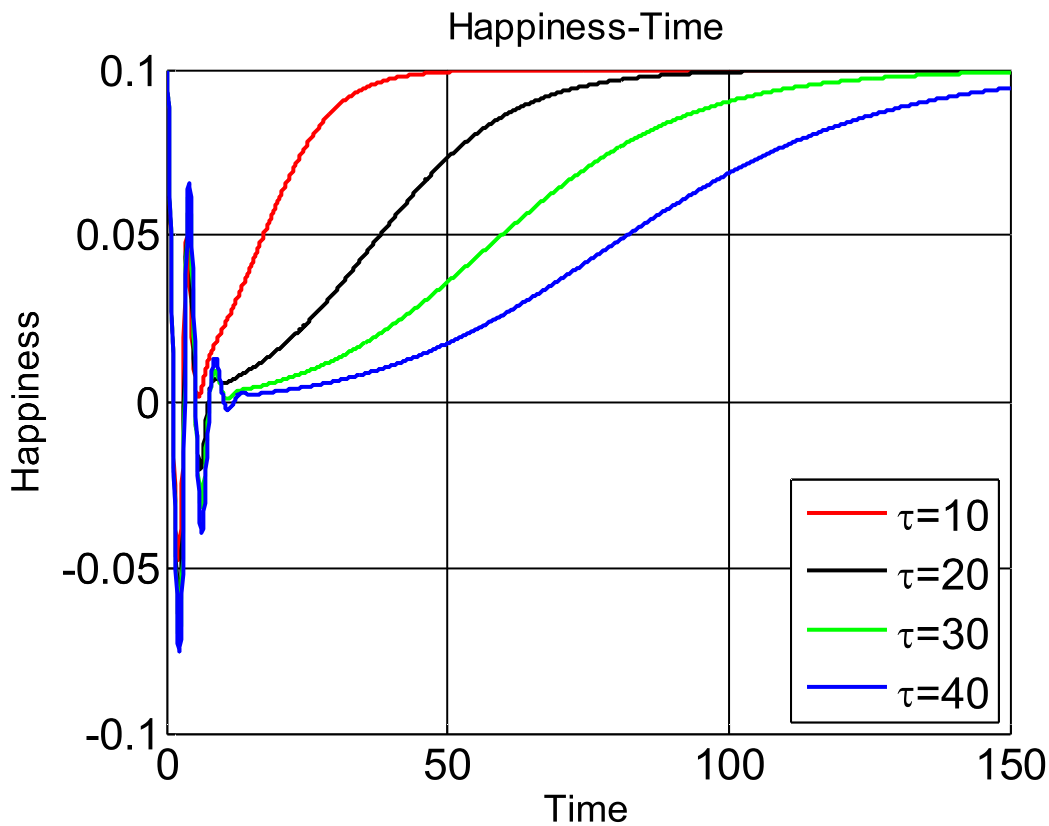

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Fig. 4. Unstable emotional behaviour in absence of treatment

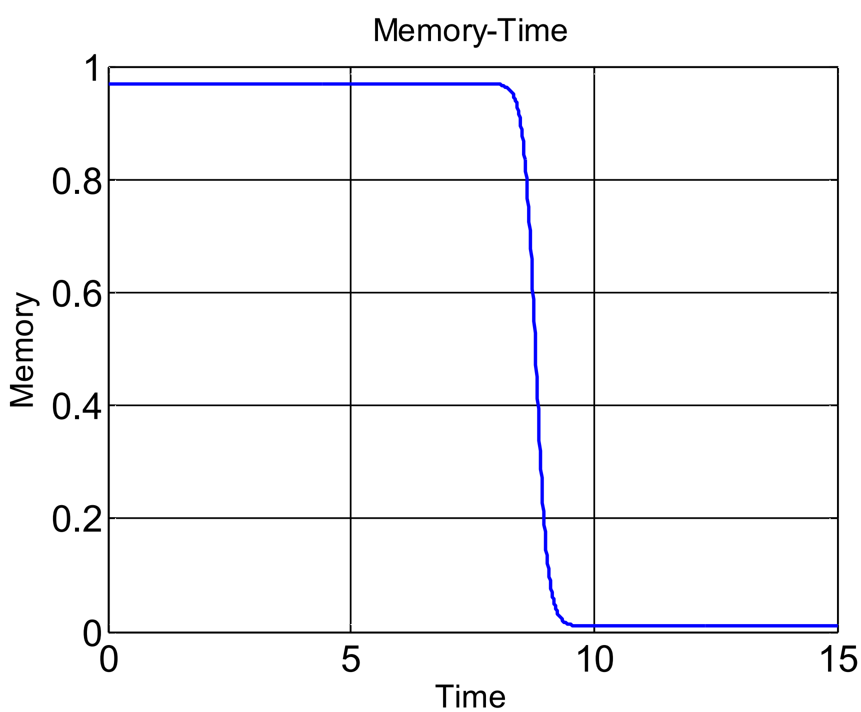


Fig. 5. Memory-Time function in ECT process

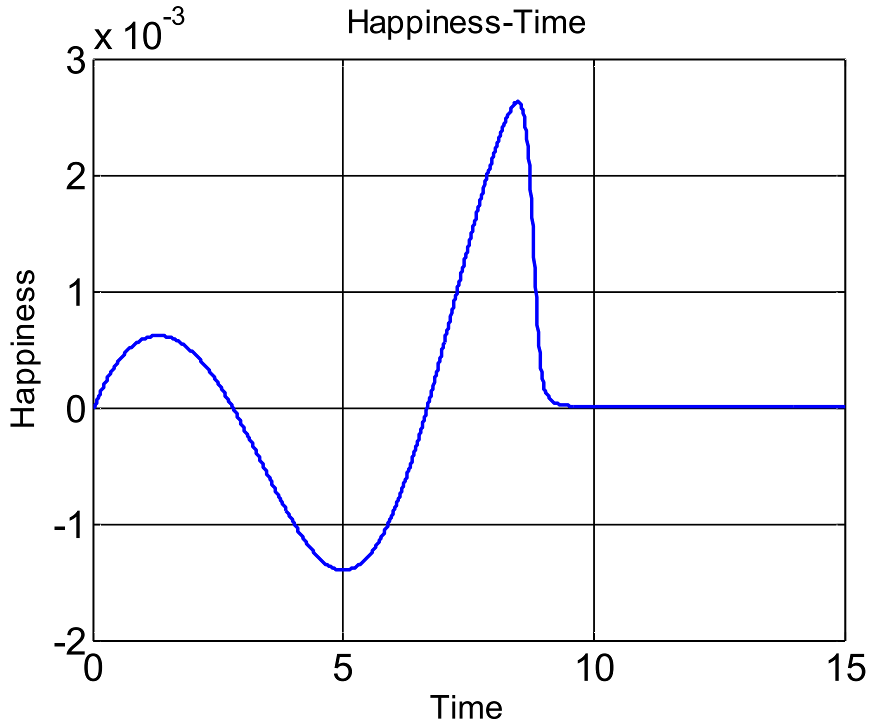
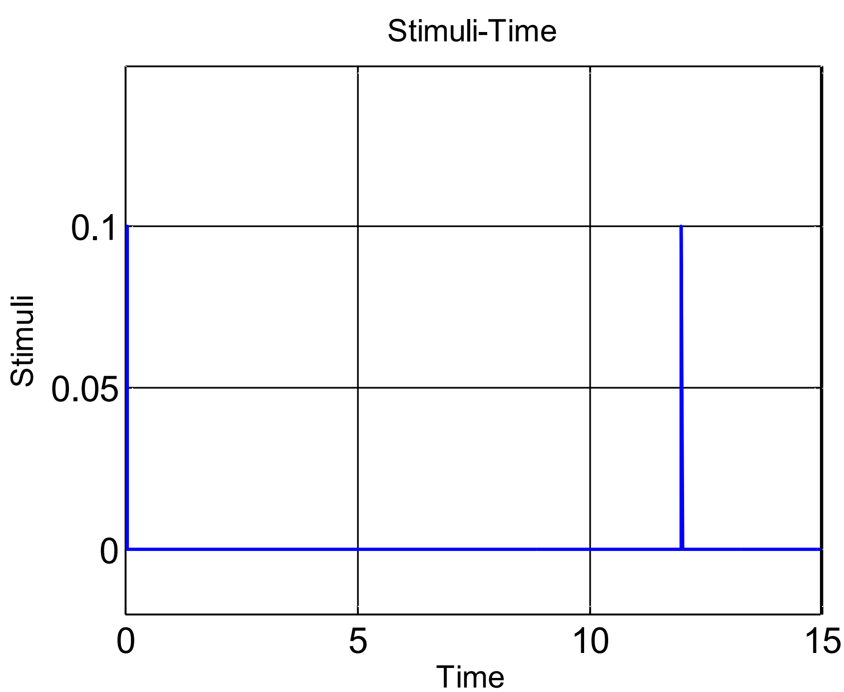


Fig. 6. Happiness-time function after applying ECT



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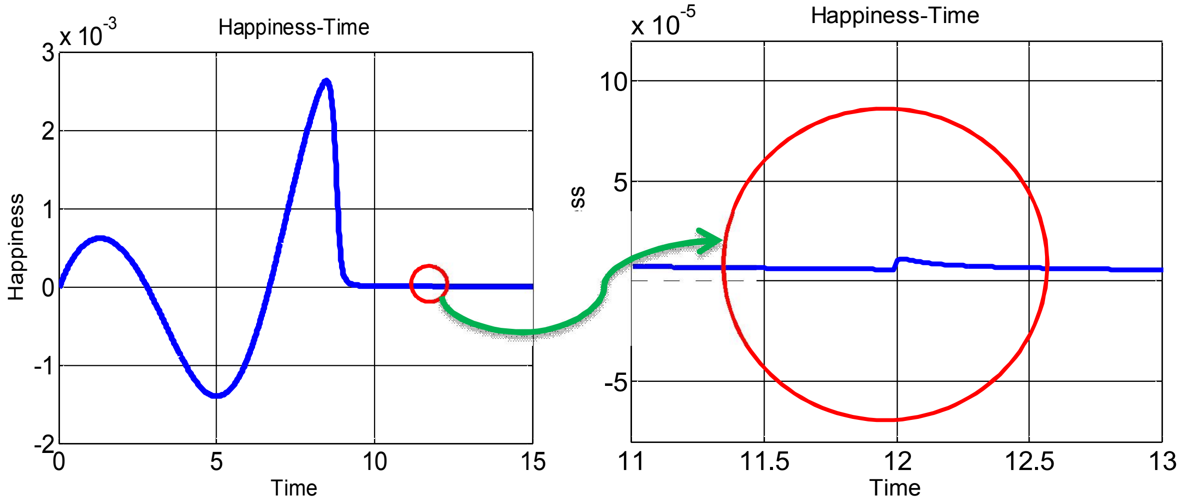

Fig. 8. Response to stimulus for a patient under ECT process of treatment

The effect of the second impulse is magnified in the right-hand graph