

Chaos and Self-Organization in Spatiotemporal Models of Ecology

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Nature is replete with spatial patterns that exhibit both organization and randomness, and these patterns evolve in a complicated and often chaotic manner. Traditionally, scientists have tried to explain such spatiotemporal behavior in terms of complex causes. In the last few decades, it has become widely accepted that simple models with a small number of variables can exhibit chaos, producing apparently random and unpredictable temporal fluctuations with sensitive dependence on initial conditions. It is now becoming evident that spatial complexity may similarly arise from very simple rules that do not require that the underlying processes be specified in detail. The heterogeneous patterns are generated internally by the homogeneous model rather than externally by the environment. Under a wide range of conditions, the rules cause the system to organize spontaneously to a self-organized-critical (SOC) state in which there are no characteristic spatial or temporal scales. The resulting patterns are fractal, and their temporal fluctuations exhibit $1/f$ power laws.

This work was motivated by a large database of forest landscapes in which tree types and density were recorded at 1-mile (1.6-km) intervals on a square grid throughout much of the Western United States prior to Euro-American settlement. The resulting spatial patterns are fractal with a dimension of about 1.6. This work describes simple cellular automata models that replicate the characteristics of the observed landscape and allow studies of the temporal evolution of these patterns. Two classes of models have been developed, one stochastic and the other purely deterministic. These models are two-dimensional and typically have periodic boundary conditions. Initial conditions can be either random or highly ordered. The properties of the observed patterns and behavior are not sensitive to the boundary conditions or initial conditions.

In the stochastic models, each cell has two or more states corresponding to the density of trees. At random times, a cell dies and is replaced by the contents of a cell chosen randomly from some circular neighborhood of radius r . The parameter r is adjusted to reproduce the morphological features of the observed landscape. When r is small, large clusters of identical cells tend to form. When r is large, there is very little organization of the landscape, and the patterns remain (or become) highly random. Intermediate values of r corresponding to about 28 neighbors give good agreement with observations as determined by several different metrics. The patterns so produced are fractal with a dimension similar to the observed value and exhibit temporal fluctuations with a $1/f^\alpha$ power spectrum with $\alpha \approx 1.6$

In the deterministic models, cells are updated synchronously from a highly ordered initial condition using deterministic rules that involve only the total number of neighbors in a given state at a given distance from the cell. Highly complex patterns are produced that combine randomness with self-organization. A genetic algorithm is used to search the vast rule space for rules that produce realistic patterns. Working backwards from the rules allows one to deduce the strength of the interaction between cells as a function of their spatial separation, thus providing information about the underlying biological processes.

Although the work described here was inspired by an application to landscape ecology, the patterns produced by the models resemble a variety of natural forms including clouds, turbulent fluids, topographical features of the Earth's surface, and migratory patterns of human and animal populations. Thus the methods ought to have wide applicability to many scientific disciplines.